Expanding systems in gauge theory/gravity duality

K. Kajantie

keijo.kajantie@helsinki.fi

University of Helsinki, Finland

27 March 2007

Work with T. Tahkokallio

Janik, Peschanski;   Nakamura, Sin, Kim

Tony Green's orders inverted:

First the hard stuff (string theory, AdS)

Then the easy stuff (thermodynamics, expansion)

AdS/CFT duality

IIB string theory on $AdS_5 \times S^5$ ($ds^2 = G_{\mu\nu} dx^\mu dx^\nu$): $\quad S = -m \int d\tau$

$$S[X^\mu, \psi^\mu, ..] = -\frac{T}{2} \int d^2\sigma \sqrt{-\det h_{ab}} \left[h^{ab} G_{\mu\nu}(X) \partial_a X^\mu \partial_b X^\nu + \epsilon^{ab} B_{\mu\nu}(X) \partial_a X^\mu \partial_b X^\nu + ... \right.$$
$$\left. - G_{\mu\nu}(X) e^\alpha_a \bar{\psi}^\mu i \rho^a \partial_\alpha \psi^\nu + ... \right] \qquad X^\mu(\sigma^1, \sigma^2)$$

is the same as

N=4 SuSy Yang-Mills (conformal field theory, no dimful parameters):

$$S[A^a_\mu, \Phi^a_i, \psi^a, \bar{\psi}^a] = \int d^4x \left[-\frac{1}{4} F^{a\,2}_{\mu\nu} + \frac{1}{2} (D_\mu \Phi_i)^2 - \frac{1}{2} \bar{\psi} i \gamma^\mu D_\mu \psi - \bar{\psi} \Phi \psi - \Phi^4 \right]$$

$(N_c^2 - 1)\times$ (2+6 bosonic + 4+4 fermionic dofs)

Conformal also on quantum level $\qquad \mu \frac{\partial g(\mu)}{\partial \mu} = \beta(g) = 0$

Problem of string theory: background (in)dependence

String theory is thus just (!!) a 2-dimensional nonlinear sigma model for the $d = 10$ fields $X^0(\sigma^1, \sigma^2), ..., X^{d-1}(\sigma^1, \sigma^2)$ + ... :

$$S[X^\mu, \psi^\mu, ..] = -\frac{T}{2} \int d^2\sigma \sqrt{-\text{det} h_{ab}} \left[h^{ab} G_{\mu\nu}(X) \partial_a X^\mu \partial_b X^\nu + ... \right]$$

It can be consistently quantised in the linear case

$$G_{\mu\nu} = \eta_{\mu\nu} = \text{diag}(-1, 1, 1, ..., 1)$$

but not for, say:

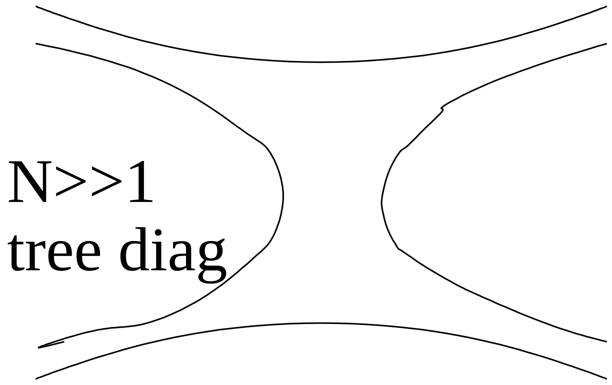
$$ds^2 = \frac{1}{\sqrt{1 + \frac{\mathcal{L}^4}{r^4}}} [-(1 - \frac{r_0^4}{r^4}) dt^2 + dx^2 + dy^2 + dz^2] + \sqrt{1 + \frac{\mathcal{L}^4}{r^4}} [\frac{dr^2}{1 - \frac{r_0^4}{r^4}} + r^2 d\Omega_5^2]$$

(this is $AdS_5 \times S_5$ for $r \ll \mathcal{L}$), let alone for general background $G_{\mu\nu}(X)$.

