Hot and dense QCD in the limit of large number of colors and flavors

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Alho - Järvinen - Kajantie - Kiritsis –Tuominen, 1210.4516 , new work

Järvinen-Kiritsis 1112.1261

1. Hot and dense QCD, $N_c \sim N_f$ large

$$e^{p(T,\mu;m_q)\frac{V}{T}} =$$

$$\int \mathcal{D}A\mathcal{D}q \; e^{-\int^{1/T} d^4x\left[\frac{1}{g^2}F^2+\bar{q}(\partial+A)q+m\bar{q}q+\mu q^\dagger q\right]}$$

- Chiral effective theories $\Phi_{ij} = \langle q_i\bar{q}_j\rangle$

- Lattice Monte Carlo (vast effort!) $V \to \infty,\ a \to 0,\ m_q \to 0$

  $\text{or } m_u, m_d, m_s, m_c, ..\ \text{physical}$

- Pert theory: $p/T^4 = 1 + g^2 + g^3 + g^4\log g + g^4 + g^5 + g^6\log g + g^6 + ..$

- Holography: >5dim gravity dual, $N_c,\ N_c g^2 \gg 1$

Make this an effective theory valid for all g^2 !!

What does one expect?

Take first $N_f \sim N_c$

At large T,μ quark-gluon plasma with chiral symmetry if $m_q=0$

in between a chiral transition at $T_\chi(\mu)$

At small T,μ a quark-gluon system with chiral symmetry broken

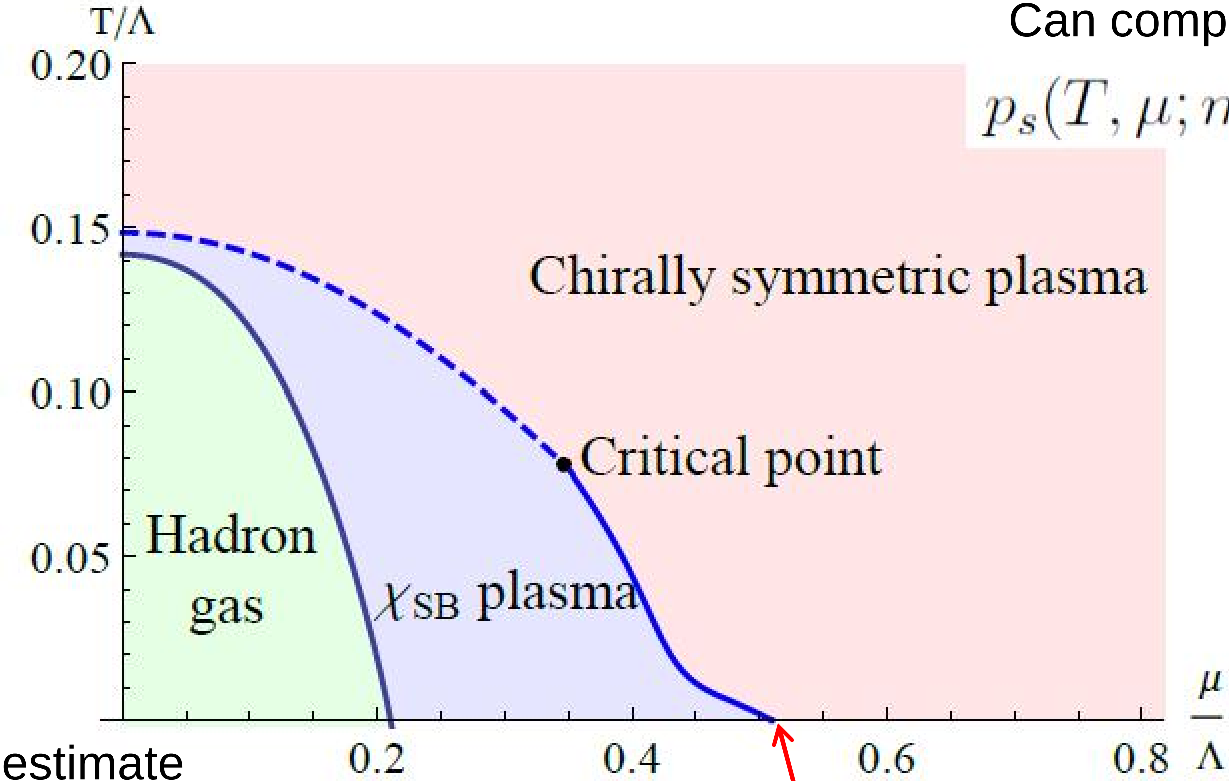
A deconfinement transition to a hadronic phase

Chiral transition has an order parameter: condensate

No order parameter, symmetry, associated with confinement!

This is what one gets:

On coexistence line T, μ, p(T,μ) equal



Can compute

$p_s(T, \mu; m_q = 0)$

Can only estimate

$p_h \sim 0$

Can compute $p_b(T, \mu; m_q = 0)$

T=0 quantum phase transition!

Reminder: coupling and mass run, are scheme dependent:

$$\lambda(\mu) \equiv \frac{N_c g^2(\mu)}{8\pi^2} \qquad x_f = \frac{N_f}{N_c}$$

We include **all** $\lambda(\mu)$

$$\mu \frac{d\lambda}{d\mu} = \beta(\lambda) = -\underbrace{\frac{11-2x_f}{3}}_{b_0}\lambda^2 - \underbrace{\frac{34-13x_f}{6}}_{b_1}\lambda^3 - ..$$

