Memory effect, supertranslations and asymptotic symmetries at null infinity

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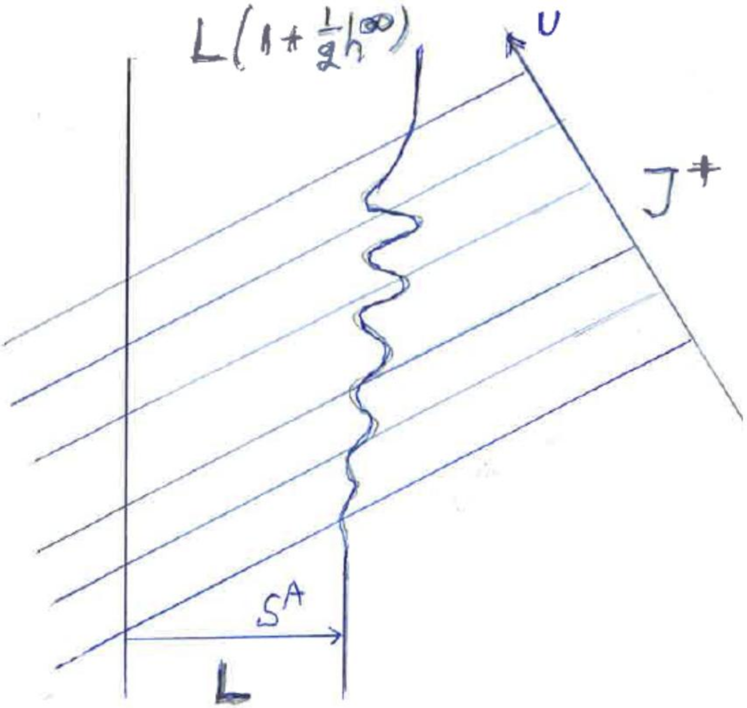
Project (Jokela-Kajantie-Sarkkinen): How do you measure supertranslations which in Hawking-Perry-Strominger 1601.00921 are said to be physically important for black hole physics?

Strominger Lectures 1703.05448

Memory effect: a burst of gravitational waves causes a permanent relative displacement of free falling detectors



$u = t\text{-}r$ = retarded time

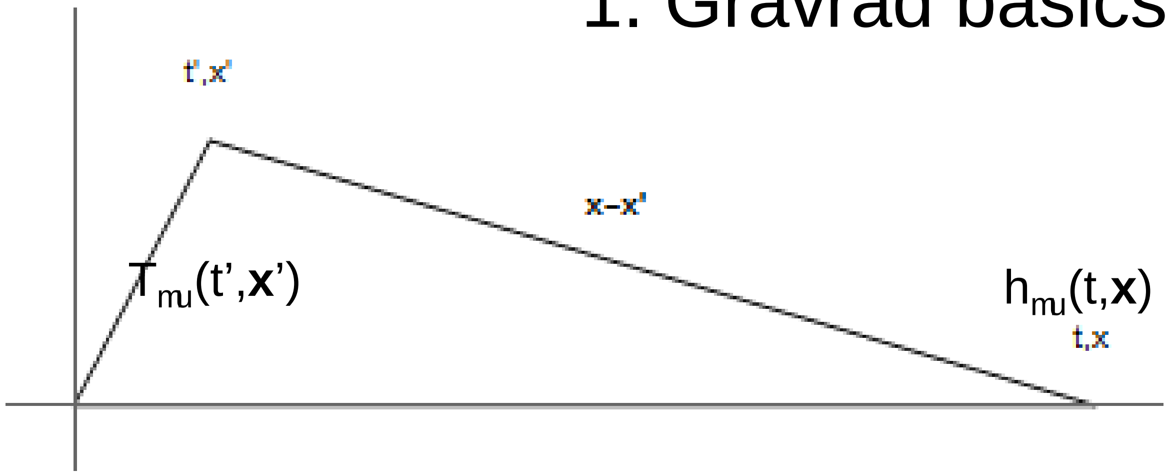
$$\theta^A = (\theta, \phi) = (z, \bar{z})$$

$$\partial_u^2 S^A = R^A{}_{uuB}\, S^B$$

Gravrad ends up in null infinity I⁺. Memory effect is related to the supertranslation symmetry of null infinity, adding Lorentz you get BMS group.