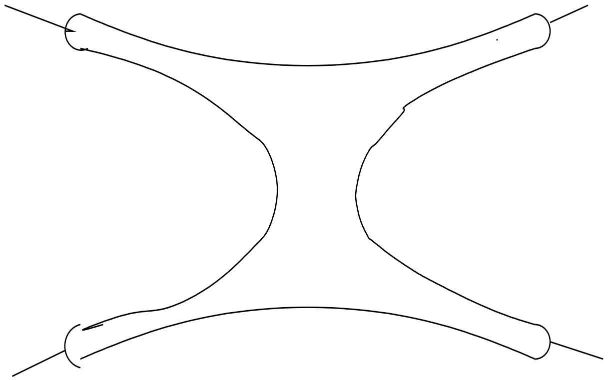
There is even no method for determining the true ground state $G_{\mu\nu}(X)$ (cf. $A_\mu(x)$ for QCD)

Since theory is not known, need approximations to get anything out.

Take number of colors large so that loop diagrams in string theory (balls with handles) do not matter, only tree diags (corrections $\mathcal{O}(1/N_c)$ incalculable):



Take the tension T so large that strings collapse to a point, equally, string excitations become so massive that only massless excitations, gravitons, gluons,..., are active $\Rightarrow$ supergravity $\Rightarrow$ $\mathrm{AdS}_5 \times \mathrm{S}_5$ (corrections $\mathcal{O}(1/g^2 N_c)$ calculable):



4d gauge field theory ⇔ 5d classical gravity concretely

$$\langle \exp\left[\int d^4x\, O(x)\phi(x,0)\right]\rangle_{\rm FT} = \exp\left\{-\int d^4x\ \int_0^{z_0} dz\, \mathcal{L}_{\rm class}[\phi(x,z)]\right\}$$

$$x^\mu = (t, x^1, x^2, x^3) \qquad\qquad x^M = (t, x^1, x^2, x^3, z)$$

LHS: All there is in the field theory, all operator expectation values:

$$\frac{\delta^2 {\rm LHS}}{\delta\phi(x,0)\delta\phi(y,0)} = \langle O(x)O(y)\rangle_{\rm FT}$$

RHS: Solve classical 5d gravity EOM for $\phi(x,z)$ with proper BC and compute the LHS.
Approximation works when the coupling of LHS is large, non-perturbative!

Key issue: holography

Dofs can match since number of dofs for gravity $\sim$ area, not volume.

Thermodynamics

Pressure of hot supersymmetric $\mathcal{N}=4$ matter for small couplings $\lambda \equiv g^2 N_c$ is, counting $2+6+7/8\times(4+4)=15\times$ color massless dofs:

$$p(T)=N_c^2\,15\frac{\pi^2}{90}T^4\left[1-\frac{3}{2\pi^2}\lambda+\frac{3+\sqrt{2}}{\pi^3}\lambda^{3/2}+a\lambda^2\log\lambda+b\lambda^2+c\lambda^{5/2}+d\lambda^3\log\lambda+...\right]$$

(Nieto computed a, b, c, Laine-Schröder d 2 years ago, unpublished)

No phase transition!

The result from AdS$_5\times$S$_5$ is

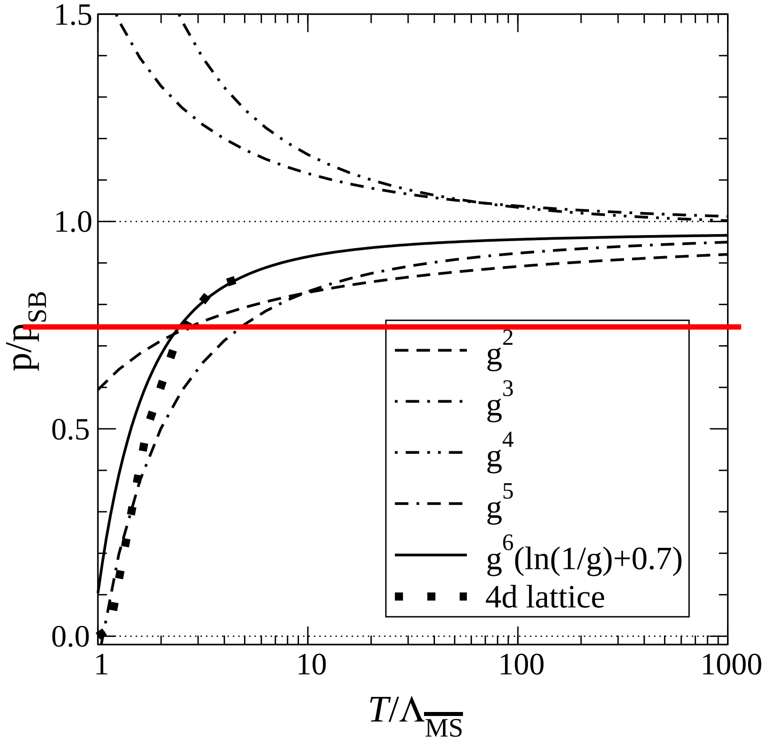
$$p(T)=\frac{\pi^2 N_c^2}{6}T^4\left[\frac{3}{4}+\frac{45\zeta(3)}{64\sqrt{2}}\frac{1}{\lambda^{3/2}}+...\right]$$