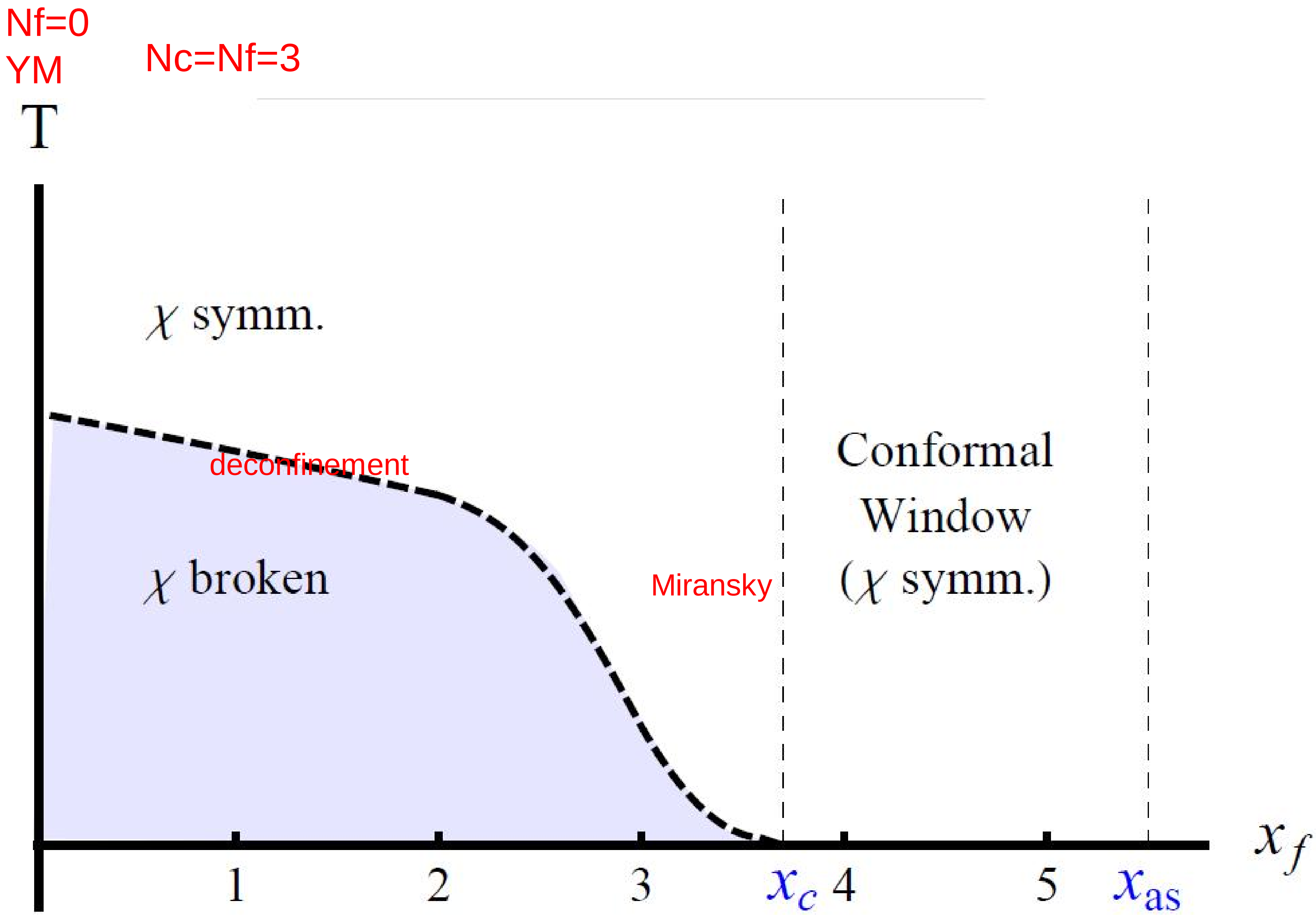
$$\mu \frac{d\log m}{d\mu} = \gamma_m(\lambda) = -\underbrace{\frac{3}{2}}_{\gamma_1}\lambda - ..$$

For $x_f > 5.5$ loose asymptotic freedom!

$$\log\mu = \int_{\lambda_0}^{\lambda(\mu)} \frac{d\lambda}{\beta(\lambda)}$$

$$m(\mu) = m_0(\log\mu)^{-\frac{9}{2(11-2x_f)}}$$

Phase diagram for larger N_f: 1210.4516



Now put here a perpendicular μ axis :

2. Gravity dual

4dim: $$\int \mathcal{D}A\mathcal{D}q\ e^{-\int^{1/T} d^4x \left[\frac{1}{g^2}F^2+\bar{q}(\partial+A)q+m\bar{q}q+\mu q^\dagger q\right]}$$

- 5dim gravity with AdS_5 boundary at z=0

$$ds^2 = b^2(z)\left[-f(z)dt^2 + d\mathbf{x}^2 + \frac{dz^2}{f(z)}\right]$$

$$b(z) \to \frac{\mathcal{L}}{z}, \quad f(0) = 1, \quad f(z_h) = 0, \quad 4\pi T = -\dot{f}(z_h)$$

temperature

entropy

$$s = \frac{b^3(z_h)}{4G_5}$$

$$\frac{N_c g^2(\mu)}{8\pi^2} \to \lambda(z)$$ dilaton

$$m \to \tau(z)$$ tachyon

$$\mu \to A_0(z)$$ potential

$$q^\dagger q = \bar{q}\gamma^0 q$$

$$S = \frac{1}{16\pi G_5} \int d^5x \, \mathcal{L},$$

$$\mathcal{L} = \sqrt{-g}\Big[R + \left[-\tfrac{4}{3}\, g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V_g(\lambda)\right]$$

Usual scalar action for dilaton

$$- V_f(\lambda,\tau)\sqrt{-\det\left[g_{ab} + \kappa(\lambda)\partial_a\tau\partial_b\tau + \omega(\lambda,\tau)F_{ab}\right]}\Big|$$

$$\sqrt{1 + ..\, \dot{\tau}^2 + ..\, \dot{A}_0^2}$$

DBI action for tachyon and potential

A_0 is cyclic $\frac{\partial L_f}{\partial \dot{A}_0} = \tilde{n}$

Tachyon action is particularly interesting; **string theory** enters

When string tension grows, strings become points

Closed string theory, containing gravity, becomes supergravity

Open string theory, containing gauge theory, goes to DBI:

Dirac-Born-Infeld

$$\mathcal{L}_{\rm DBI} = -\frac{1}{\ell^4}\sqrt{-\det(g_{\mu\nu} + D_\mu\tau D_\nu\tau + \ell^2 F_{\mu\nu})}$$

$$\overline{\tau\equiv 0}\; -\frac{1}{\ell^4}\sqrt{1-\ell^4(E^2-B^2)-\ell^8(E\cdot B)^2} = \tfrac{1}{2}\,(E^2-B^2)+\tfrac{1}{2}\,\ell^4(E\cdot B)^2+\ldots$$

$$\ell^2 = 1/T = 2\pi\alpha'$$

Physics in these functions:

Z<<1 UV asymptotic freedom, small λ

Z>>1 IR confinement

$$\lambda(z) = \frac{1}{b_0 \log(1/\Lambda z)} + \cdots$$

grows towards IR

$$\lambda_h = \lambda(z_h)$$

parameter!

$$\tau(z) = m\,(\log\frac{1}{\Lambda z})^{-\frac{3}{2b_0}}\, z + \langle \bar{q}q\rangle\,(\log\frac{1}{\Lambda z})^{\frac{3}{2b_0}}\, z^3 + ..$$

$$\tau_h = \tau(z_h)$$

fixes m_q=0

Solve $\dot{A}_0$ from $\frac{\partial L_f}{\partial \dot{A}_0} = \tilde{n}$

$$A_0(z) = \mu + \int_0^z dz\,\dot{A}_0(z) = \mu - nz^2 + ..$$

chemical potential
number density

$$n = \frac{\tilde{n}}{4\pi} s = \tilde{n}\frac{b_h^3}{16\pi G_5}$$

The five functions $b(z)$, $f(z)$, $\lambda(z)$, $\tau(z)$, $A_0(z)$ are obtained as solutions of Einstein's equations shooting from the horizon

Two types of tachyon solutions:
$\tau = 0$: chirally symmetric, no condensate
τ nonzero: chirality broken, nonzero condensate

As in lattice Monte Carlo, particularly time consuming is fixing $m_q = 0$. One has to choose $\tau(z_h)$ properly:

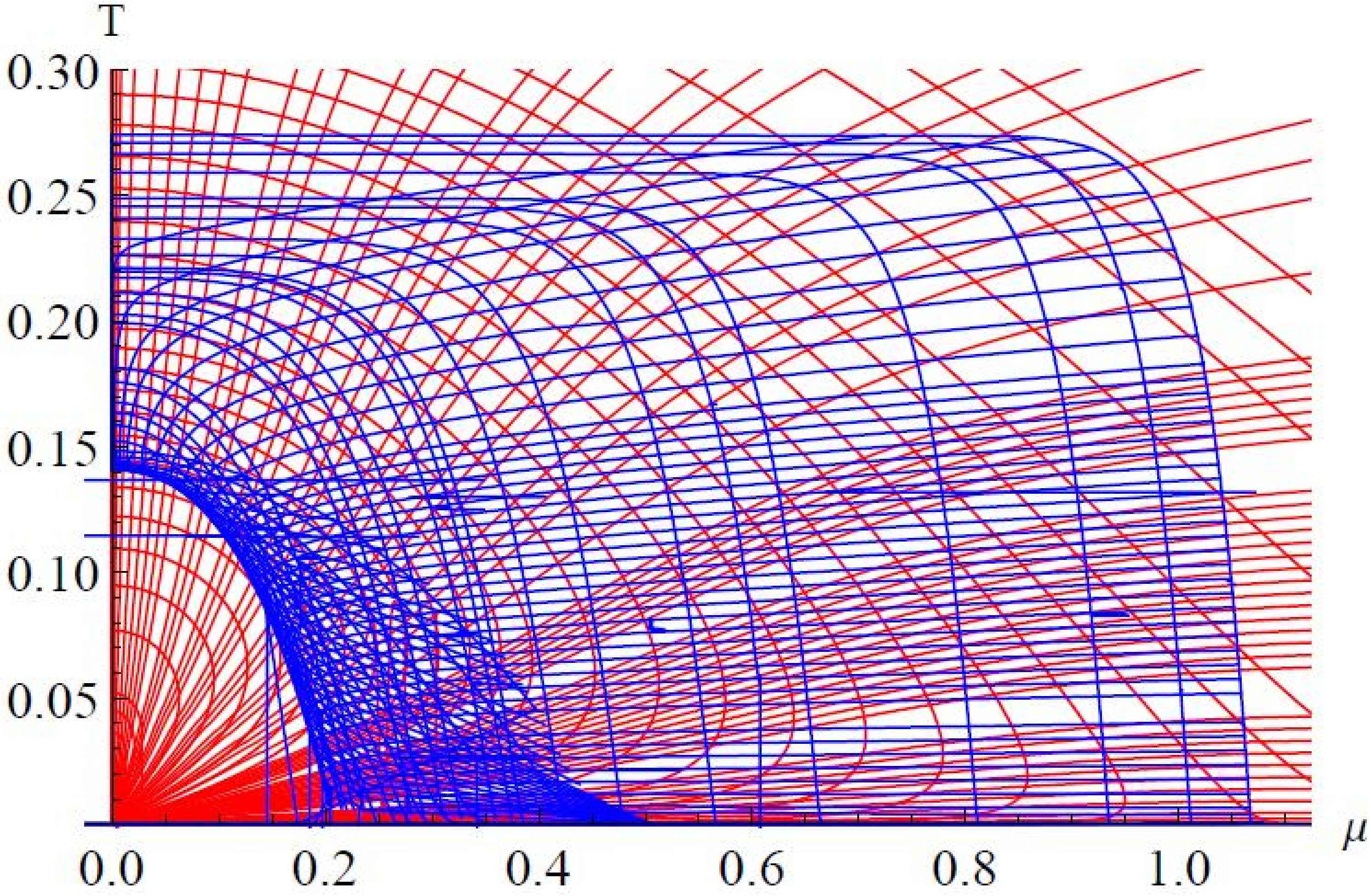
Two parameters: $\lambda_h, \ \tilde{n}$

