Quantum Gravity (= QG)

K. Kajantie HIP 28 May 2015 9.00-10.30

(lectures in Finnish, notes in English)

There is no theory of QG

Make yourself eternally famous by discovering one!

Classical gravity = General Relativity and Quantum Field Theory = Standard Model work enormously well, joining them is the problem!

Where could one find QG effects? Black holes , very early universe, cosmological constant, single gravitons, gravitational radiation, wave function of the universe/multiverse,...

1. $c,\ \hbar,\ G,\ G\hbar$

c Special relativity SR you need when v/c is non-negligible

$$c = 299792458\,\mathrm{m/s} \quad c^2 \approx 10^{17}$$

$\hbar$ Quantum mechanics QM you need when

$$p \cdot x \sim \hbar = 1.0546 \cdot 10^{-34}\,\mathrm{J\,s} = 6.582 \cdot 10^{-25}\,\mathrm{GeV\,s}$$

$$\hbar c = 197.327\,\mathrm{MeV\,fm} = 0.229\,\mathrm{K\,cm}$$ QFT

G General relativity GR you need for large O(1)

$$\frac{2GM}{c^2 r} \equiv \frac{r_s}{r} \qquad r_s = r_{\mathrm{Schwarzschild}} = 2GM/c^2$$

$$r_s(\mathrm{Earth}) = 0.88\,\mathrm{cm} \quad r_s(\mathrm{Sun}) = 2950\,\mathrm{m}$$

$$\frac{r_s}{r_{\mathrm{earth}}} = 1.3 \cdot 10^{-9}$$

When do you need QuantumG?

Planck units:

$$\begin{aligned} \text{mass} &= \sqrt{\hbar c/G} = 2.177 \cdot 10^{-8}\,\mathrm{kg} \\ \text{length} &= \sqrt{\hbar G/c^3} = 1.616 \cdot 10^{-35}\,\mathrm{m} \\ \text{time} &= \sqrt{\hbar G/c^5} = 5.391 \cdot 10^{-44}\,\mathrm{s} \\ \text{power} &= c^5/G = 3.63 \cdot 10^{52}\,\mathrm{W} \end{aligned}$$

You expect to see QG effects at these scales, and the only places in nature seem to be early Big Bang and around Black Holes, possibly gigantic ones.

$$M_p = \sqrt{\frac{\hbar c}{G}} = 21.8\mu\mathrm{g} = 1.22 \cdot 10^{19}\,\mathrm{GeV/c^2} = 543\,\mathrm{kWh}$$

Kepler motion

General relativity corrects Newton by a factor $\frac{r_s}{r}$

Quantum general relativity might involve the Compton wave length of the system

$$L_{\mathrm{Compton}} = \frac{\hbar}{Mc}$$

$$r_s L_{\mathrm{Compton}} = \frac{2G\hbar}{c^3} \equiv 2L_p^2$$

so the overall correction in QG would be $\frac{L_p^2}{r^2}$

Truly infinitesimal!

2. Quantum mechanics coexists very well with classical gravity:

Neutron, mass m, in the potential mgz on the table

$$E = \frac{p^2}{2m} + mgz \underbrace{=}_{p \sim \hbar/z} \sim \frac{\hbar^2}{2mz^2} + mgz \Rightarrow -\frac{\hbar^2}{mz^3} + mg = 0$$

$$z_0 = \left(\frac{\hbar^2}{2m^2 g}\right)^{1/3} = 5.87\,\mu\mathrm{m}, \quad E_0 = mgz_0 \cdot 2.338 = 1.407\,\mathrm{peV}$$