Interpolate nicely!

Experimental evidence for the $\frac{3}{4}$:

Pressure of QCD matter$/aT^4$

The famous factor 3/4



Points: Lattice Monte Carlo, Curves: Perturbation theory

Phase transition at $T_c \approx \Lambda_{\rm QCD}$ with large increase in n:o of dofs.

Argument: Near T_c the gauge system is necessarily strongly interacting and also $\alpha_s(T)$ nearly constant, conformally invariant. The 3/4 gives average behavior. Good fit!

How do you get the 3/4 from AdS$_5$?

According to an authority in string theory:

Pochinski, cosmicvariance.com/2006/12/07/: Physicists have found that some of the properties of this plasma are better modeled (via duality) as a tiny black hole in a space with extra dimensions than as the expected clump of elementary particles in the usual four dimensions of spacetime.

Here is this tiny (10^{-15}m) black hole ($z = 0$ is **boundary**, $z > 0$ is **bulk**)

$$\text{5d AdS with BH in bulk} = \frac{\mathcal{L}^2}{z^2}\left[-\left(1-\frac{z^4}{z_0^4}\right)dt^2 + d\mathbf{x}^2 + \frac{dz^2}{1-z^4/z_0^4}\right] \qquad T_{\text{Hawk}} = \frac{1}{\pi z_0}$$

$$\text{solves} \quad R_{MN} - \frac{1}{2}R g_{MN} = \frac{6}{\mathcal{L}^2} g_{MN} \sim T^{\text{bulk}}_{MN} \qquad \text{NO } T^{\text{brane}}_{MN}!$$