Bondi-Metzner, Sachs 1960-62

1. Gravrad basics:



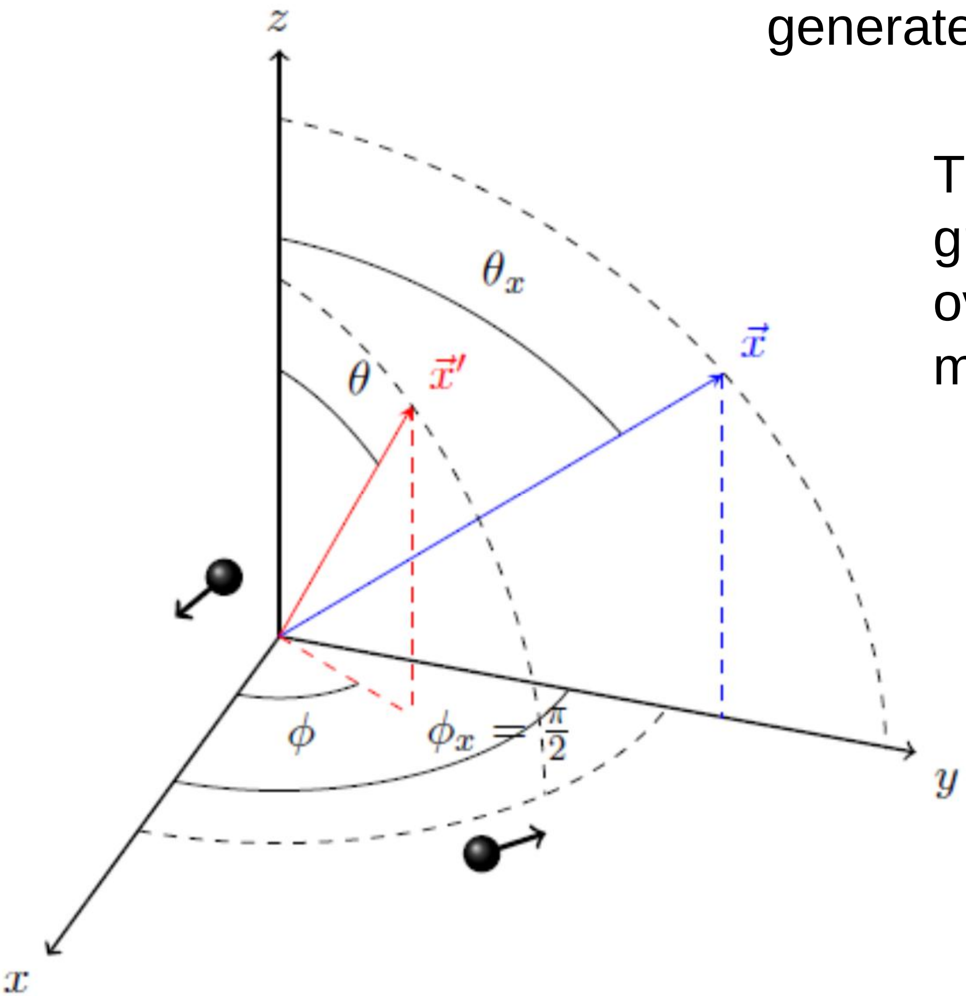
$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + \mathcal{O}(h^2) \qquad (\partial_t^2 - \nabla^2) h_{\mu\nu}(t, \mathbf{x}) = 16\pi G T_{\mu\nu}$$

$$h_{\mu\nu}(t, \mathbf{x}) = 4G \int \frac{d^3\mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|} T_{\mu\nu}(t - |\mathbf{x} - \mathbf{x}'|, \mathbf{x}') = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_{11} & h_{12} & 0 \\ 0 & h_{12} & -h_{11} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

*TT=Transverse traceless gauge, **x**=(0,0,x)*

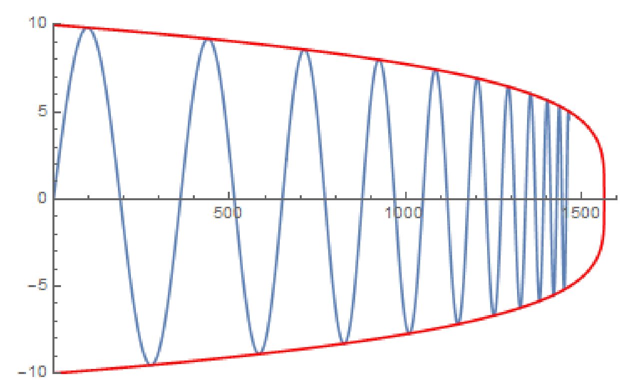
$$t_{03} = \frac{c^2}{16\pi G} (\dot{h}_{11}^2 + \dot{h}_{12}^2)$$

Merging BH binary: radiation from radiation



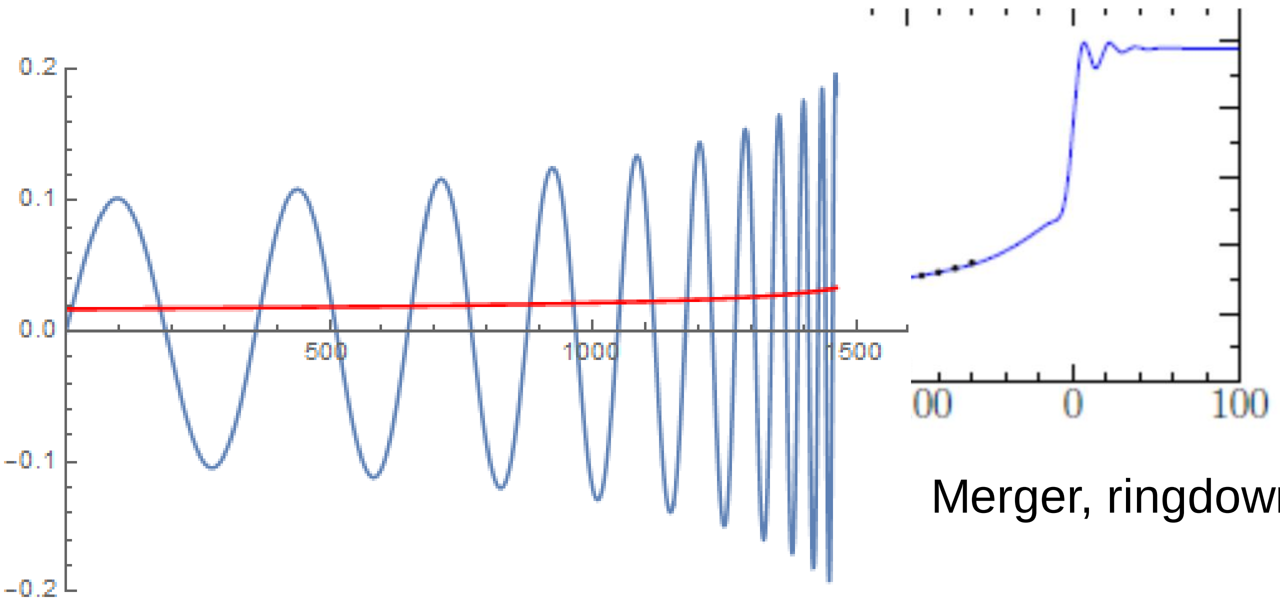
Binary M+M on circular orbit, radius R, generates gravrad at **x**', R decays