$$10^{-12}\,\mathrm{eV}$$

$$v \approx 1.6\,\mathrm{cm/s}$$

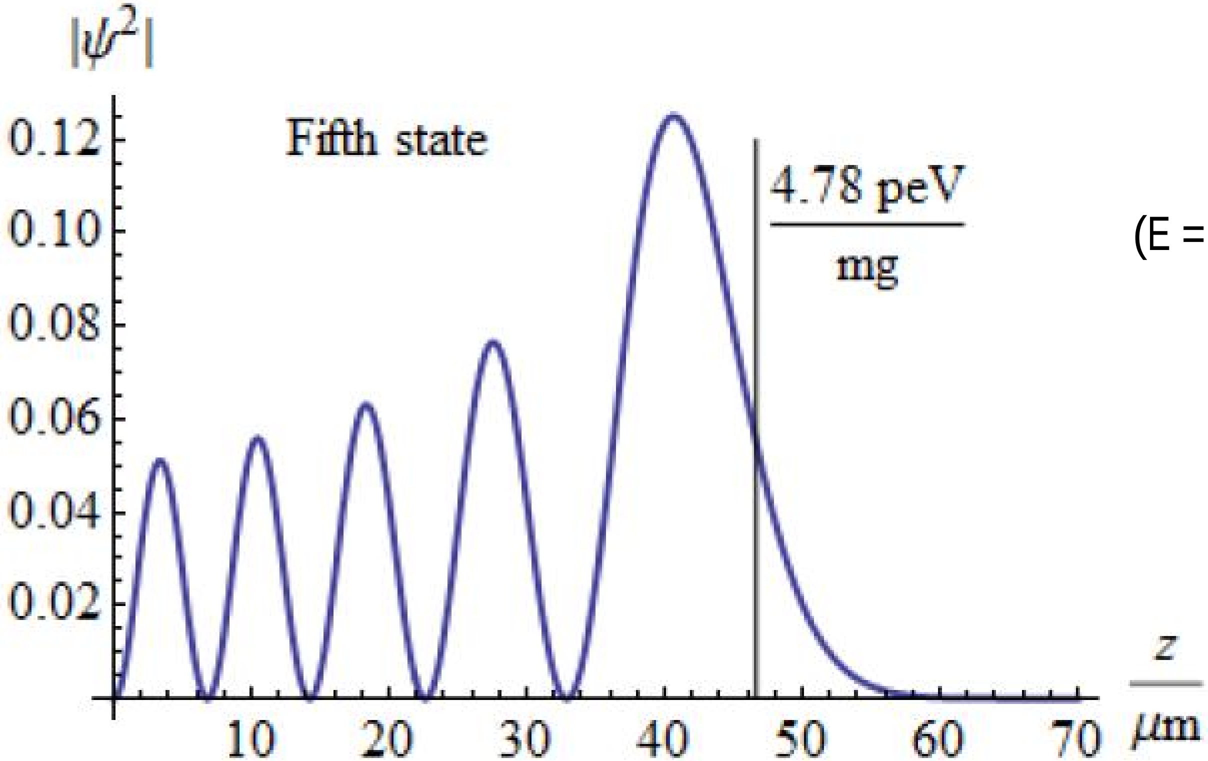
Ultracold neutrons needed!

$$\psi(0) = 0$$

$$\frac{\hbar^2}{2m}\psi''(z) + mgz\psi(z) = E\psi(z)$$

$$\psi_i(z) = N_i \, \mathrm{Ai}(\frac{z}{5.87\mu\mathrm{m}} - \lambda_i)$$

$$\lambda_i = 2.3381, \ 4.0879, \ 5.5205, \ 6.7867, \ 7.9441, ..$$



(E = mg z)

"quantum bounce"

Waste of time, but I cannot resist plotting some more since it is so easy with Mathematica:

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Show[Plot[1/Sqrt[10.2225] (AiryAi[z/5.87-29.9318])^2,
{z,0,195},PlotRange-> All,AxesLabel->{Style["z/\[Mu]m",16],
Style[ "|\[Psi]^2|",18]},TicksStyle->Directive[FontSize-> 16],
PlotStyle-> Thick],Graphics[Line[{{175.7,0},{175.7,0.08}}]],
Graphics[Text[Style["(18.01peV)/mg",12],{183,0.084}]],
Graphics[Text[Style["35th state",16],{80,0.085}]]]
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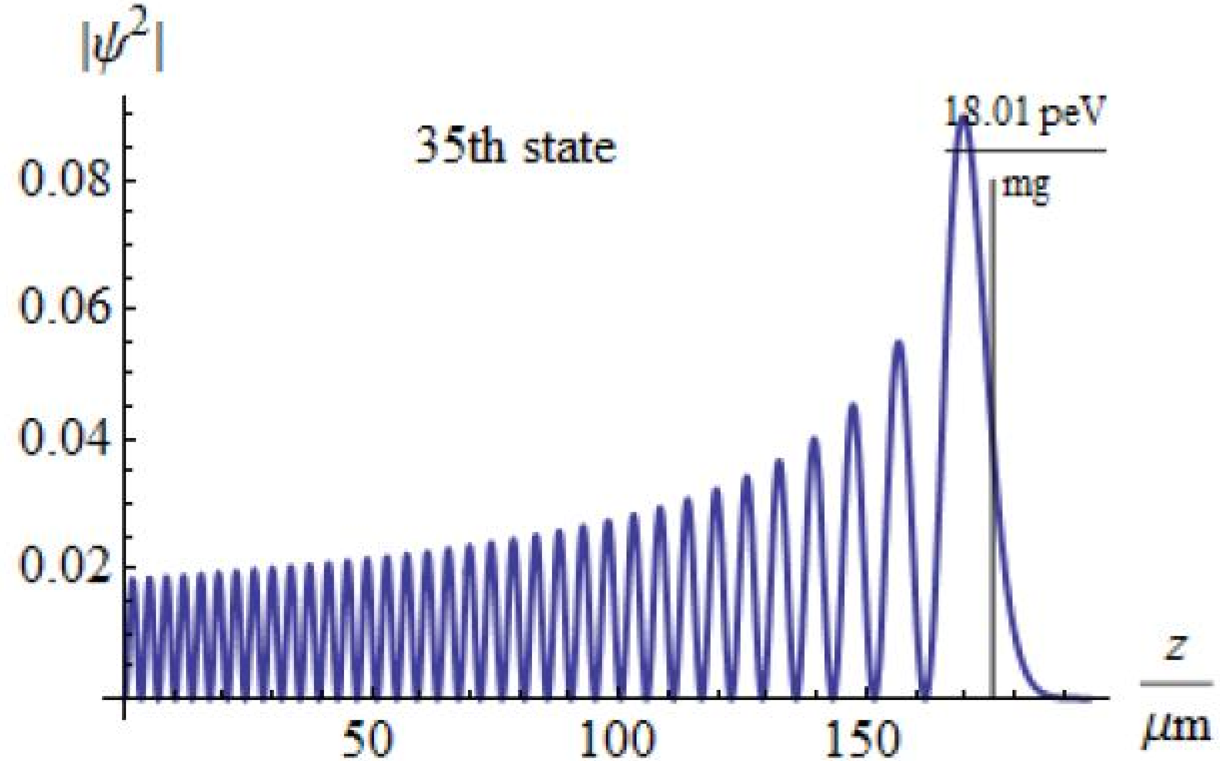


3. Special relativity and QM coexist as relativistic quantum field theory:

Key question with great implications to QG: is there supersymmetry?

Point: in QFT the Higgs mass generically is, for dimensional reasons, of the form

$$m_H^2 = m_{H0}^2 + g^2 m_{\text{new}}^2 \log \frac{m_{\text{new}}}{m_h} + \ldots$$

New BSM particles, m_{new} > TeV

where are these corrections experimentally? Two solutions:

- There are lots of new particles which cancel each other: supersymmetry. This is what huge crowds of theorists liked; in 2 y we will see if this is so!

- There are no new particles: no correction, problem solved… until M_P and QG

Desert between 100 and 10^{19} GeV, with lots of life: dark matter, ΔB!??

4. Classical Einstein gravity

Carroll, Weinberg

Newton: $\phi(x)$

Action

Dynamics: $S[q,q'] = \int dt \left(\frac{1}{2} q'(t)^2 - V(q(t))\right)$

GR: $$S[g_{\mu\nu}(x)] = \frac{1}{16\pi G} \int d^4x \sqrt{-\det g_{\mu\nu}} \left(g^{\mu\nu} R^{\alpha}{}_{\mu\alpha\nu} + 2\Lambda\right)$$

S[g,g',g'']

$$R_{\mu\nu\alpha\beta} = \partial\Gamma - \partial\Gamma + \Gamma\Gamma - \Gamma\Gamma$$
$$\Gamma = g(\partial g + \partial g - \partial g)$$