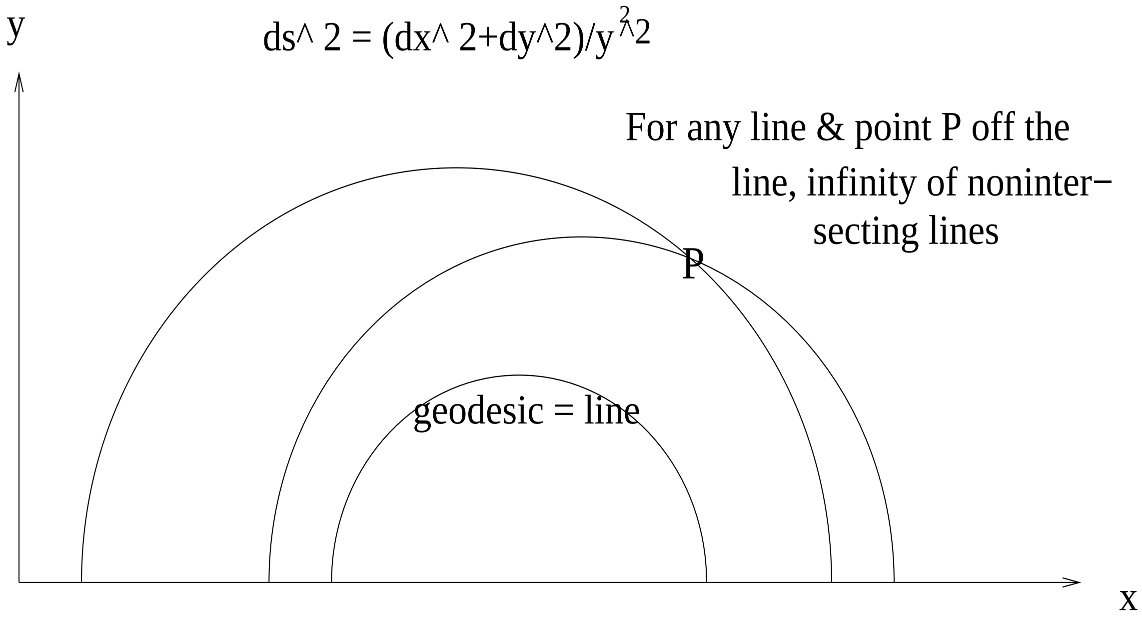
Compare

$$\text{4d Black Hole :} \quad ds^2 = -\left(1-\frac{r_s}{r}\right)dt^2 + \frac{1}{1-r_s/r}dr^2 + r^2 d\Omega^2$$

$$\text{Solves} \quad R_{\mu\nu} = 0, \quad T_{\text{Hawk}} = \frac{1}{4\pi r_s}$$

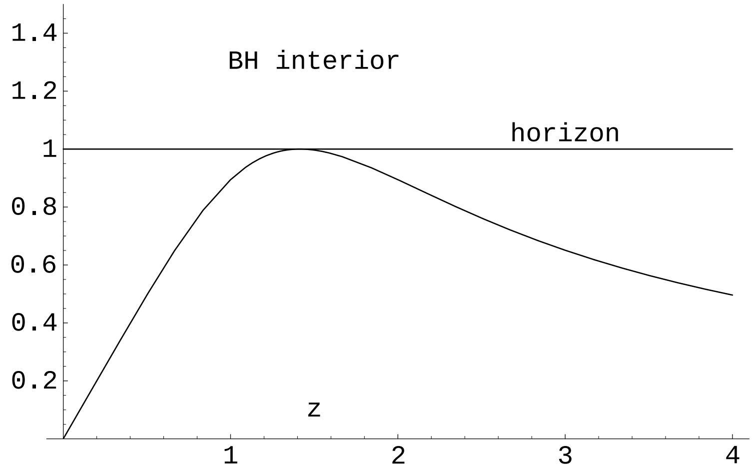
Poincare plane: model of non-Euclidian geometry

$$ds^2 = \frac{1}{y^2}(dx^2 + dy^2)$$



$$s = \int ds = \int dy \frac{1}{y}\sqrt{1 + x'(y)^2} \Rightarrow \frac{d}{dy}\left[\frac{x'(y)}{y\sqrt{1+x'^2}}\right] = 0 \Rightarrow (x-a)^2 + y^2 = c^2$$

The energy momentum tensor $T_{\mu\nu}$ can be read from the boundary value of the 5d metric, transformed so that $g_{55} = 1/z^2$ (transform $z^2 \longrightarrow z^2/(1+z^4/4z_0^4)$:



$$ds^2 = \frac{\mathcal{L}^2}{z^2}\left[-\frac{(1-z^4/(4z_0^4))^2}{1+z^4/(4z_0^4)}dt^2 + \left(1+\frac{z^4}{4z_0^4}\right)(dx_1^2+dx_2^2+dx_3^2)+dz^2\right]$$

$$\equiv \frac{\mathcal{L}^2}{z^2}\left\{\left[g_{\mu\nu}(x,0) + \underbrace{g^{(4)}_{\mu\nu}(x)}_{\sim T_{\mu\nu}} z^4 + \ldots\right] dx^\mu dx^\nu + dz^2\right\}$$

$$\Rightarrow T_{\mu\nu} \sim \mathrm{diag}(3,1,1,1)\frac{1}{z_0^4}$$

Static boundary energy momentum tensor

Magnitude is fixed:

$$\mathcal{L}^2 = g\alpha' = \frac{g}{2\pi T} \Rightarrow \frac{\mathcal{L}^3}{G_5} = \frac{2N_c^2}{\pi}$$

$$T_{\mu\nu} = \frac{\mathcal{L}^3}{4\pi G_5} g^{(4)}_{\mu\nu} = \begin{pmatrix} 3aT^4 & 0 & 0 & 0 \\ 0 & aT^4 & 0 & 0 \\ 0 & 0 & aT^4 & 0 \\ 0 & 0 & 0 & aT^4 \end{pmatrix} \qquad a = \frac{\pi^2 N_c^2}{8}$$

What about systems in expansion, their gravity duals = ?

