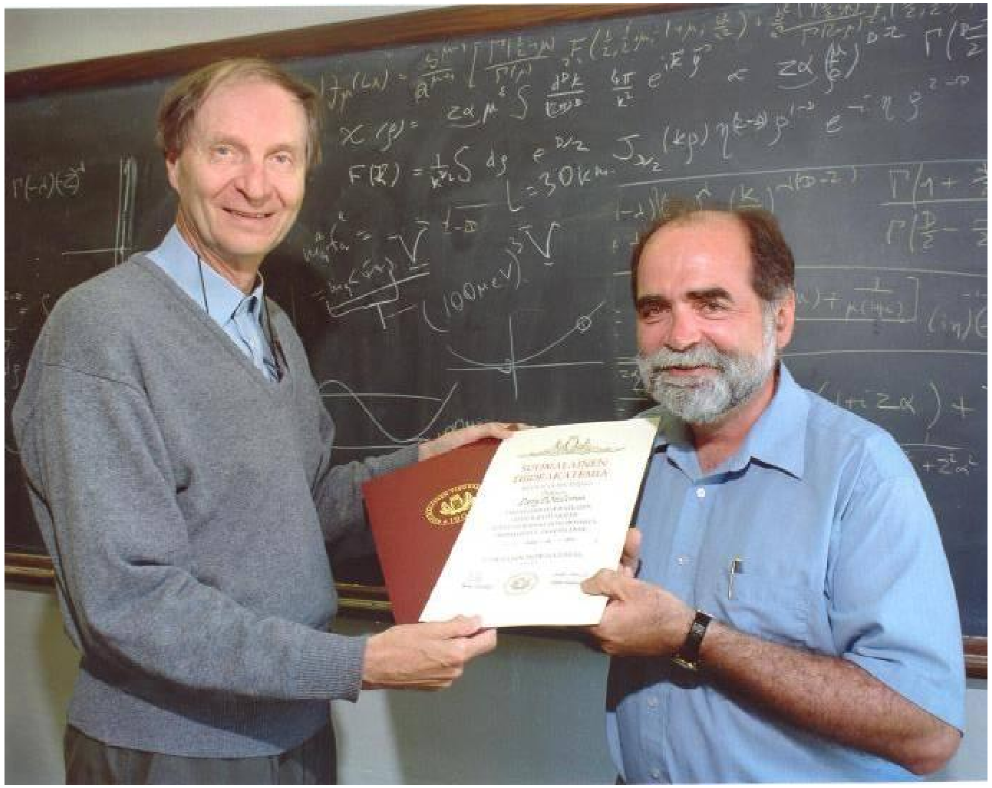
The equation of state of hot QCD: the magic of AdS/QCD or hard work?

Keijo Kajantie

University of Helsinki

Saclay, 23 April 2009



Dilemma

The heart of QCD is running coupling, Λ_{QCD}, breaking of conformal invariance, confinement

The heart of gauge/gravity duality, AdS/CFT, is conformal invariance

Can one find the gravity dual of QCD, i.e., some metric + other fields in > 4d space, with which one could reproduce QCD results?

Discuss two examples for finite T QCD:

- Spatial string tension $\sigma(T)$
- Pressure $p(T)$

Spatial string tension σ(T)

Finite T QCD: 3d space + imaginary time $0 < \tau < 1/T$

Measure string tension in the 3d spatial sector for varying T, get σ(T)

But can also measure σ in the 3d spatial sector without any 4th dim, string tension in 3d SU(3) Yang-Mills

$$\sqrt{\sigma_s} = 0.553(1) g_M^2 \qquad g_M^2 = g^2(T)T$$

$$\frac{T}{\sqrt{\sigma_s}} = 1.81 \frac{1}{g^2(T)} = 0.25 \left[\log \frac{T}{\Lambda_\sigma} + \frac{51}{121} \log \left(2 \log \frac{T}{\Lambda_\sigma}\right)\right]$$

$$\Lambda_\sigma = ?$$

Full hot QCD $\longleftrightarrow$ EQCD $\longleftrightarrow$ MQCD

$$\boxed{\text{QCD} \equiv \text{4d YM} + \text{quarks; } |\mathbf{k}| \sim g^2T, gT, 2\pi T}$$

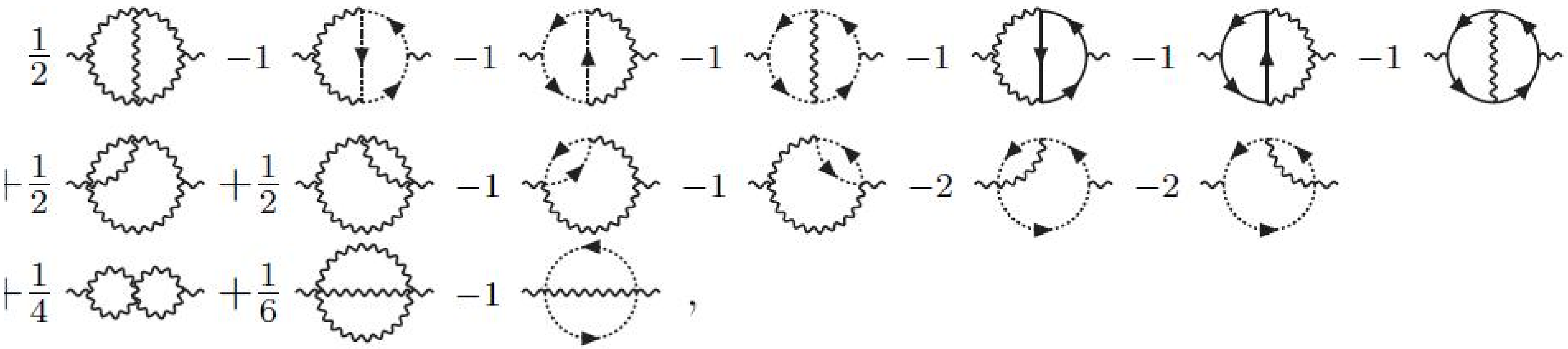
$\Downarrow$ perturbation theory (1)

$$\boxed{\text{EQCD} \equiv \text{3d YM} + A_0\text{; } |\mathbf{k}| \sim g^2T, gT}$$