The Tij of this gravrad sources further gravrad observed at **x**. Integrating over lifetime of binary gives rise to memory effect at **x**



$$h_{\theta\theta}^{TT}(t,\mathbf{x}) = -h_{\phi\phi}^{TT} = \frac{GE_{\text{kin}}}{c^4 r} \cdot \frac{1}{24}(17 + \cos^2\theta_x)\sin^2\theta_x$$ + oscill

Memory, like Newtonian 1/r-potential



Merger, ringdown numerically

Inspiral computed analytically

How do you observe the red curve from the sum?

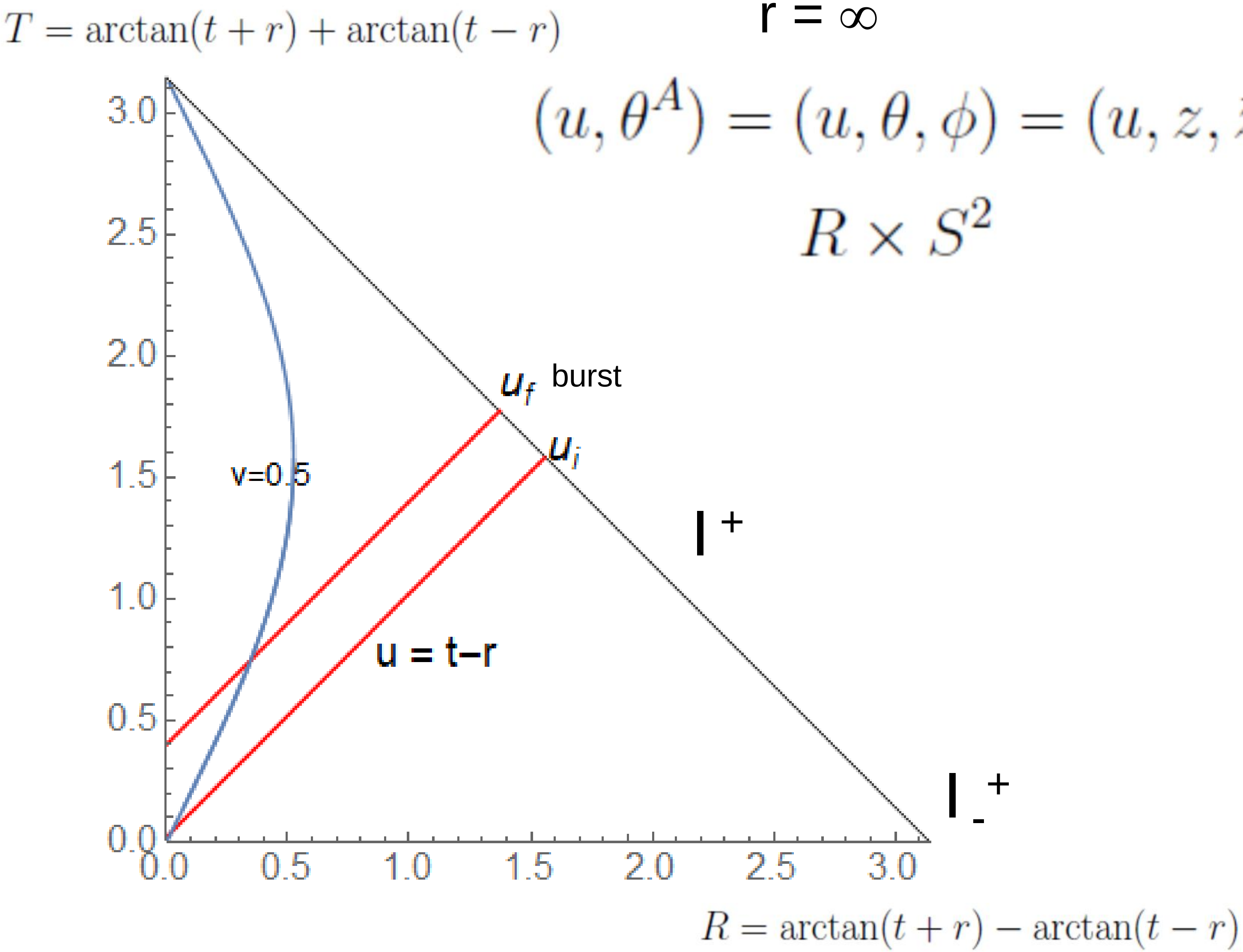
2. Null infinity: where photons and gravitons end up