We have solved the AdS_3 case (1+1d expansion; under study) and the 1+3d case if the boundary metric is cosmological FRW – not Minkowski!

Viscosity/entropy

Another celebrated result is

$$\frac{\eta}{s} = \frac{\hbar}{4\pi}\left[1 + \frac{135\zeta(3)}{16\sqrt{2}\lambda^{3/2}} + ...\right]$$

obtained by evaluating the correlator:

$$\eta = \lim_{\omega\to 0}\frac{1}{2\omega}\int dtd^3x\, e^{i\omega t}\langle T_{12}(x)T_{12}(0)\rangle \qquad \int d^4x\, T_1^2(x)g_2^1(x, z=0)$$

Air ($s = S/V \sim N/V \sim 1\mathrm{kg}/m_p/\mathrm{m}^3 \sim 10^{27}/\mathrm{m}^3$):

$$\frac{\eta}{s} \approx \frac{10^{-5}}{10^{27}} \gg \frac{10^{-35}}{4\pi}$$

Kinetic theory:

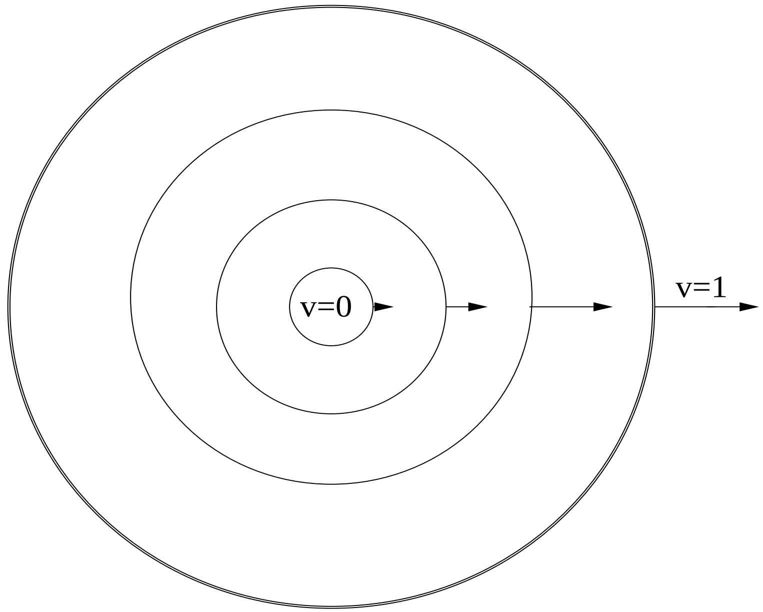
$$\frac{\eta}{s} \approx \frac{p\tau_c}{s} \approx \frac{T^4\tau_c}{T^3} = T\tau_c > \hbar$$

Experimental fact: QCD matter observed in heavy ion collisions at RHIC/BNL has T up to $5T_c$ (strongly coupled!!) and flows nearly ideally.

Spherical bang, $\eta_{\mu\nu} = (-1, 1, ..., 1)$: matter expands

$T_{\mu\nu} = (\epsilon + p)\frac{x_\mu x_\nu}{\tau^2} + p g_{\mu\nu}$

Fixed time t:



Similarity flow in $1+3$: $\mathbf{v} = \frac{\mathbf{x}}{t}\theta(t - |\mathbf{x}|)$, $u^\mu = (\gamma, \gamma\mathbf{v}) = \frac{x^\mu}{\tau}$, $\tau = \sqrt{t^2 - \mathbf{x}^2}$

$$\epsilon'(\tau) + \frac{3}{\tau}(\epsilon + p) = 0 \overset{p=\epsilon/3}{\Rightarrow} \epsilon(\tau) = \frac{\epsilon_0}{\tau^4} = \frac{\epsilon_0}{(t^2 - \mathbf{x}^2)^2}$$