$\Downarrow$ perturbation theory (2)

$$\boxed{\text{MQCD} \equiv \text{3d YM; } |\mathbf{k}| \sim g^2T}$$

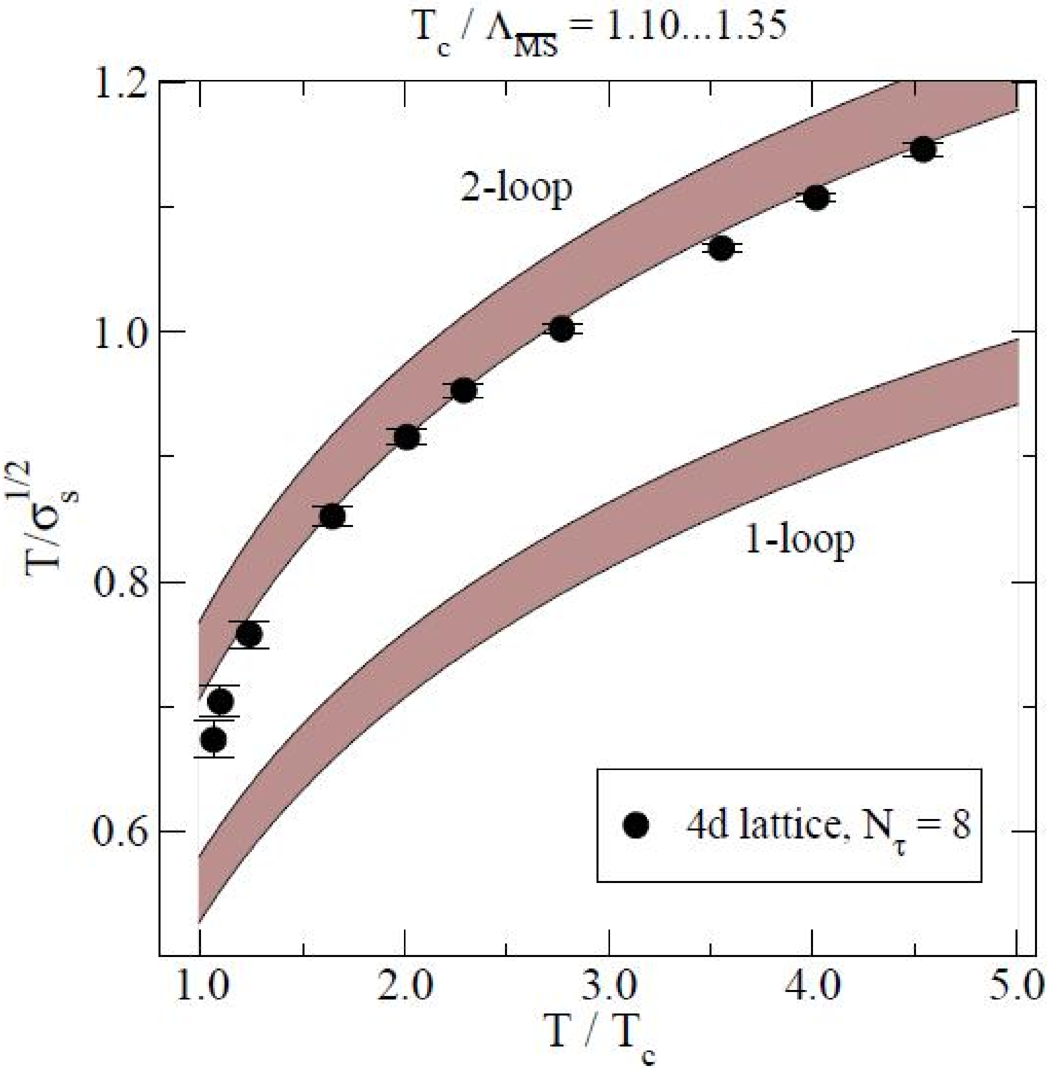
$$g_{\mathrm{M}}^2 = g_{\mathrm{E}}^2 \left[1 - \frac{1}{48}\frac{g_{\mathrm{E}}^2 C_A}{\pi m_{\mathrm{E}}} - \frac{17}{4608}\left(\frac{g_{\mathrm{E}}^2 C_A}{\pi m_{\mathrm{E}}}\right)^2\right]$$



$$g_{\mathrm{E}}^2 \equiv T\left\{g^2(\bar{\mu}) + \frac{g^4(\bar{\mu})}{(4\pi)^2}\left[\alpha_{\mathrm{E}7} + \beta_{\mathrm{E}3}\epsilon + \mathcal{O}(\epsilon^2)\right] + \frac{g^6(\bar{\mu})}{(4\pi)^4}\left[\gamma_{\mathrm{E}1} + \mathcal{O}(\epsilon)\right]\right.$$

Get a quantitative 2-loop prediction:

Laine-Schröder
hep-ph/0503061



Reproducible
well defined

3 loop ?
Lattice cont ?