$$T(\lambda_h, \tilde{n}), \quad \mu(\lambda_h, \tilde{n})$$

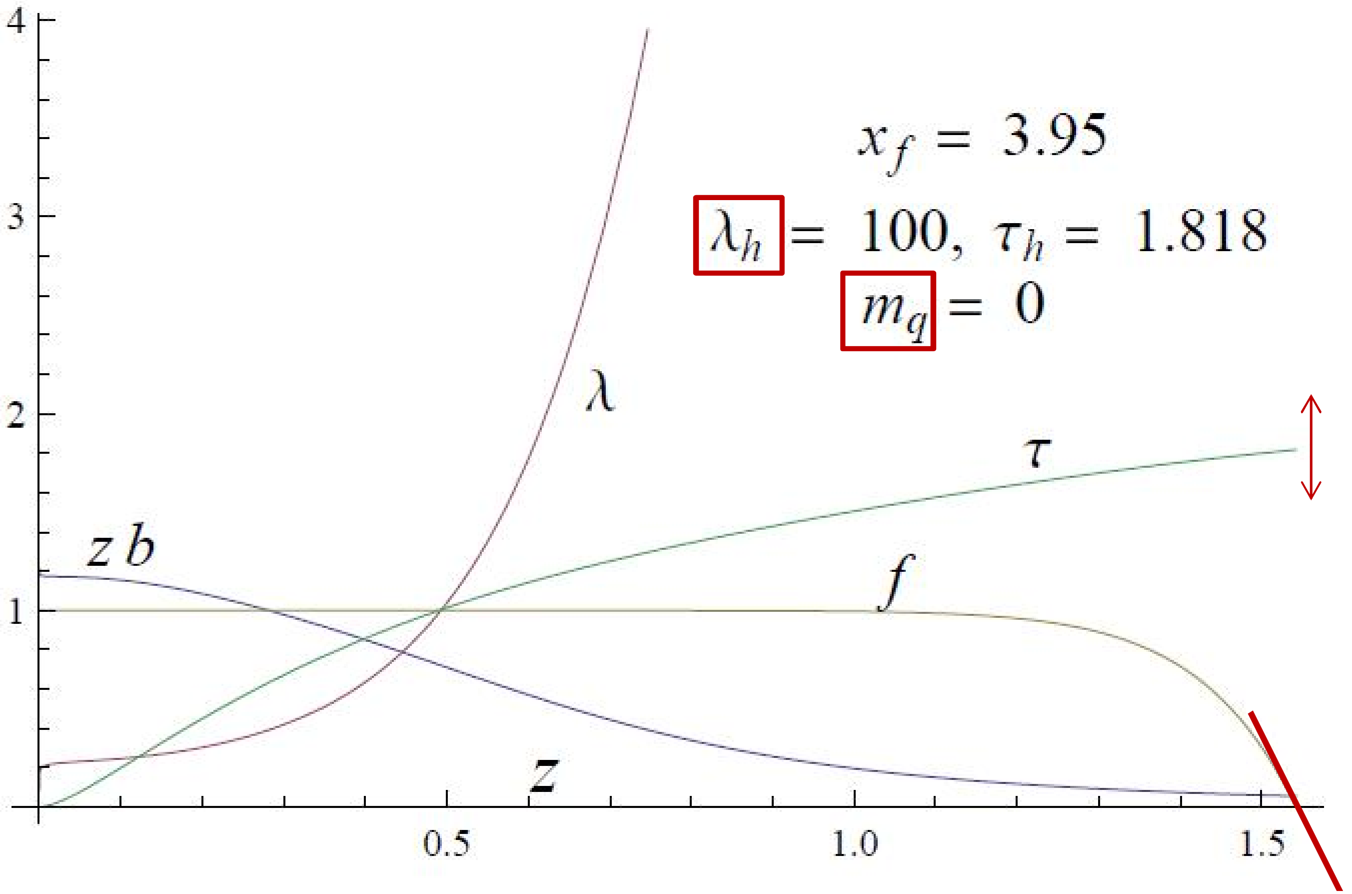
$$dp = s dT + n d\mu \Rightarrow p_s(\lambda_h, \tilde{n}), \, p_b(\lambda_h, \tilde{n})$$