Big bang, $g_{\mu\nu} = (-1, r(t), r(t), r(t))$: space expands

$$\epsilon'(t) + \frac{3\,\dot{r}(t)}{r(t)}(\epsilon + p) = 0 \qquad r(t) = \frac{t}{t_0} \text{ gives the above}$$

One solution in 1+3 dimensions: $r = r(t)$

$$ds^2 = \frac{\mathcal{L}^2}{z^2}[-a(t,z)dt^2 + b(t,z)d\mathbf{x}^2 + dz^2], \qquad R_{MN} + 4g_{MN} = 0$$

$$a(t,z) = \frac{\left[\left(1 - \frac{r''}{4r}z^2\right)^2 - \left(\frac{r''}{4r} - \frac{r'^2}{4r^2}\right)^2 z^4 - \frac{1}{4r^4 z_0^4} z^4\right]^2}{\left[\left(1 - \frac{r'^2}{4r^2}\, z^2\right)^2 + \frac{1}{4r^4 z_0^4} z^4\right]} \xrightarrow[r(t)=1]{} \frac{(1 - z^4/(4z_0^4))^2}{1 + z^4/(4z_0^4)}$$

$$b(t,z) = r^2 \left[\left(1 - \frac{r'^2}{4r^2}\, z^2\right)^2 + \frac{1}{4r^4 z_0^4} z^4\right] \xrightarrow[r(t)=1]{} 1 + \frac{z^4}{4z_0^4}$$

Boundary metric and $T_{\mu\nu}$: $\qquad g_{\mu\nu}(x,0) = (-1, r^2(t), r^2(t), r^2(t)) =$ flat FRW

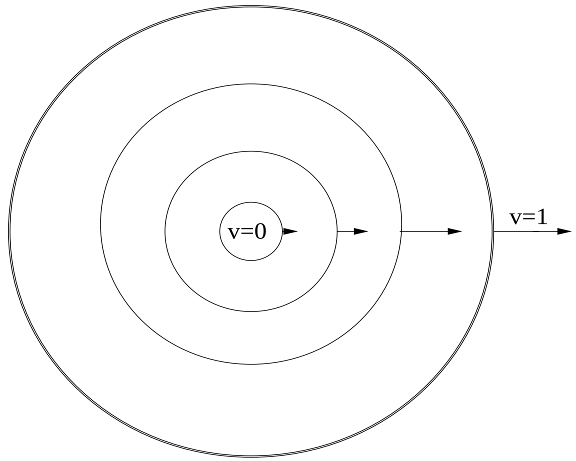
$$T_{tt} = \epsilon(t) = \frac{3N_c^2}{8\pi^2}\left(\frac{1}{z_0^4 r^4} + \frac{r'^4}{4r^4}\right) \sim \frac{T_0^4}{r^4(t)} + \frac{1}{t^4} \sim \underbrace{T^4(t)}_{\text{radiation}} + \underbrace{t^{-4}}_{\text{curvature}},$$

$$T_1^1(t) = \frac{1}{3}\,\epsilon(t) - \underbrace{\frac{N_c^2}{8\pi^2}\frac{r'^2 r''}{r^3}}_{\text{trace anomaly}/3}$$

Introducing $T_{\mu\nu}^{\rm brane}(z=\epsilon)$, brane gravity, $\Rightarrow$ Einstein's equations for $r(t)$.

$$\text{Choosing} \qquad r(t) = \frac{t}{t_0} \quad \Rightarrow \epsilon(t) = \frac{\epsilon_0}{t^4}$$

we have gravity dual of matter in the center of spherical bang:



Thermalisation condition: temperature can be defined for

$$t_0/z_0 = \pi T t_0 > 1 = \hbar$$

We are still missing a solution with flow with shear $\Rightarrow$ shear viscosity η.

Conclusions

- For hot equilibrium SYM systems gauge/gravity duality has made predictions (pressure, shear viscosity, quark energy loss) in qualitative agreement with experimental results on hot QCD matter
- For time dependent systems with flow, in only local thermal equilibrium, we do not (yet?) have the gravity dual for non-trivial configurations (1+1d or rest frame of 1+3d is very special)
- Brane cosmology is an example of what one can do in this set-up for a larger system: the whole universe