Gluons? Dress to massive glueballs

$r = \infty$

$$(u, \theta^A) = (u, \theta, \phi) = (u, z, \bar{z})$$

$$R \times S^2$$



Celestial sphere S^2

Coordinates q^A, A=1,2

$$\theta^A = (\theta, \phi) = (z, \bar{z}) \qquad z = \frac{1}{\tan\frac{1}{2}\theta} e^{i\phi}$$

$$ds^2 = h_{AB}\, d\theta^A d\theta^B = d\theta^2 + \sin^2\theta d\phi^2 = \frac{4}{(1+z\bar{z})^2} dz d\bar{z}$$

$$h_{AB} = \begin{pmatrix} 1 & 0 \\ 0 & \sin^2\theta \end{pmatrix} \quad \begin{pmatrix} 0 & \gamma \\ \gamma & 0 \end{pmatrix}$$

Covariant derivative D_A, $\quad D^2 = D_A D^A = -\mathbf{L}^2$

3. Photons at null infty: U(1) memory is a kick

Bieri-Garfinkle 1307.5098

Sarkkinen helda.helsinki.fi/handle/10138/231490

Gauss $\nabla \cdot \mathbf{E} = \rho \Rightarrow$

$$-\partial_u E_r + D_A E^A = \rho(u, \theta^A)$$

Time integral over pulse gives a kick:

$$\ddot{x} = eE \quad \Rightarrow \quad \Delta\dot{x} = e\int dt\, E$$

U(1) memory = kick $eM^B(q^A)$ can be solved from

$$D_B \underbrace{\int_{u_i}^{u_f} du\, E^B}_{=M^B(\theta^A)} = \underbrace{\int_{u_i}^{u_f} du\, \rho(u, \theta^A)}_{\text{charge flux in }\theta^A}$$

Need massless charged ples!

U(1) covariant discussion, only **E,B**, no potentials A_m

What happens if we use potentials A_m and fix the gauge?

Choose A_r=0 everywhere and A_u=0 at r=infty. A_B remains with gauge invariance $A_B \rightarrow A_B + \partial_B \chi$

$$M_B = \int du E_B = \int du F_{uB} = \int du\, \partial_u A_B = A_B(u_f) - A_B(u_i)$$

~ super-translation!

A physical measurement fixes a gauge transformation.

Proclamation: these ”large” (=nonzero!) gauge trafos at null infinity are not redundancies but form a new symmetry of ED vacuum at null infty.
I see no new physics, only gauge choice

Abhorrent to a lattice QCD person

Symmetry built in a trivially cons current $J^\mu = \nabla_\nu (F^{\nu\mu} \chi(\theta^A))$ $\quad Q_\chi = \int d\Omega\, r^2 F^{r0} \chi$

Gives a way of rederiving soft photon emission amplitudes.

Past null to future null infty.

4. Gravitons at null infinity

The memory effect can be computed in analogy with U(1) case by replacing Lorentz force by the geodetic dev eq

$$\partial_u^2 S^A = R^A{}_{uuB}\, S^B$$

Bieri-Garfinkle 1312.6871

and Maxwell by (C=Weyl)

$$\nabla^\mu C_{\mu\nu\alpha\beta} = 4\pi G\, \left(\nabla_\alpha T_{\nu\beta} - \nabla_\beta T_{\nu\alpha}\right)$$

Entirely covariant, no ”gauge choice”. Supertranslations never mentioned.

Where is the symmetry group at null infinity and what are its gauge transformations?

5. Gravity near spatial infinity

Vacuum $T_{mn}=0$ $R_{mn}=0$ Schwarzschild in u,r,q^A

$$ds^2 = -(1-\frac{2M}{r})du^2 - 2du\,dr + r^2 h_{AB} d\theta^A d\theta^B$$

$$d\theta^2 + \sin^2\theta d\phi^2 = \frac{4}{(1+z\bar{z})^2} dz d\bar{z},$$

$R_{mn}=0$ but R_{mnab} is nonzero = Weyl tensor C_{mnab}

DoF counting: 10 comps of g_{mn} are determined by 10 Einstein eqs $R_{mn}=8pGT_{mn}$
(6 dynamical, 4 gauge) The extra 10 comps of Riemann are hidden in Weyl

6. Gravity near null infinity

r to infty at constant u = t-r

How do you modify Minkowski to exhibit effects of gravrad?

u r q^A

$$g_{\mu\nu} = \begin{pmatrix} -1 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & h_{AB} \end{pmatrix}, \quad g^{\mu\nu} = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & h^{AB} \end{pmatrix}$$

Bondi gauge: choose 4 gauge conds $g_{rr} = g_{rA} = \partial_r \det h_{AB} = 0$

Sachs, PR 128, 2851 (1962)

$$g_{\mu\nu} = \begin{pmatrix} g_{uu} & g_{ur} & g_{uA} \\ g_{ur} & 0 & 0 \\ g_{uA} & 0 & r^2 h_{AB} \end{pmatrix}$$

Zeroes imply that null geodesics are u = const, q^A=const

Complete gauge fixing, only 6 dynamical dofs remain!