That was the hard QCD work. What about magical AdS?

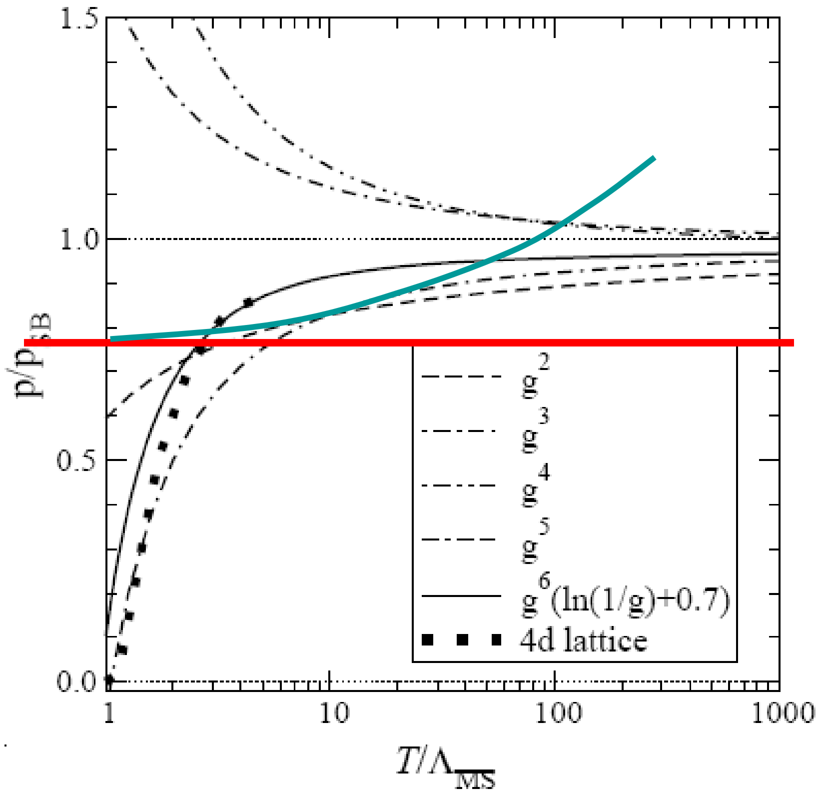
BH in 5d asymptotically (z→0) AdS_5

$$ds^2 = \frac{\mathcal{L}^2}{z^2}\left[-\left(1 - \frac{z^4}{z_0^4}\right)dt^2 + d\mathbf{x}^2 + \frac{dz^2}{1 - z^4/z_0^4}\right]$$

$$T_{\text{Hawk}} = \frac{1}{\pi z_0} \qquad S = \frac{A}{4G_5} \quad = V_3 \cdot \frac{\pi^2 N_c^2}{2} T^3$$

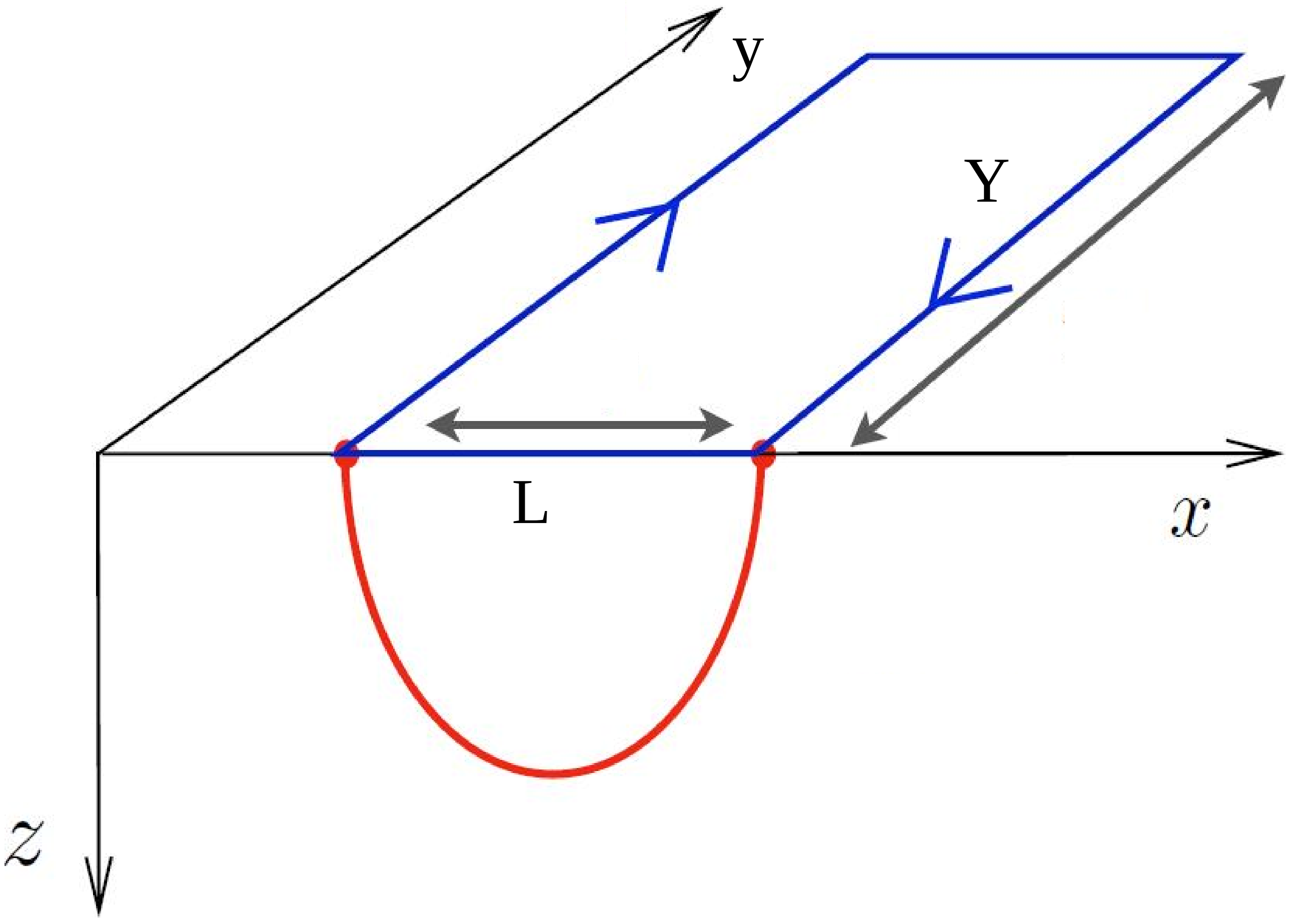
⟶ The famous 3/4 :