Extrema give classical $g_{\mu\nu}$:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - \Lambda g_{\mu\nu} = 0 \left(= 8\pi G T_{\mu\nu},\ T_{\mu\nu} = \frac{-2}{\sqrt{g}} \frac{\delta S_{\text{matter}}}{\delta g^{\mu\nu}}\right)$$

Inv under local coordinate transformations affecting x

Compare theory of gluons, inv under SU(3) affecting a,b,c:

$$S[A^a_\mu(x)] = \frac{1}{4} \int d^4x \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu - \frac{g}{\sqrt{\hbar c}} f_{abc} A^b_\mu A^c_\nu\right)^2$$

Only consistent as a quantum theory (confinement, gluons -> color singlet glueballs) and quantisation is perfectly understood

Schwarzschild metric

Solve $R_{\mu\nu} = 0$ (no mass, no time dep!) in coordinates $x^\mu = (t, r, \theta, \phi)$

Carroll, Ch 5

$$g_{\mu\nu}(x) = \begin{pmatrix} -(1-\frac{r_s}{r}) & 0 & 0 & 0 \\ 0 & \frac{1}{1-r_s/r} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{pmatrix}$$

$$d\tau^2 = (1-\frac{r_s}{r})dt^2 - \frac{1}{1-r_s/r}dr^2 - r^2\,d\Omega^2$$

Computing radial geodetic motion and comparing with Newton at large r (weak field) you fix $r_s=2GM/c^2$

Black hole with horizon at $r=r_s$

Exercises

1. Find eom for radial motion of a particle with total energy E

$$\left(\frac{dr}{cd\tau}\right)^2 = \frac{\mathbf{p}^2}{(mc)^2} + \frac{r_s}{r} \qquad \frac{dt}{d\tau}\left(1 - \frac{r_s}{r}\right) = \frac{E}{mc^2}$$

2. Find $\tau = \tau(r) \quad t = t(r)$ for radial motion starting with p=0 at r=infinity

$$\tau(r) = \frac{2}{3}\left(r^{3/2} - 1\right) \quad t(r) = \frac{2}{3}\left(r^{3/2} + 3r^{1/2} + \log\frac{\sqrt{r}-1}{\sqrt{r}+1}\right)$$

3. The only way to find what is behind the horizon of the big Milky Way BH is to go and have a look. You start from infinity, how long does it take from the horizon to the singularity (26 s) ? You fall feet first, what is the difference in accelerations experienced by your feet and head (g_{earth}/8000 for 2m height)? Assume no firewall.

http://www.eftaylor.com/general.html

http://jila.colorado.edu/~ajsh/insidebh/intro.html

A weak field solution of $R_{\mu\nu} = 0$: gravitational waves $R^{\mu}_{00\nu} \neq 0$!!

Carroll sect 7.4

Transverse traceless gauge:

Flat background!

$$g_{\mu\nu} = \eta_{\mu\nu} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \epsilon_{11} & \epsilon_{12} & 0 \\ 0 & \epsilon_{12} & -\epsilon_{11} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{-ipx}, \quad p = (E, 0, 0, E)$$

A spin 2 graviton moving in the z direction, polarisation determined by ε_{ij}

$$T_{\mu\nu} = \frac{p_\mu p_\nu}{8\pi G}(|\epsilon_{11}|^2 + |\epsilon_{12}|^2)$$

Seem to get gigantic fluxes for $p_0 = \omega =$ Hz: $\frac{\mathrm{Hz}^2}{G} \cdot c \approx 10^{45} \frac{\mathrm{GeV}}{\mathrm{m^2 s}}$

The amplitude ε_{ij} had better be small, otherwise crazy; for binary BH of size R at dist X ly:

$$\epsilon \sim \frac{r_s}{R}\frac{r_s}{r} = \frac{1}{10}\frac{10\,\mathrm{km}}{X\,\mathrm{ly}} = \tfrac{1}{X}\,10^{-13}$$

X = 10^8 !?

Ligo may soon see gravitational radiation but that is not yet QG!

Detecting one graviton would be a big step towards QG

Seeing a single graviton is theoretically possible, but eliminating neutrino background would require so much shielding that the detector collapses into a black hole!

Rothman-Boughn gr-qc/0601043

So we do not find correct QG by measuring graviton-graviton scattering!

Gluon-gluon scattering can be measured very indirectly and confirms QCD!