Key point: approach to flatness when r increases:

With order 1/r corrections:

$$
\begin{array}{cccc} u & r & z & \bar{z} \end{array}
$$

$$
g_{\mu\nu} = \begin{pmatrix} -1 + \frac{2Gm(u,z,\bar{z})}{r} & -1 & \partial^z c(u,z,\bar{z}) & \partial^{\bar{z}} \bar{c}(u,z,\bar{z}) \\ -1 & 0 & 0 & 0 \\ \partial^z c(u,z,\bar{z}) & 0 & rc(u,z,\bar{z}) & r^2\gamma \\ \partial^{\bar{z}} \bar{c}(u,z,\bar{z}) & 0 & r^2\gamma & r\bar{c}(u,z,\bar{z}) \end{pmatrix}
$$

Gravrad is built in the next-to-leading functions $m(u, q^A)$ and symmetric traceless tensor $C_{AB}(u, q^A)$ on S^2

$$
r^2 h_{AB} + rC_{AB} = \begin{pmatrix} rc & r^2\gamma \\ r^2\gamma & r\bar{c} \end{pmatrix} \qquad \gamma = \frac{2}{(1+z\bar{z})^2}
$$

$$
C_{AB} = (D_A D_B - \tfrac{1}{2} D^2 h_{AB})C = \begin{pmatrix} C_{zz} & 0 \\ 0 & C_{\bar{z}\bar{z}} \end{pmatrix} = \begin{pmatrix} D_z^2 C & 0 \\ 0 & D_{\bar{z}}^2 C \end{pmatrix}
$$

$$
\psi_i = \partial_i s
$$

So far only gauge fixing! But aiming at correct radiation zone physics

The analogue of U(1) invariance for ED
is the set of diffeomorphisms leaving this invariant=the symmetry group at null infinity

$$x^\mu \to x'^\mu = x^\mu + \xi^\mu(x)$$

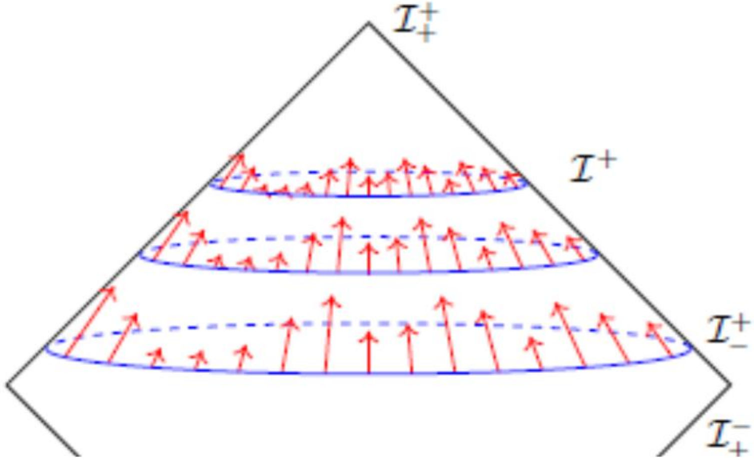
BMS group

$$(u, r, \theta^B) \to \underbrace{(u + \alpha(\theta^A))}_{\text{supertranslation}}, \, r + \tfrac{1}{2}D^2\alpha, \, \theta^B - \tfrac{1}{r}\, D^B\alpha)$$

$$\mathcal{L}_\xi g_{rr} = 2 g_{ur}\partial_r \xi^u = 0$$ etc

u coordinate is shifted in each direction on S^2 by a direction dependent amount: we are infinitely far and clocks cannot be synchronised (?)

Energy conserved at each q^A



Strominger 1703.05448

Also m and C_{AB} are transformed under supertranslations:

$$C_{AB} \to C_{AB} - 2(D_A D_B - \tfrac{1}{2}h_{AB}D^2)\alpha(\theta^A) + ..$$

Gravity memory after gauge fixing

Take geodetic deviation equation

$$r^2\,\partial_u^2 S_A = R_{AuuB}\,S^B$$

compute for the Bondi gauge   Simple!

$$R_{uAuB} = -\tfrac{1}{2} r\,\partial_u^2 C_{AB} + \mathcal{O}(r^0)$$

integrate $\partial_u^2 S_A = \frac{1}{2r}\partial_u^2 C_{AB}\,S^B$ over $u_i<u<u_f$

and get the displacement memory $\Delta S_A = \frac{1}{2r}\Delta C_{AB}\,S_i^B$

Physical measurement has fixed the ”large” gauge trafo, supertranslation $a(q^A)$

Crucial difference: U(1) holds in the bulk, BMS on the boundary

7. BMS group, symmetry group of null infty

Poincare: semidirect product of Lorentz and translations

$$(\Lambda_1, a_1)(\Lambda_2, a_2) = (\Lambda_1\Lambda_2, \Lambda_1 a_2 + a_1)$$

Wigner representations: massive, massless particles with discrete spins

Banerjee 1801.10171: little group of null momentum direction!