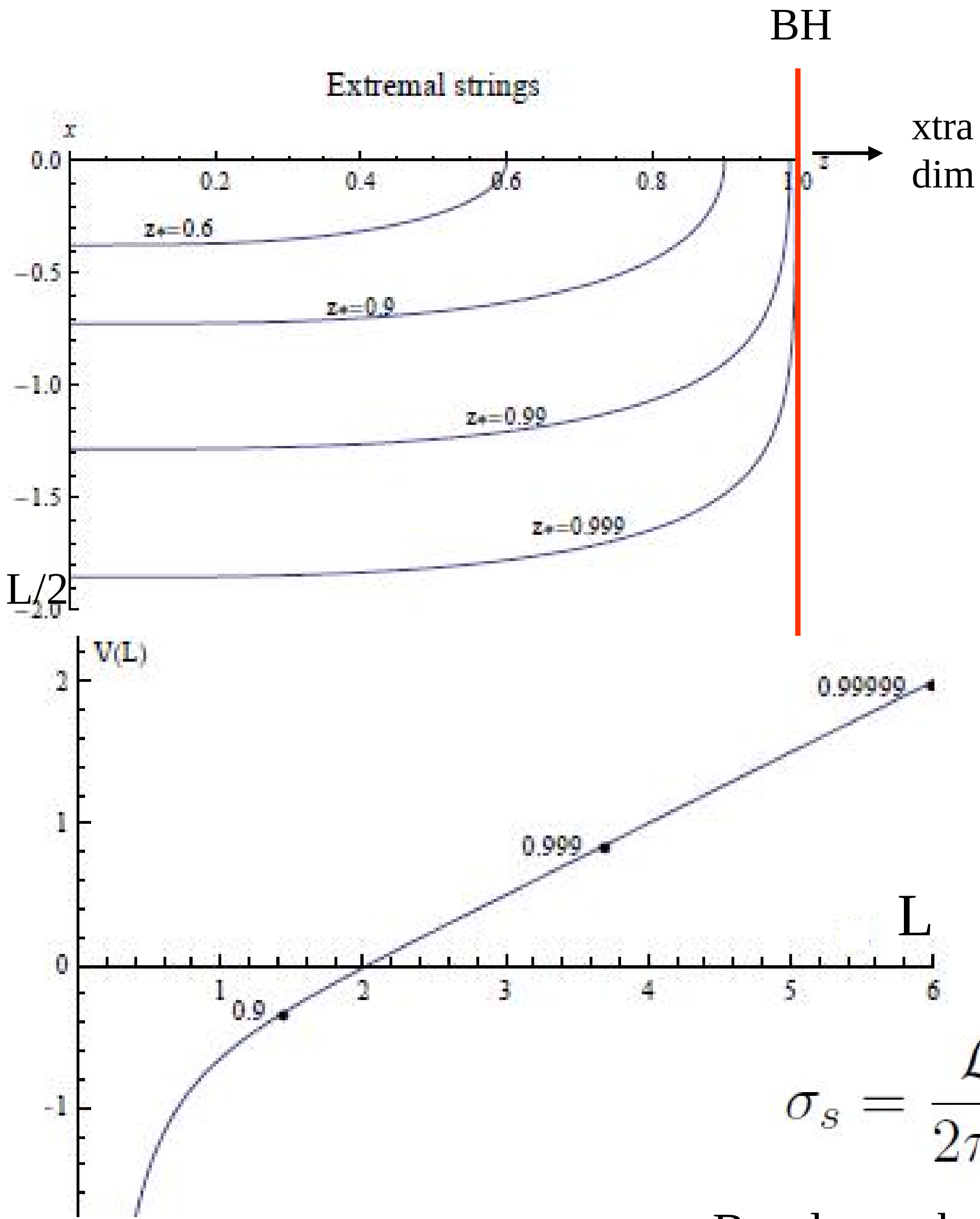
p(T) in AdS/CFT vs perturbation theory



Spatial string tension in AdS/QCD:

<Wilson loop> : value of extremal action of string sheet hanging from the loop to 5th dim





To get V(L) at large L the extr string must hang deep close to black hole

$$\sigma_s = \frac{\mathcal{L}^2}{2\pi\alpha'}(\pi T)^2 = \sqrt{g^2 N_c}\,\frac{\pi}{2}T^2$$

But do not know the parameters!