3. Results

$$p(\lambda_h, \tilde{n}) \Rightarrow p(T, \mu)$$



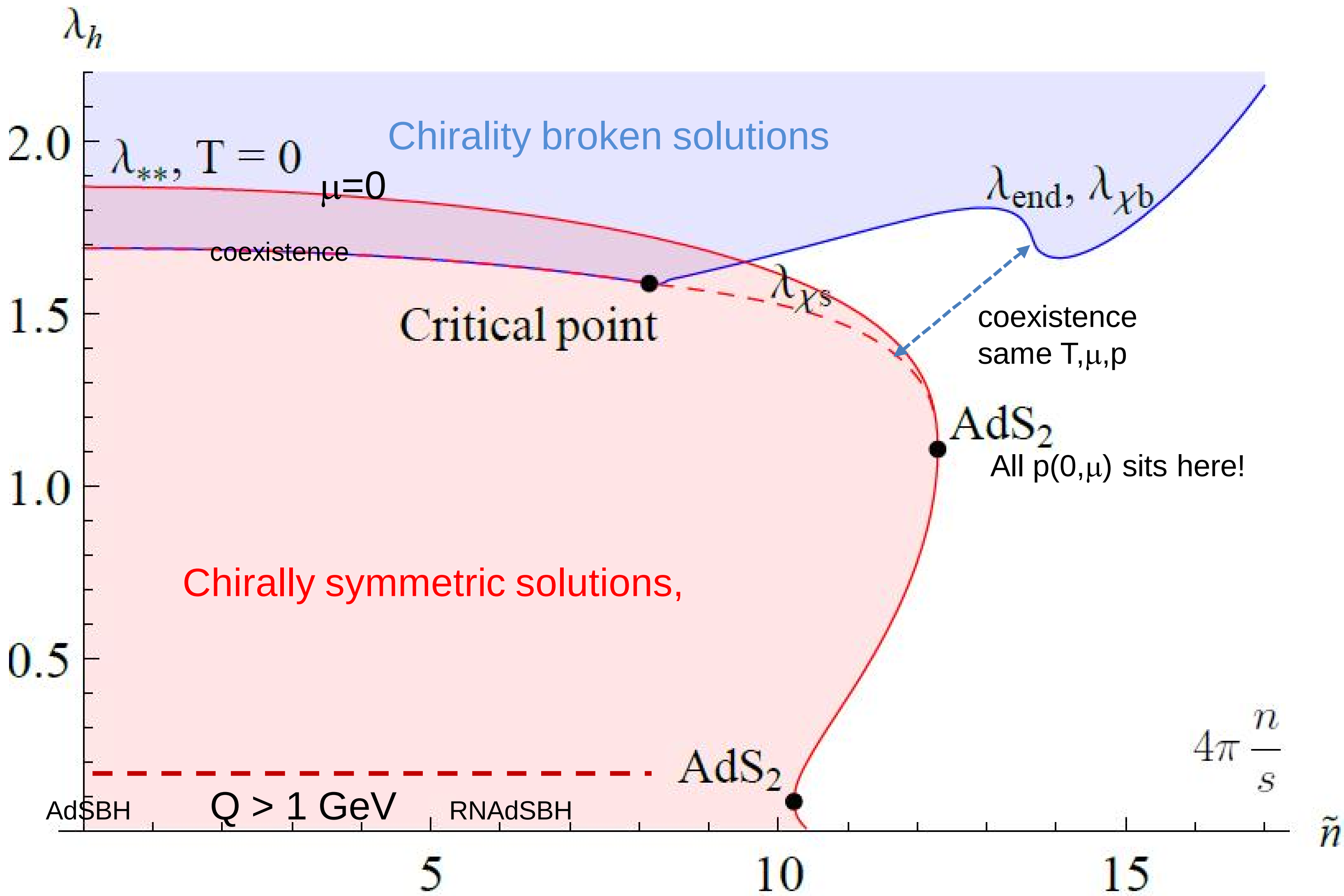
Typical bulk field configuration:



Tune $\tau(z_h)$ so that near $z=0$ $\tau(z) \sim z^3$

$$4\pi T = -f'(z_h)$$

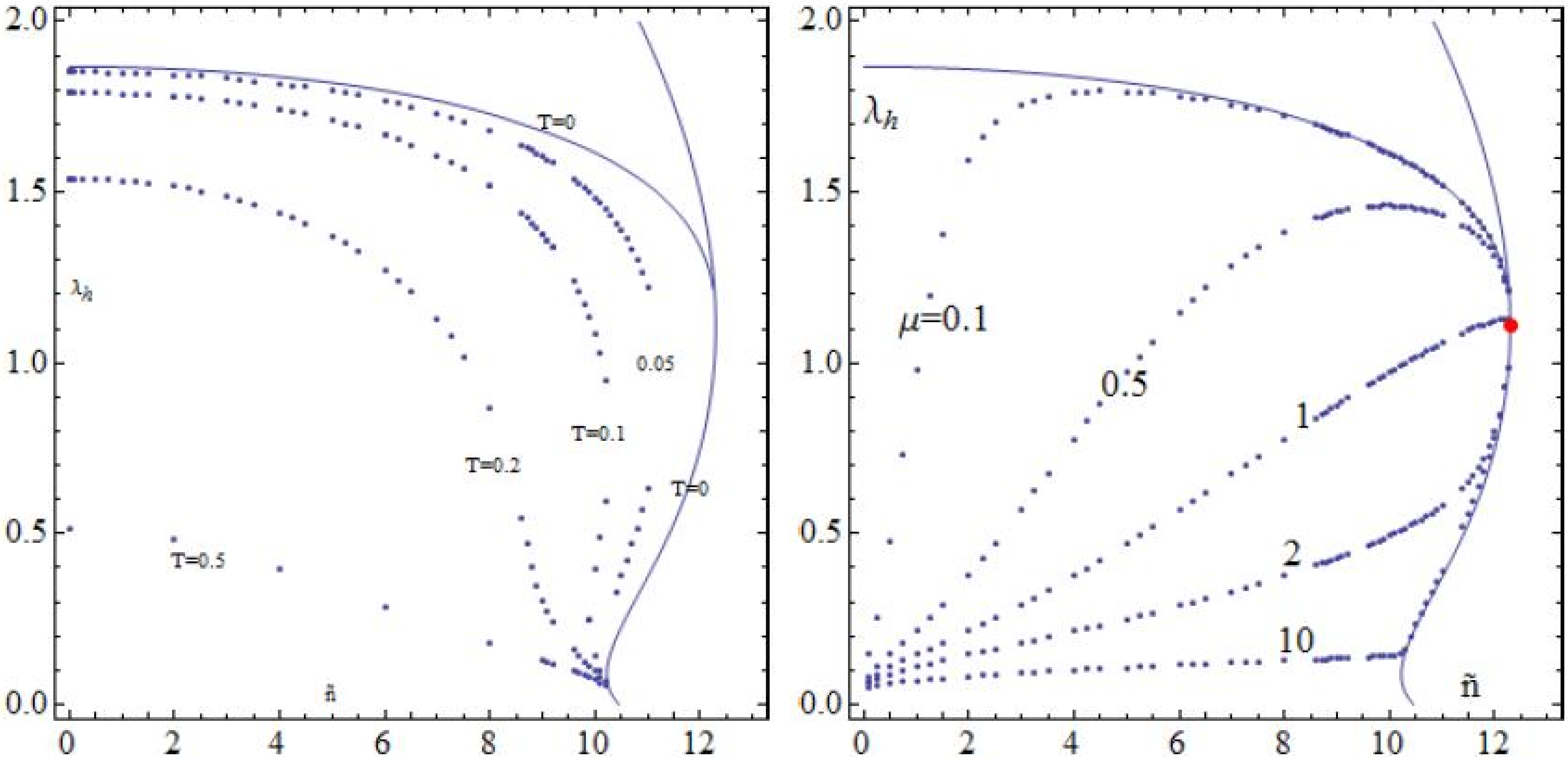
Physical region



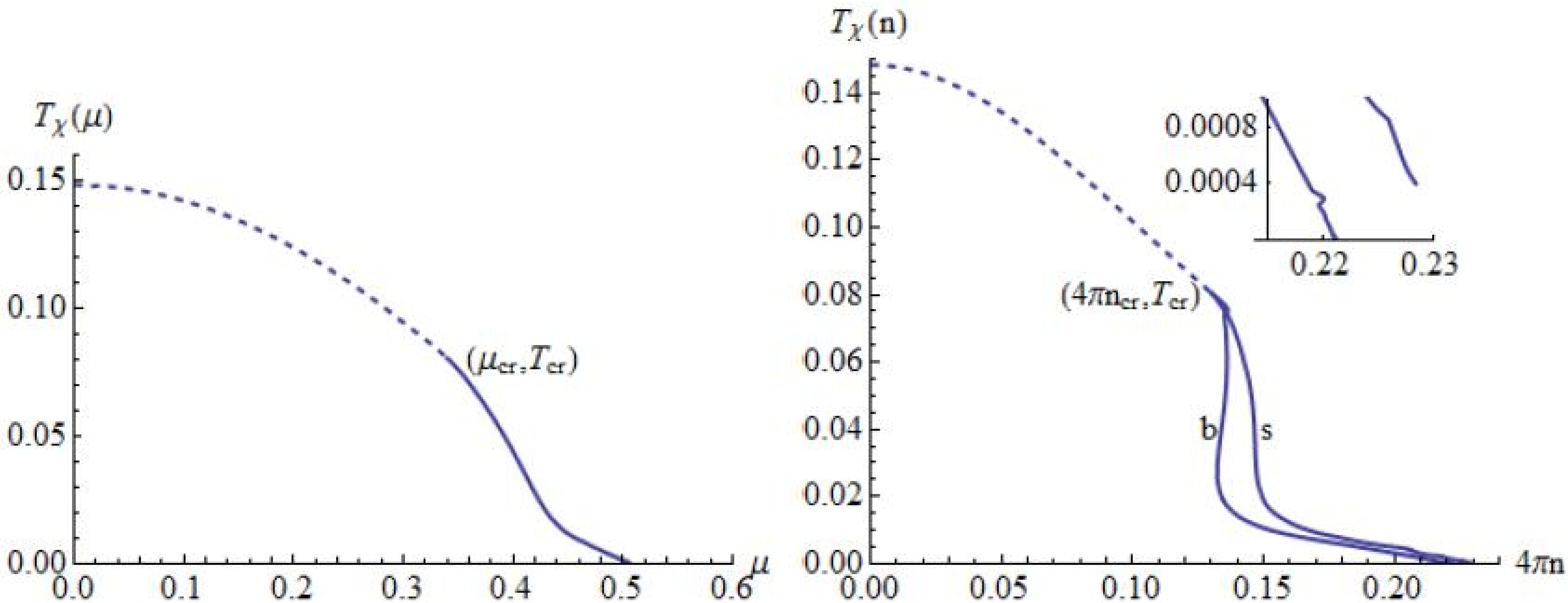
~ charged Reissner-Nordström BHs

Constant T,μ on $\lambda_h,\ \tilde{n}$ plane

(chirally symmetric sols)

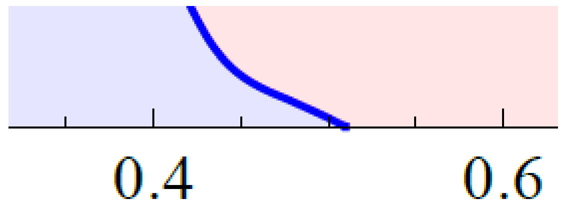


$$T_\chi(\mu) \Rightarrow T_\chi(n)$$



Entropy finite at T=0!!

T=0



chirality broken state
at λ_h=infty

T=0
μ = 0.506
p = 0.0016

symmetric state at AdS_2 point

$$T = 0 \Rightarrow f'(z_h) = 0 \qquad ds^2 = -z^2 dt^2 + \frac{dz^2}{z^2} + d\bar{x}^2 \quad \mathrm{AdS}_2 \times R^3$$

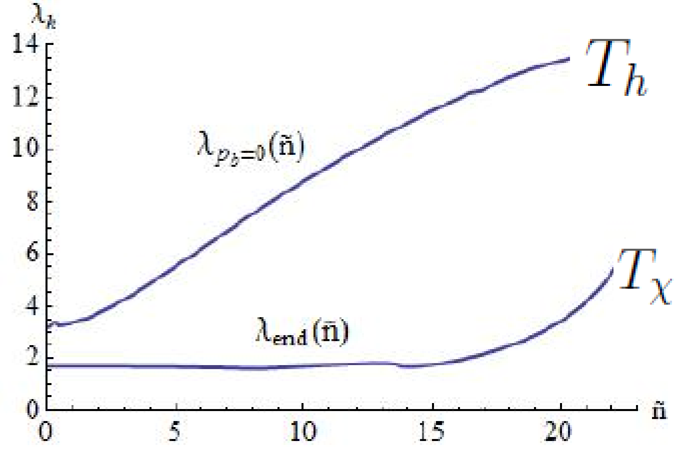
Actually the expansion for getting all T=0 solutions is more complicated:

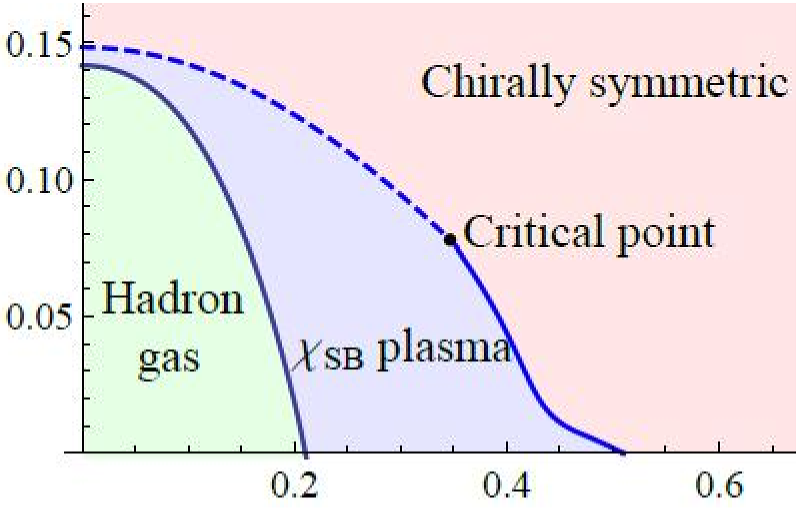
$$f(z) = z^2 + z^{2+\mathrm{noninteger}} + \ldots$$

why is entropy finite? No baryon operator, nuclear matter..
No qq operator, color-flavor locking, etc

4. Deconfinement

The condition $p_b(\lambda_h, \tilde{n}) = p_{\text{hadr}} \approx 0$





Gives us T_h

$$2N_c^2 + \frac{7}{8}4N_fN_c \quad \text{vs} \quad N_f^2 \quad \text{dofs}$$

5. Conclusions

- This model is an effective theory connecting strong coupling holography to the weak coupling region

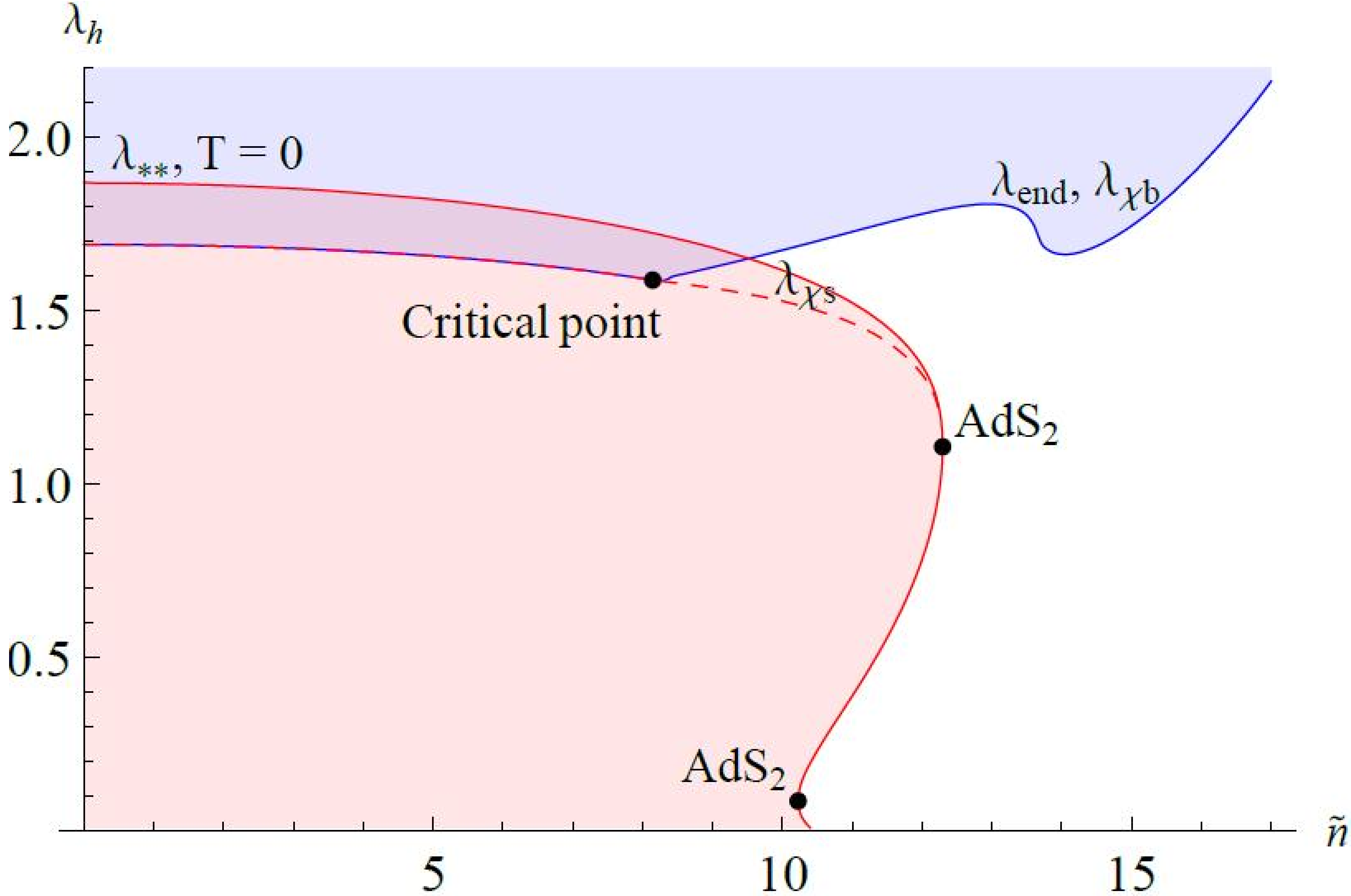
- The subtle interplay between confinement/chiral symmetry and charged black holes with and without tachyons produces a coexistence line with a critical point. Quite impressive

- The potentials $V_g(\lambda)$, $V_f(\lambda)$ are constrained but not completely: predictive power is limited. Offers a **framework, alternatives**

- Not a cheap simple way to solve QCD!

- Much to do: more and better numbers, other potentials, larger Nf, more on T=0, other BSM theories (technicolor!), correlators, magnetic fields, theta vacua, baryons....