BMS group: semidirect product of Lorentz=SL(2,C) and supertranslations

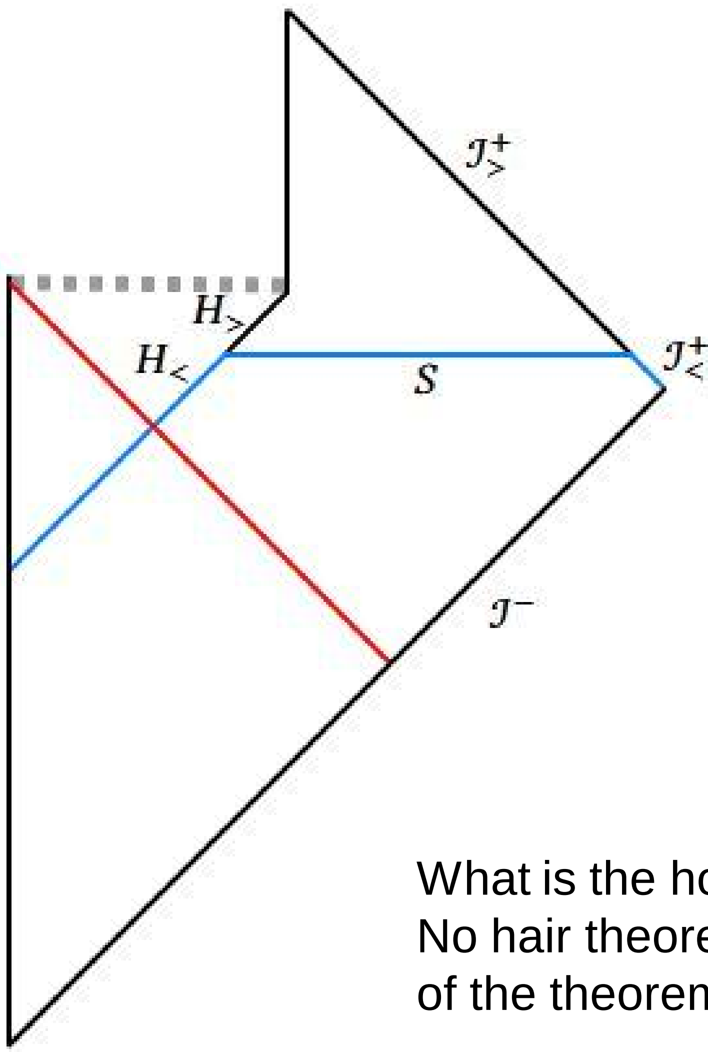
In $\Lambda_1 \alpha_2(z_2, \bar{z}_2)$ L_1 also affects z_2

Reps have been worked out but they do not have the interpretation of particles in the bulk since this is an asymptotic boundary symmetry

Pretty detail: 4 ordinary translations are ell=0,1 supertranslations ~ Y_l^m

8. From null infinity to black holes

Hawking-Perry-Strominger
1601.0092, 1611.09175
Kolekar-Louko
1703.10619, 1709.07355



Idea:

I+ and H are both null surfaces
R x S^2 moving with c

I+ has an asympt BMS symmetry
incl supertranslations

Could one extend the same symm
to H and use sutras and its
charges to characterise BHs?

What is the horizon measurement corresp to memory?
No hair theorem tells no such exists – unless conditions
of the theorem are violated.

Conclusions

Memory effect as such is straightforward to analyse both for ED and GR

In ED it can be analysed in terms of U(1) gauge symmetry and the magnitude can be related to a gauge transformation. No breaking of gauge invariance.

In GR it can also be related to a carefully defined symmetry of null infinity and its symmetry transformations: Lorentz times supertranslations. A supertranslation is a ”gauge transformation” and its magnitude can be determined by observing the memory effect

I find it hard to believe that this could have anything to do with understanding where info of what you threw to the BH is after the BH has disappeared

Leftovers

Memory1: $\mathbf{v} = 0 \to \mathbf{v}$ (of galaxy, graviton, whatever)

Weinberg, Gravitation 10.4 1972

$$h_{ij}(t,\mathbf{x}) = 4G \int \frac{d^3x'}{|\mathbf{x}-\mathbf{x}'|} \frac{p_j p_j}{E} \delta^3(\mathbf{x}' - \mathbf{v}(t - |\mathbf{x}-\mathbf{x}'|))$$

$$= 4G \frac{p_i p_j}{E} \frac{1}{|\mathbf{x}| - \mathbf{v}\cdot\mathbf{x}} = \frac{4GE}{r(1-v_z)} \begin{pmatrix} \frac{1}{2}(v_x^2 - v_y^2) & v_x v_y & 0 \\ v_x v_y & -\frac{1}{2}(v_x^2 - v_y^2) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

TT projection

Magnitude: $$\frac{r_{\text{Schw}}}{r} \gamma\, v^2 \sim 10^{3-5-16-6} \sim 10^{-24} \ldots.$$

Soft graviton emission amplitude from an outg graviton line of mom x:

$$h_{ij} = 4G \frac{p_i\, p_j}{p \cdot x}$$

Gravrad at the point x' =(q,f ,r):

Keplerian!

$$h^{TT} = \frac{r_s^2}{2rR}\begin{pmatrix} -\frac{1}{2}\left(1+\cos^2\theta\right)\cos 2(\Omega u-\phi) & \cos\theta\sin 2(\Omega u-\phi) & 0 \\ \cos\theta\sin 2(\Omega u-\phi) & \frac{1}{2}\left(1+\cos^2\theta\right)\cos 2(\Omega u-\phi) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$r_s = \frac{2GM}{c^2} \to 2M$$

has an energy flow

$$\frac{dL}{d\Omega} = \frac{1}{16\pi}\frac{r_s^5}{(2R)^5}\left[\tfrac{1}{2} + 3\cos^2\theta + \tfrac{1}{2}\cos^4\theta - \tfrac{1}{2}\cos(4\Omega u - 4\phi)\right].$$

$$L = \int d\Omega \frac{dL}{d\Omega} = \frac{2}{5}\frac{M^5}{R(t)^5} = -\frac{dE}{dt} \to \frac{dR}{dt} = -\frac{8}{5}\frac{M^3}{R^3}$$

which sources a new grav wave

$$h_{ij}(t,\mathbf{x}) = 4G\int d^4x'\,\delta(t-t'-|\mathbf{x}-\mathbf{x}'|)\frac{1}{|\mathbf{x}-\mathbf{x}'|}\frac{dL}{r'^2 d\Omega'}n_i' n_j'.$$