Bottom-up models

- Hard, $z < z_0$ ”bag model”

- Soft, insert exp[c z^2]

- Dynamical, generate z-dep from Einstein for metric + scalar

Kiritsis et al 0812.0792 etc, total of 330 pages

$$S = \frac{1}{16\pi G_5}\left\{\int d^5x\,\sqrt{-g}\left[R - \tfrac{4}{3}(\partial_\mu\phi)^2 + V(\phi)\right]\right\}$$

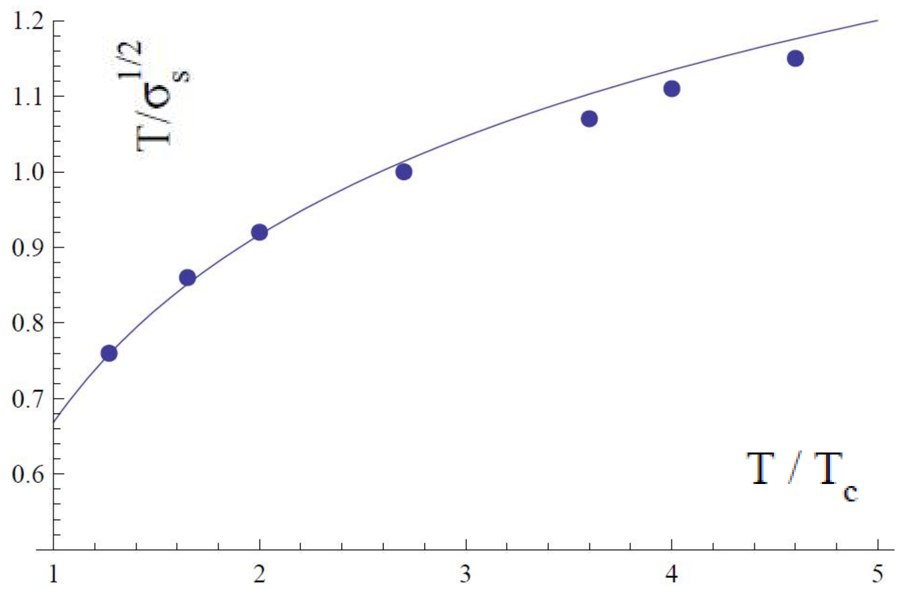
$$ds^2 = b^2(z)\left[-f(z)dt^2 + d\mathbf{x}^2 + \frac{dz^2}{f(z)}\right], \qquad \phi = \phi(z)$$

3 Einstein eqs + $g^2 = \lambda(z) = e^{\phi(z)}$, $\beta(\lambda) = b\frac{d\lambda}{db}$ ➞ Four functions of z: b, f, φ, V(φ)

Dual of a theory with any beta function!

$$\frac{T}{\sqrt{\sigma_s}} = \underbrace{\sqrt{\frac{2}{\pi\sqrt{g^2 N_c}}}}_{\text{conformal}} \underbrace{\left(1+\frac{4}{9\log(\pi T/T_c)}\right)\log^{2/3}}_{\text{conf inv breaking}} \frac{\pi T}{T_c}$$

Magnitude unknown, fits shape:



Modified AdS can fit, but is this more?

QCD pressure p(T): the hard work part

The action really is

$$H = \frac{1}{g^2}\left(\sum_{i,j} a_{ij}x_i x_j + \sum_{i,j,k} b_{ijk}x_i x_j x_k + \dots\right).$$

but one inserts $y_i = x_i/g$ and gets

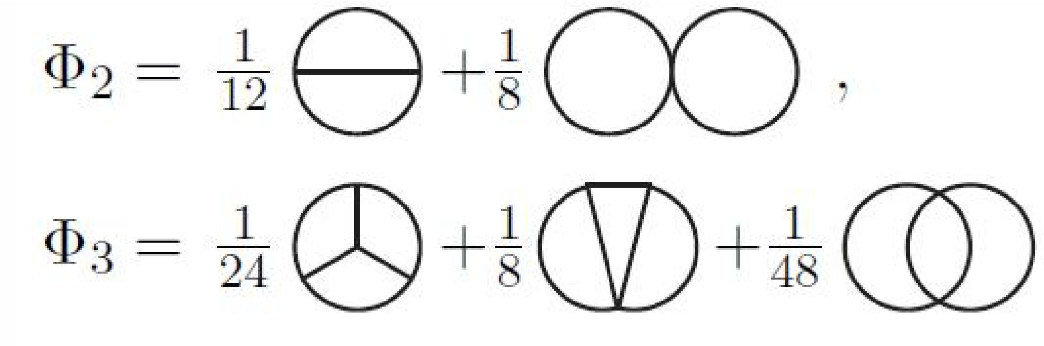
$$H = \left(\sum_{i,j} a_{ij}y_i y_j + g\sum_{i,j,k} b_{ijk}y_i y_j y_k + \dots\right)$$