5. Hawking radiation

Maybe this is QG!

BHs have temperature $$T_H = \frac{\hbar c}{4\pi r_s} \sim \frac{10^{23}\,\mathrm{kg}}{M}\,\mathrm{K}$$ entropy $$S = \frac{c^3}{\hbar}\frac{A}{4G} \equiv \frac{A}{4L_P^2}$$

BH thermo: $$E = Mc^2 \quad dE = T_H\,dS$$ Carlip 1410.1486

Prototype derivation: Accelerated mirror in 1+1d Birrell-Davies sect 4.4 Carroll sect 9.5

Traschen gr-qc/0010055

BHs are unstable $$\frac{dM}{dt} \sim -r_s^2 T_H^4 = -\frac{\hbar c^4}{60\cdot 265\pi}\frac{1}{G^2M^2} \qquad M(t) = M_0\left(1-\frac{t}{t_{\mathrm{ev}}}\right)^{1/3}$$

Here decay to photons

$$t_{\mathrm{ev}} = 20\cdot 256\pi\frac{M_0^3}{M_p^3}t_p \sim 10^{-16}s\,\frac{M_0^3}{\mathrm{kg}^3}$$

In nature $M_{BH} > M_{sun}$?
Milky Way BH is BIG: $$M \sim 4\cdot 10^6\,M_{\mathrm{sun}} \quad r_s = 12\cdot 10^6\,\mathrm{km} \approx \frac{1}{10}\,\mathrm{au}$$

Very active research topic, information paradox, firewalls,.... Harlow 1409.1231

BH info paradox:

How do you unitarily evolve a quantum state over the formation and decay of the BH:

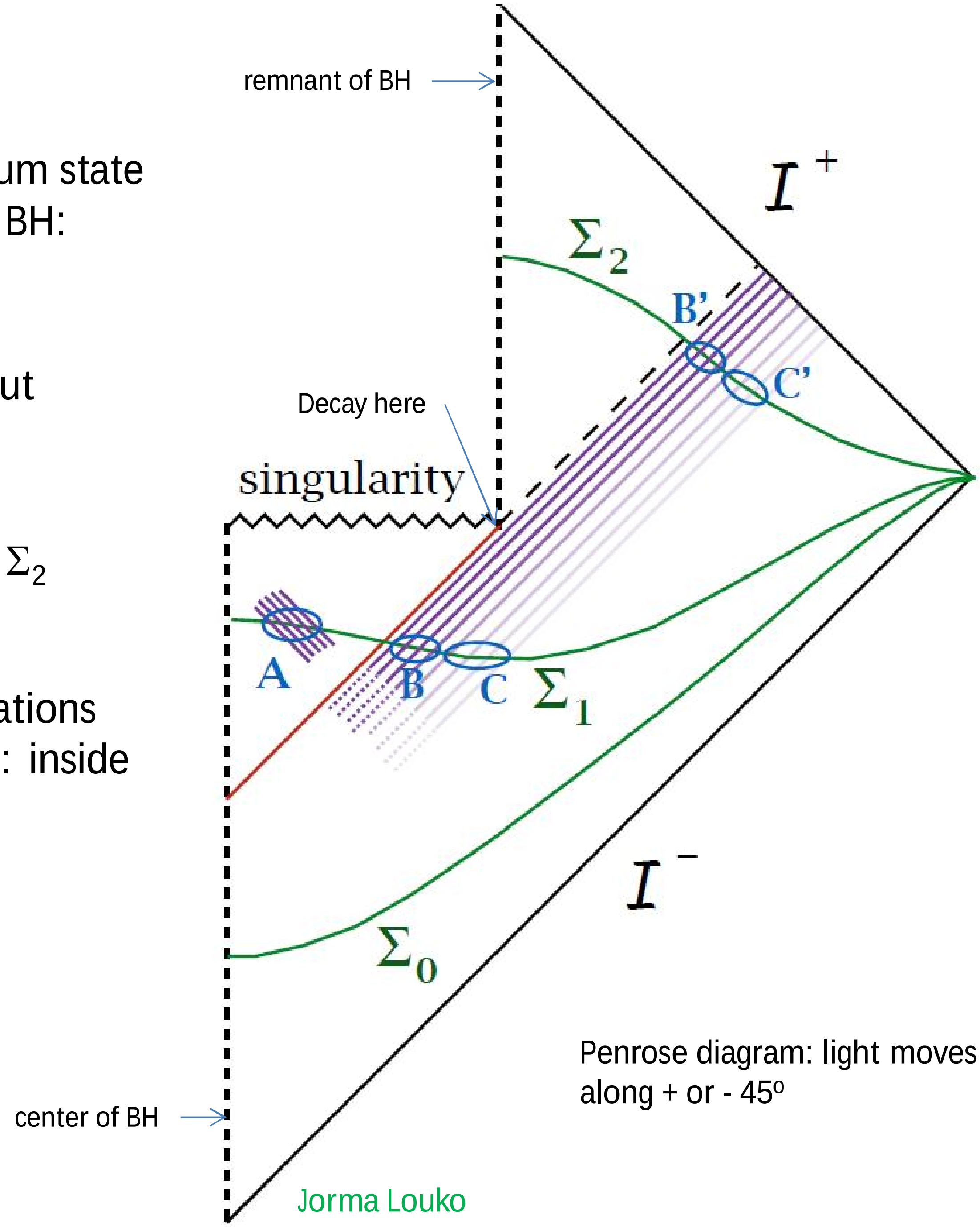
$$\psi(\Sigma_2) = U\,\psi(\Sigma_0)$$

Seemed paradoxical: pure state in, out comes radiation.

Resolution: radiation entangled on Σ_2

A consequence on Σ_1 is that correlations in and out of BH have to be broken: inside is basically inaccessible. Firewall!

These are just words!
Do not respect them!



Penrose diagram: light moves along + or - 45°

Jorma Louko

One expects full understanding of black holes will involve QG: interior, microstates, physics at horizon, evaporation, evolution, interaction between metric and Hawking radiation

Maybe the area of a BH is quantised: Medved 0906.2641

$$A_n = 8\pi L_p^2 \cdot n, \quad n = 0, 1, 2, \ldots \quad S_n = 2\pi n$$

BHs be stabilised by putting in a box or by going to Anti de Sitter space with a boundary

Holography: our 4d world sits at the boundary of 5d AdS space

Ex. Compute the fate of a BH with T_H=100 MeV ($M_0=10^{11}$ kg, t_{ev}=3 Ga). When has T grown to 100 GeV and how much of M has then been radiated (at 3Ga-3a, 0.999M_0).

Ex. Motivate S ~ A/G by arguing that for a system of particles of mass m one has S ~ M/m but one has to make sure that the Compton wave length of particles has to be < r_s. Check correct hbar and c.

6. Weak fields: Quantum Field Theory of gravitons

Linearize action around Minkowski space: $(-+++)$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Carroll 7.1, Weinberg 10.1

$$\int d^4x \, h^{\mu\nu}[-\partial^2\eta_{\mu\alpha}\eta_{\nu\beta}+\partial_\mu\partial_\alpha\eta_{\nu\beta}+\cdots+\eta_{\mu\nu}(\partial^2\eta_{\alpha\beta}-\partial_\alpha\partial_\beta)]\, h_{\alpha\beta}+\mathcal{O}(h^3)+\mathcal{O}(h^4)$$

Inverse graviton propagator

Very complicated 3- and 4-graviton vertices

Proceed as in perturbative QFT: finite to 1loop, divergent to 2 loops

Need new term $\sqrt{-g}R^{\mu\nu}{}_{\alpha\beta}R^{\alpha\beta}{}_{\gamma\delta}R^{\gamma\delta}{}_{\mu\nu}$ to cancel divergence: unrenormalisable

Goroff-Sagnotti NPB266(1986)709

Tree level gg->gg: gigantic computation, very compact result! Elvang, Scattering amplitudes, 1209.1607

More weak fields: supersymmetry, supergravity

Theory with just $g_{\mu\nu}$ diverges: maybe you can help by adding more particles?