Overflow



λ_h
2.0
1.5
1.0
0.5
λ_{**}, T = 0
$\lambda_{\text{end}}, \lambda_{\chi b}$
λ_{χ^s}
Critical point
AdS$_2$
AdS$_2$
5
10
15
$\tilde{n}$

Here is the gravity action:

$$S = \frac{1}{16\pi G_5}\int d^5x\,\sqrt{-g}\,\mathcal{L} = S[g_{\mu\nu},\lambda,\tau]$$

$$\mathcal{L} = R + \left[-\tfrac{4}{3}\,g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V_g(\lambda)\right]$$

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$$-\,x_f V_f(\lambda) e^{-\mu^2\tau^2}\sqrt{1 + g^{zz}(1+\lambda(z))^{-4/3}\tau'(z)^2}$$

Matched to β function near λ=0

$$V_g(\lambda) = \frac{12}{\mathcal{L}_0^2}\left[1 + \frac{88\lambda}{27} + \frac{4619\lambda^2}{729}\,\frac{\sqrt{1+\ln(1+\lambda)}}{(1+\lambda)^{2/3}}\right]$$

remnant of

$$e^{-\phi}R + .. \rightarrow R + ..$$

confinement at large λ

$$\mathrm{EOM}:\ \frac{\delta S}{\delta g^{\mu\nu}} = 0,\qquad \frac{\delta S}{\delta\lambda} = 0,\qquad \frac{\delta S}{\delta\tau} = 0$$

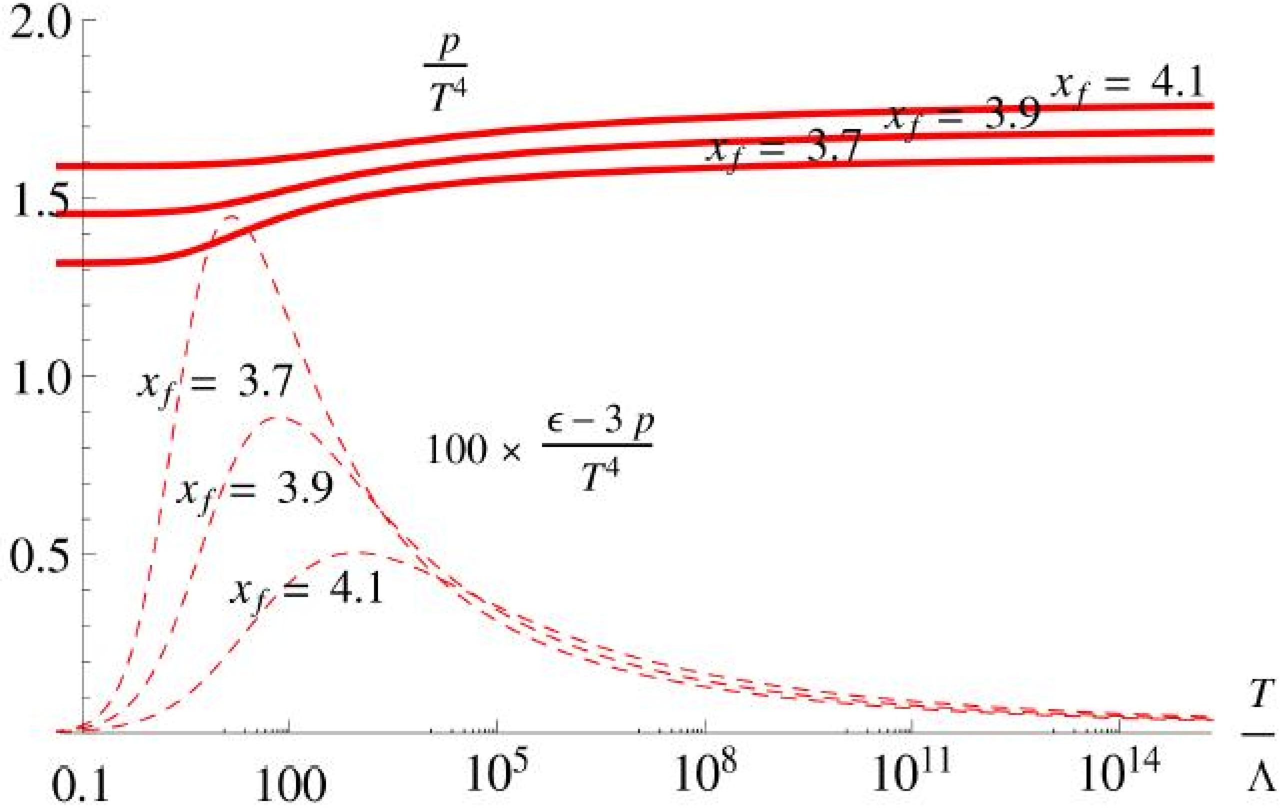
Simple thermal argument for x_c=4:

At low T dominant, easiest-to-excite modes are N_f^2 Goldstone bosons (usual chiral symmetry breaking)

At some Tc these melt into $2N_c^2 + \frac{7}{8}4N_fN_c$ gluons and quarks

Assume latent heat is ~ difference between these numbers. One enters the conformal regime when latent heat vanishes.

It vanishes at $N_f = 4\ N_c$



Normalised to
SB at T=infty

’t Hooft, Witten, Veneziano limits

’t Hooft limit: λ fixed, N_c large

Chiral anomaly:

$$\partial_\mu(\bar{\psi}\gamma^\mu\gamma_5\psi) \sim g^2 N_f\, \tilde{F}_{\mu\nu}F^{\mu\nu} = \frac{\lambda}{N_c} N_f\, \tilde{F}_{\mu\nu}F^{\mu\nu}$$

Witten: at fixed λ, N_f large N_c switches of the anomaly

Veneziano: keep N_f/N_c fixed at large N_c

Gauge/gravity duality

$-4 \qquad \Delta_- \qquad 4-\Delta_-$

$$\langle \exp\left[i \int d^4x \, \phi_0(x) \, \mathcal{O}(x)\right] \rangle$$

$$\exp\left[i \int d^4x \int dz \sqrt{-g} \, \mathcal{L}_{\rm grav}[g_{\mu\nu}, \ldots, \phi(x,z)]\right]$$

$$\phi(x,z) = \phi_0(x) z^{\Delta_-} + \langle \mathcal{O} \rangle z^{4-\Delta_-} + \ldots$$

Dofs of gravity ~ area, not volume!

AdS_5 has boundary at z=0 and scale L

N_c, g^2N_c large