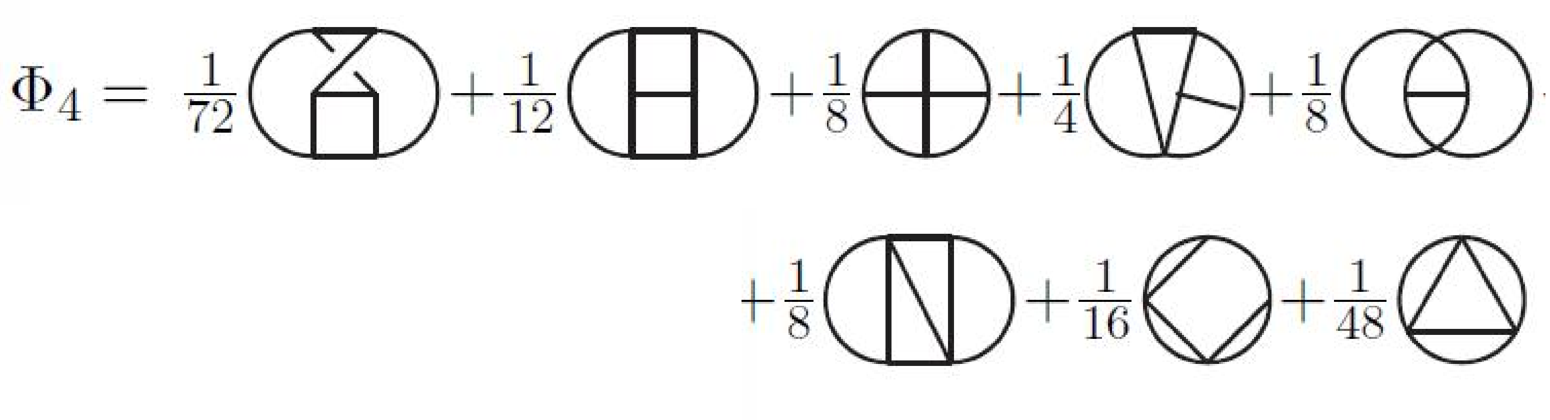
expands, integrates, gets p(T)=T^4 *

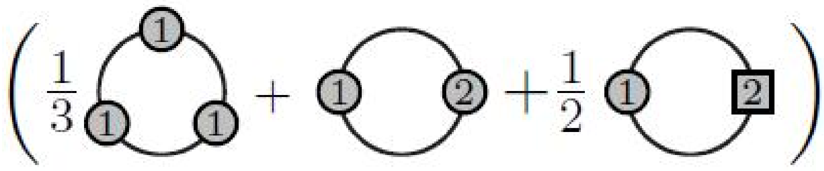
$$c_0 + c_2 g^2 + c_3 g^3 + (c_4' \ln g + c_4)g^4 + c_5 g^5 + (c_6' \ln g + c_6)g^6 + \mathcal{O}(g^7)$$

[c_2 Shuryak 78, c_3 Kapusta 79, c_4' Toimela 83, c_4 Arnold/Zhai 94, c_5 Zhai/Kastening 95, Braaten/Nieto 96, c_6' KLRS 03], c_6 ??

Expanding in g get diagrams of type:

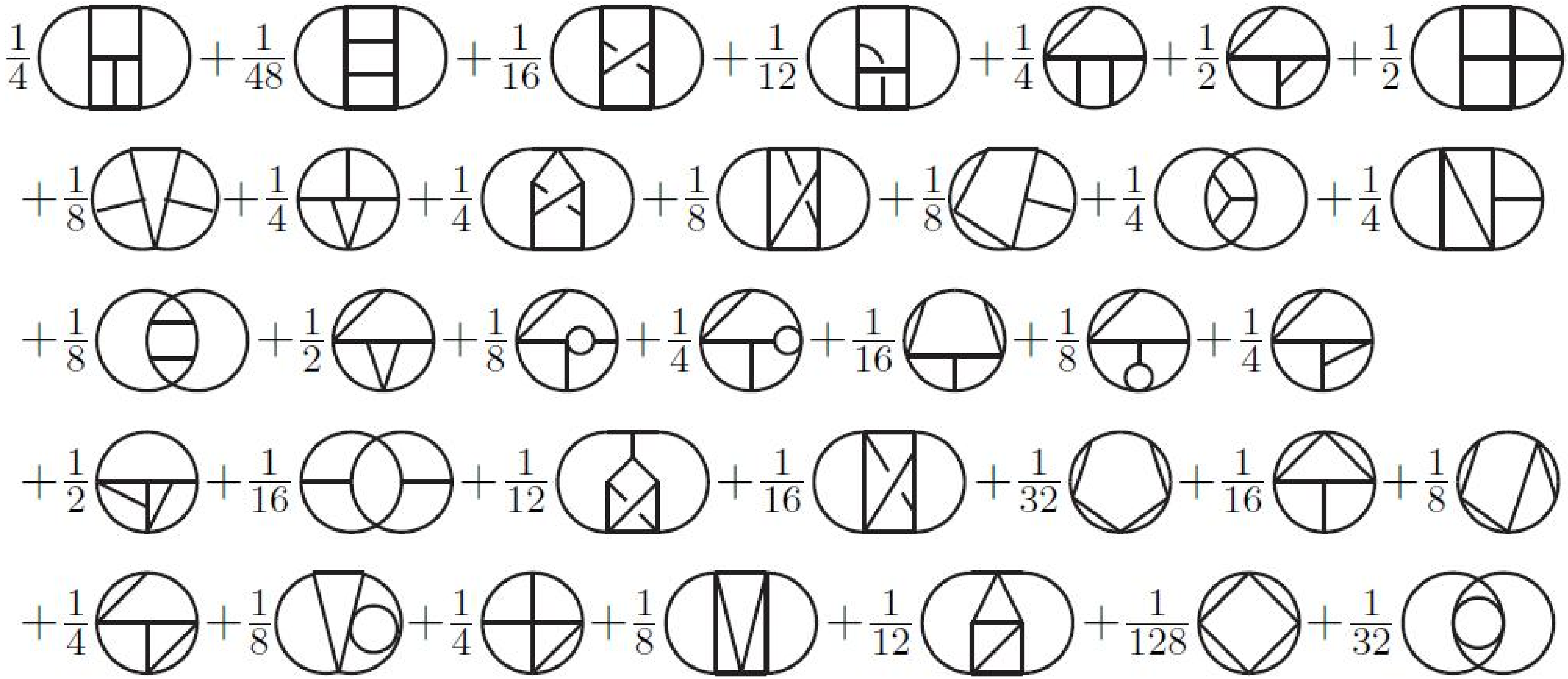






All topologically distinct 5-loop vacuum diags;

Kajantie-Laine-Schröder
hep-ph/0109100



Exercise in futility (mathematics): generalise to n loops

No wonder QCD matter becomes strongly interacting!

Computation has to be organised according to the pattern

QCD $\equiv$ 4d YM + quarks; $\|\mathbf{k}\| \sim g^2T, gT, 2\pi T$

$\Downarrow$ perturbation theory (1)

EQCD $\equiv$ 3d YM + A_0; $\|\mathbf{k}\| \sim g^2T, gT$

$\Downarrow$ perturbation theory (2)

MQCD $\equiv$ 3d YM; $\|\mathbf{k}\| \sim g^2T$

Get expansion of type

$$
\begin{aligned}
1 + g^2_{(1)} \quad & + g^4_{(1)} \ln \quad & + g^6_{(1)} (\ln + [\text{pert}]_1) + \ldots \\
+ g^3_{(2)} & + g^4_{(2)} \ln + g^5_{(2)} & + g^6_{(2)} (\ln + [\text{pert}]_2) + \ldots \\
& & + g^6_{(3)} (\ln + [\text{non-pert}]) +
\end{aligned}
$$

All but the last term on first line is known!

The Linde coefficient c_6

$$c_0 + c_2 g^2 + c_3 g^3 + (c_4' \ln g + c_4) g^4 + c_5 g^5 + (c_6' \ln g + c_6) g^6 + \mathcal{O}(g^7)$$

Define g to be the standard 2-loop MSbar running coupling

c_6 gets contributions from infinite number of loops: is non-perturbative: lattice MC has to be used

Stages of computation:

πT	$1 + g^2_{(1)}$		$+ g^4_{(1)} \ln$		$+ g^6_{(1)}(\ln + [\text{pert}]_1) + \dots$
E: gT		$+ g^3_{(2)}$	$+ g^4_{(2)} \ln$	$+ g^5_{(2)}$	$+ g^6_{(2)}(\ln + [\text{pert}]_2) + \dots$
M: g^2 T					$+ g^6_{(3)}(\ln + [\text{non-pert}]) +$

The nonperturbative contribution, HKLRS hep-lat/0412008

-free energy of 3d SU(N) gauge theory:

$$\frac{1}{V}\ln\left[\int\mathcal{D}A_k\,\exp\left(-S_{\rm E}\right)\right]_{\overline{\rm MS}}$$

$$= \quad g_3^6\frac{d_A N_{\rm c}^3}{(4\pi)^4}\left[\left(\frac{43}{12}-\frac{157}{768}\pi^2\right)\ln\frac{\bar\mu}{2N_{\rm c}g_3^2}\ - 0.2\pm0.8\ \right]$$

Lattice and continuum are matched so that $\bar{\mu}$ is THE MSbar scale!

Needed 4-loop lattice perturbation theory in 3d, thought to be impossible, was solved with numerical stochastic lattice perturbation theory
DiRenzo, Laine, Miccio, Schröder, Torrero hep-ph/0605042

Contribution from the scale gT

-free energy of 3d SU(N) gauge + adjoint scalar theory:

KLRS hep-ph/0304048

$$\frac{1}{V}\ln\int \mathcal{D}A_k\mathcal{D}A_0\,\exp\Big(-S_{\mathrm{E}}\Big)$$

$$= \cdots + g^6\,\frac{d_A N_c^3}{(4\pi)^4}\left[\left(\frac{43}{4}-\frac{491}{768}\pi^2\right)\ln\frac{\bar\mu}{2m(\bar\mu)} - 1.391512\right]$$

$$1.391512 = \frac{311}{256}+\frac{43}{32}\ln 2 + \frac{11}{3}\ln^2 2 - \frac{461}{9216}\pi^2 + \frac{491}{1536}\pi^2\ln 2 - \frac{1793}{512}\zeta(3)$$

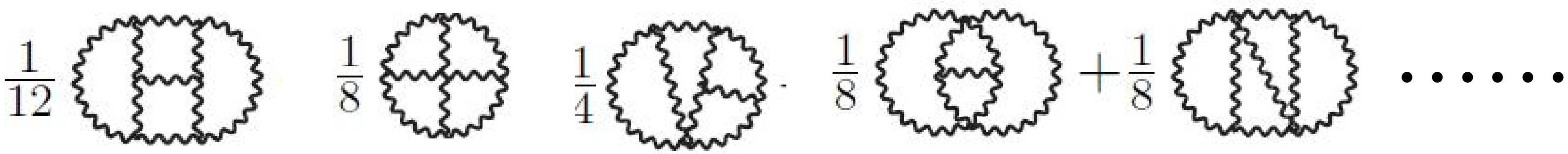
Again: $\bar\mu$ is THE MSbar scale!