$T_{ij}(t',\mathbf{x}')$

Conserved symm
current

$$J^{\mu} = \nabla_{\nu}(F^{\nu\mu}\chi(\theta^{A}))$$

$$\nabla_{\mu}J^{\mu} = 0$$

With associated
charge

$$Q_{\chi} = \int d\Omega\, r^{2} F^{r0}\chi$$

One claims gauge inv of vacuum is broken -> Goldstone bosons

I see no breaking of U(1) gauge invariance, only gauge choice

Fundamental eq of gravrad: Einstein, R_{uu} = 0:

$$m'(u) = -\frac{1}{8G}(\partial_u C_{AB})(\partial_u C^{AB}) + \frac{1}{8G} D^2(D^2+2)\partial_u C$$

Change in m(u)

gravrad flux at I$^+$

$$T^{\rm rad}_{uu} = \frac{1}{r^2\, 32\pi G}\,\partial_u C_{AB}\,\partial_u C^{AB}$$

and a change in C which after pulse has passed adds up to a change in C_{AB}:

Integrating over pulse: magnitude of $D^2(D^2+2)DC$ is computed from Dm (ordinary gravrad) and flux of gravrad to null infty

C labels physically inequivalent (different memory) vacua at null infty

$D^2(D^2+2)=-l(l+1)(l+2)(l-1)$

so you cannot determine the $Y_1^m(q,f)$ modes these are ordinary translations!!

In ED U(1) a kick was measured:

(*) $$\Delta \dot{x} = e \int du E \equiv e M$$

and related to a gauge transformation:

(**) $$M_B = \int du E_B = \int du F_{uB} = \int du \, \partial_u A_B = A_B(u_f) - A_B(u_i)$$

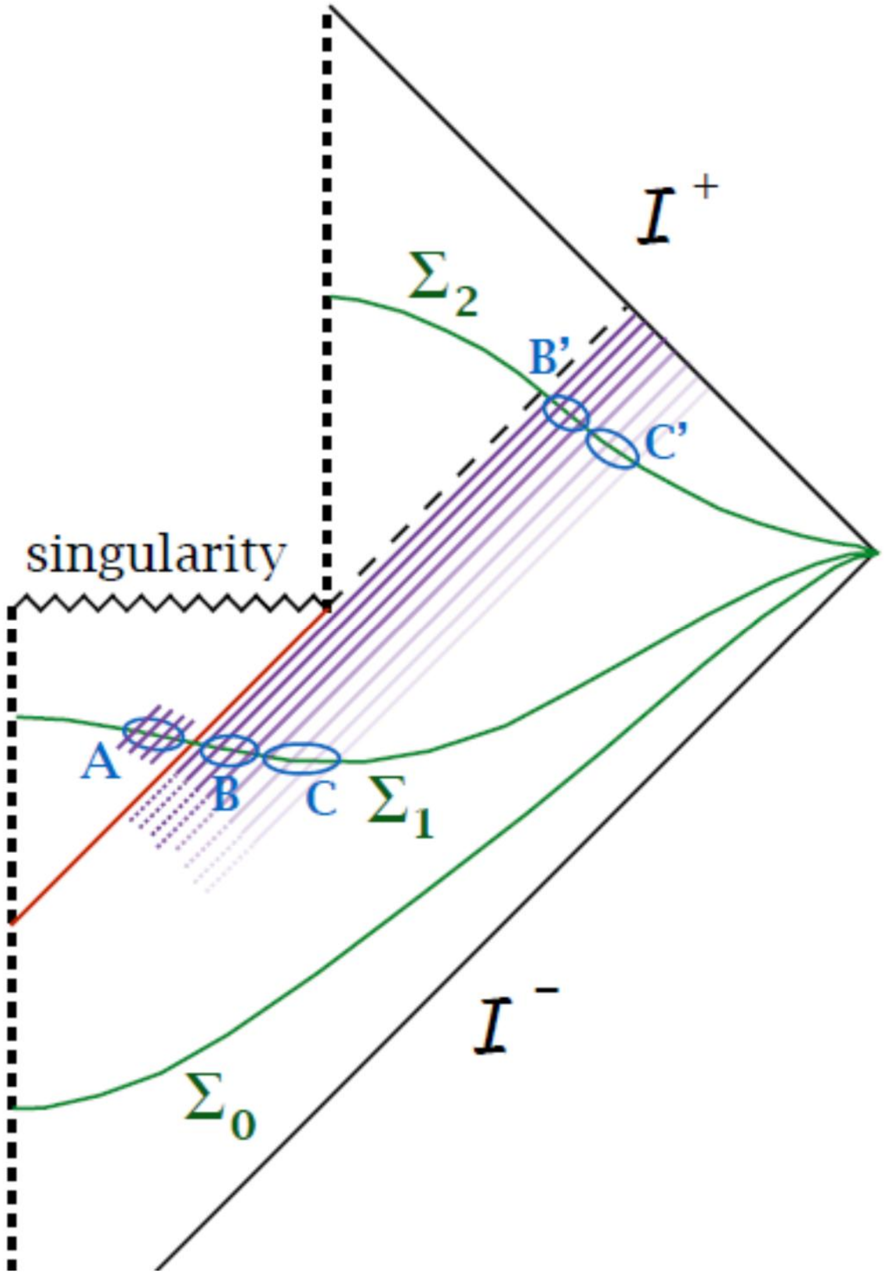
Now we have a "gauge transformation", supertranslation. It is measured via the memory effect