As a starter, add spin 3/2 gravitino

$$\int d^4x \sqrt{-g}\,(R - \epsilon^{\mu\nu\alpha\beta}\psi_\mu\gamma_5\gamma_\nu i\nabla_\alpha\psi_\beta)$$

Finally get to $\mathcal{N} = 8$ susy: 1 spin 2, 8 spin 3/2, 28 spin 1, 56 spin ½, 70 spin 0

Extensive theory program for proving that this very symmetric theory
Is perturbatively finite Bern-Carrasco-Dixon-Johansson-Roiban 1103.1848
Kallosh, 1412.7117

Even if this is true, this is not physical QG – unless supersymmetry is discovered!

7. Canonical quantisation

$$L(q,\dot{q}) \qquad p = \frac{\partial L}{\partial \dot{q}} \qquad [p,q] = i\hbar$$

Choose a spacelike surface and split $g_{\mu\nu} = \begin{pmatrix} N & N_a \\ N_a & g_{ab} \end{pmatrix}$

Coordinates q are g_{ab}, momenta p will be extrinsic curvature K_{ab} of the surface.
N and N_a become constraints between g_{ab} and K_{ab} ADM: Arnowitt-Deser-Misner

Hamiltonian formalism can be worked out Wald, Appendix E.2, but the 00 constraint is too complicated for practical use

Ashtekar variables: take a square root of g_{ab} and put a piece of Christoffel to K_{ab}:

$$g_{ab} = \delta_{ij}\, E_a^i E_b^j \qquad\qquad A_a^i = \epsilon^{ikl}\Gamma_{akl} - iE^{bi}K_{ab}$$

Constraints become identities!

Loop quantum gravity: Take as variables Tr path ordered exponential of A^i_a: Rovelli, Smolin

$$W_C[A] = \mathrm{Tr}\,\mathrm{P}\exp\left[-\int_C dx^a A_a^i(x)T_i\right]$$

SU(2) generators T_i

like lattice QCD! But no as.freedom!

Pullin 1505.02089: ADM,Ashtekar,LQG

8. Asymptotic safety

Weinberg 1976

Asymptotic freedom: $g(\mu)$ runs to 0 when μ grows

GR is perturbatively non-renormalisable: no running gravity coupling $G(\mu)$

Maybe nevertheless

$$G(k) = \frac{G}{1 + \xi_0 G\, k^2}$$

so that at large k, in UV, $G(k)k^2$ runs into a nontrivial UV fixed point

$$G(k)k^2 \to \frac{1}{\xi_0}$$

And maybe more generally a suitably defined set of coupling constants
for all interactions run into this kind of UV fixed point: final unification!

9. Path integral quantisation, Causal dynamical triangulation

Why not do like in QCD:

$$e^{iW[T^{\mu\nu}]} = \int \mathcal{D}g_{\mu\nu}\, e^{iS[g_{\mu\nu}]+iT^{\mu\nu}g_{\mu\nu}}$$

generates all expectation values of products of $g_{\mu\nu}$ = full quantum theory

So you do not fix the metric (”gauge”) but sum over all of them

Problem: cannot define the path integral, cannot go Euclidian, ++++.

Instead of summing over all $g_{\mu\nu}$, one may try to sum over all geometries, diffeomorphism equivalent classes of smooth metrics $g_{\mu\nu,}$ triangulations (Regge calculus) Ambjorn-Görlich-Jurkiewicz-Loll, 1203.3591, 1302.2173

10. (Super)string theory

Quantised motion of (open or closed) strings has mass spectrum of type

$$M^2 = \frac{1}{\alpha'}(N-1)$$ only param tension = $\frac{1}{2\pi\alpha'} \sim M_p^2$

Lowest state N=1 has spin 2: graviton! In the limit α' = 0 strings become points: supergravity

$$S = \frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{-g}\, e^{-2\phi} \left(R + 4(\nabla\phi)^2 - \frac{1}{2} H^{\alpha\beta\gamma} H_{\alpha\beta\gamma} + \ldots\right) + \mathcal{O}(\alpha' R)$$

So there is much more than just R: extra states, supersymmetry, 10 dimensions

For some extremal BH solutions one can even count the string states leading to S=A/(4G).

Immense effort devoted to string theory in 1970…2000 but this is not the solution to QG!

11. Conclusions

This was a superficial list of ideas, each of which is basically a research field

One has the feeling that the final theory of QG is still very far. Too little help from observation!

But it is there: go and find it!

Literature

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http://math.ucr.edu/home/baez/hamiltonian/hamiltonian.html