Contribution from scale πT

$$1 \; + g^2_{(1)} \qquad + g^4_{(1)} \ln \qquad + g^6_{(1)} (\ln + [\text{pert}]_1)$$

is unknown!!

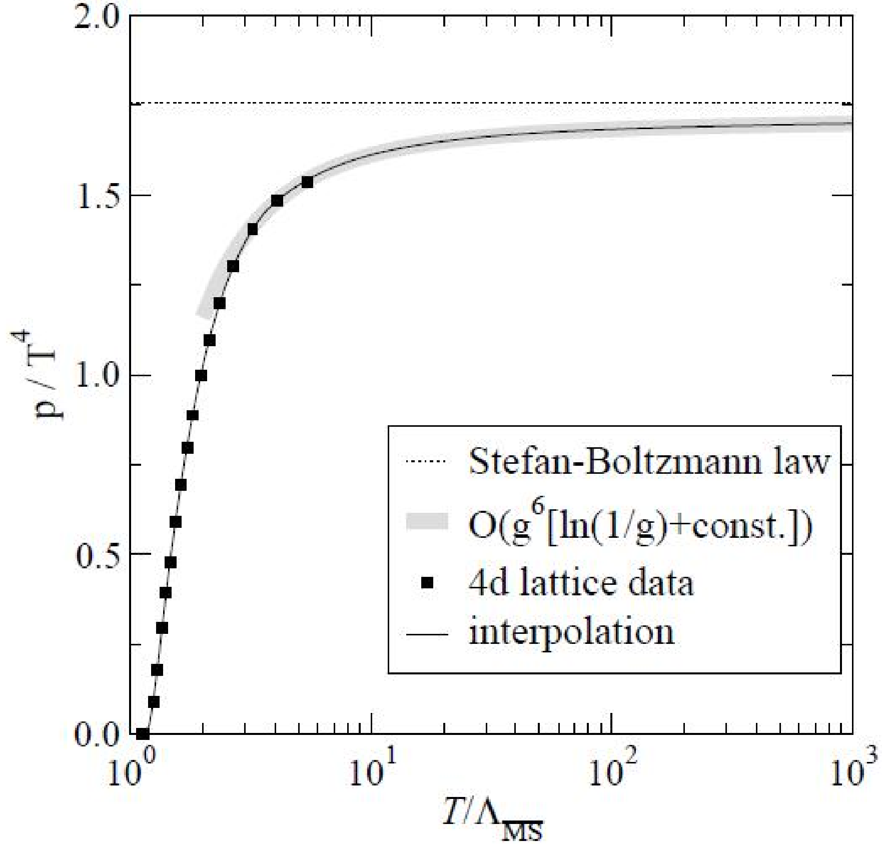
Have to do a 4loop sum-integral computation:



Strict MSbar, sums over n, integrals in 3-2ε dimension

Symbolic techniques not yet fully developed!

One can fit the constant to agree with data:



but how does this compare with the outcome of computation?

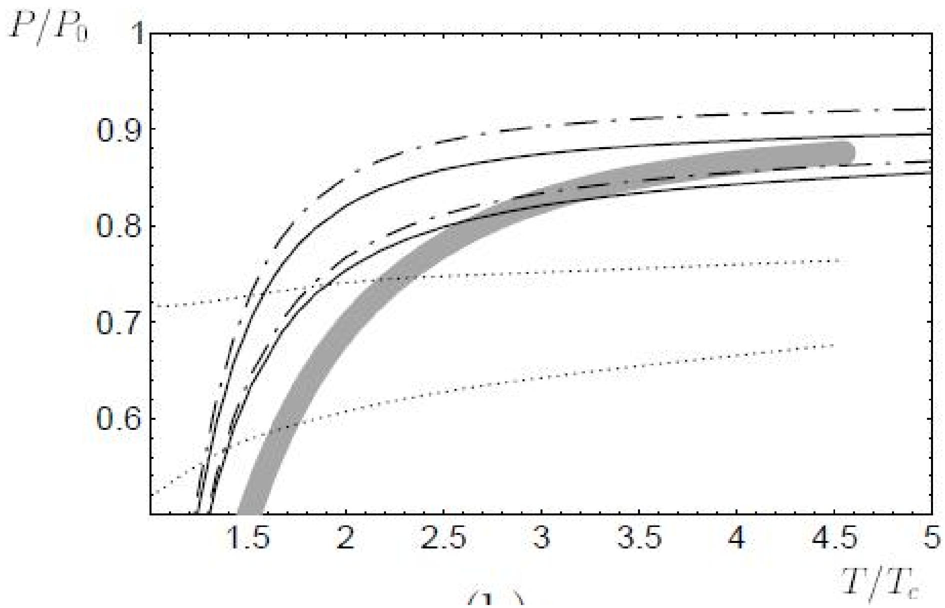
Goal:

Some day the number for c_6 should be out

But what then?

At least: work out g^7 and show it is small….

J-P et co have set up a self-consistent resummation scheme and produced good fits to the same data hep-ph/9910309



Our human effort was bigger by a factor ~ 40 and the number of flops by a factor of ~ 10^{15}

so our result had better be better!