(^) $$h_{AB} \Delta S^B = \frac{1}{2r} \Delta C_{AB} \, S_i^B \qquad \partial_u^2 S^A = R^A{}_{uuB} \, S^B$$

(**) $$C_{AB} \to C_{AB} - 2(D_A D_B - \tfrac{1}{2} h_{AB} D^2)\alpha(z, \bar{z}) + \alpha \partial_u C_{AB}$$

+ analogue of Gauss = uu component of Einstein

$$\Delta C_{AB} = (D_A D_B - \tfrac{1}{2} h_{AB} D^2) \Delta C$$

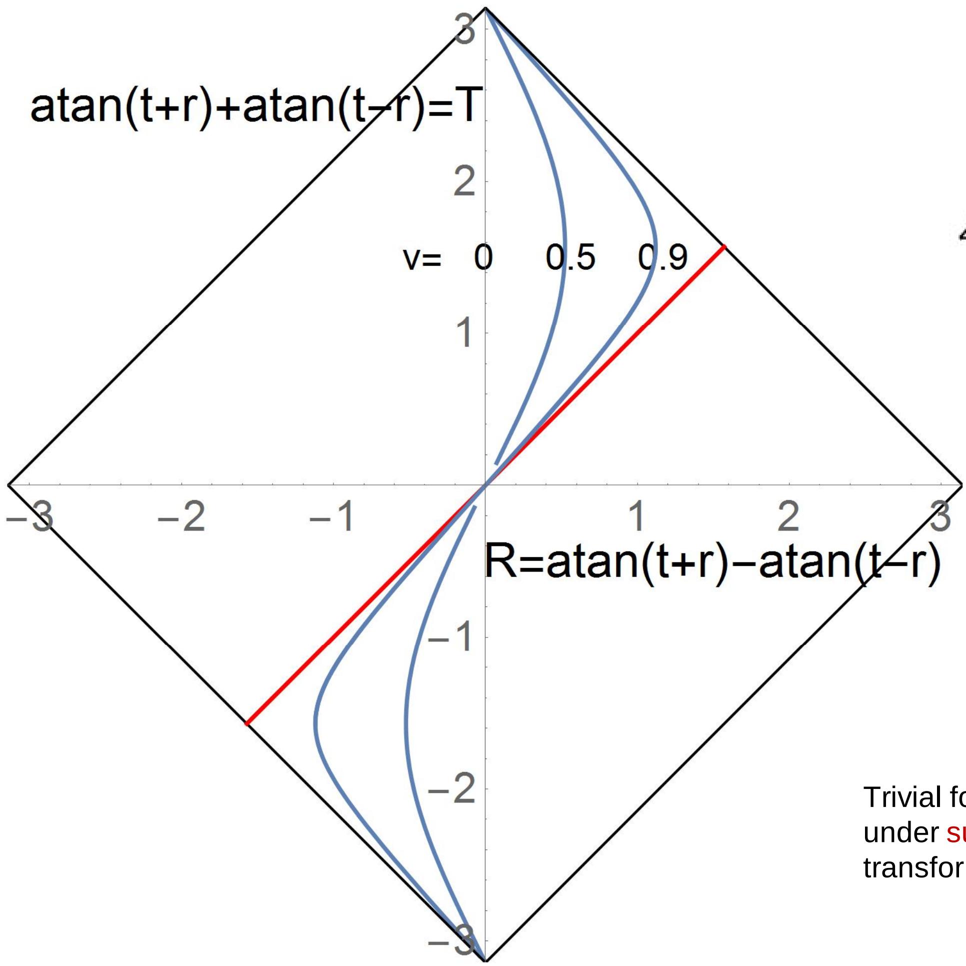


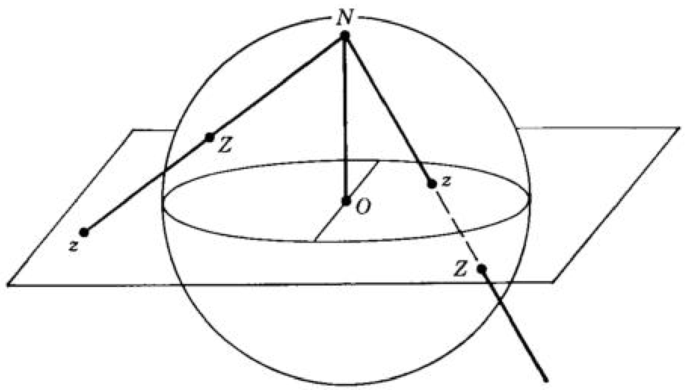
$\mathcal{I}^+$
Σ_2
B'
C'
singularity
A
B
C
Σ_1
$\mathcal{I}^-$
Σ_0

Null infinity

$$-dt^2 + dr^2 + r^2 d\Omega^2$$

$$ds^2 = \frac{1}{(\cos T + \cos R)^2}\left(-dT^2 + dR^2 + \sin^2 R\, d\Omega^2\right)$$





Trivial for U(1). In gravity there will be a non-trivial symmetry under supertranslations u -> u+f(q^A) where also the "gauge transformation" f can be fixed by measuring memory.