Phases and phase transitions of QCD with large N_f

$N_c=3>>1$ $=0,3,6,9,..$

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Helsinki 10 February 2015

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QCD thermodynamics

$$e^{p(T,\mu;m_q)\frac{V}{T}} =$$

Maximum p dominates!

$$\int \mathcal{D}A\mathcal{D}q \; e^{-\int^{1/T} d\tau d^3x \left[\frac{1}{g^2}F^2 + \bar{q}(\partial + A)q + m_q\bar{q}q + \mu q^\dagger q\right]}$$

Color N_c, Flavor N_f, QCD scale Λ_{QCD}

$m_q = 0$ to have chiral symmetry

$\mu=0$ in this talk

$QCD_{physical}$: m_u, m_d, m_s, m_c, m_b, m_t

Chiral symmetry:

Action $\int d^4x \left[\frac{1}{g^2}F^2 + \bar{q}(\partial + A)q\right]$

has $\mathrm{SU}_L(N_f)\times \mathrm{SU}_R(N_f)$ symmetry, while solutions may only have $\mathrm{SU}_V(N_f)$ symmetry

⇒ $N_f^2 - 1 \approx N_f^2$ massless Goldstone bosons, "pions"

(in SM EW sector N_f=2 and 3 gb's are eaten by W and Z)

Order parameter: $\langle \bar{q}q \rangle$

Chiral phase transition at some $T_c \approx \Lambda_{\mathrm{QCD}} \approx m_{\mathrm{min}}/5$ $\quad m_\pi = 0!$

No order parameter for confinement-deconfinement, unless N_f=0

Values of N_f:

$$\mu\frac{\partial g(\mu)}{\partial \mu} = -\frac{1}{16\pi^2}\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right)g^3 + ...$$

As freedom lost if $x_f \equiv \frac{N_f}{N_c} > \frac{11}{2}$

Massless dofs at T >>T_c $2N_c^2 + \frac{7}{2}N_cN_f$

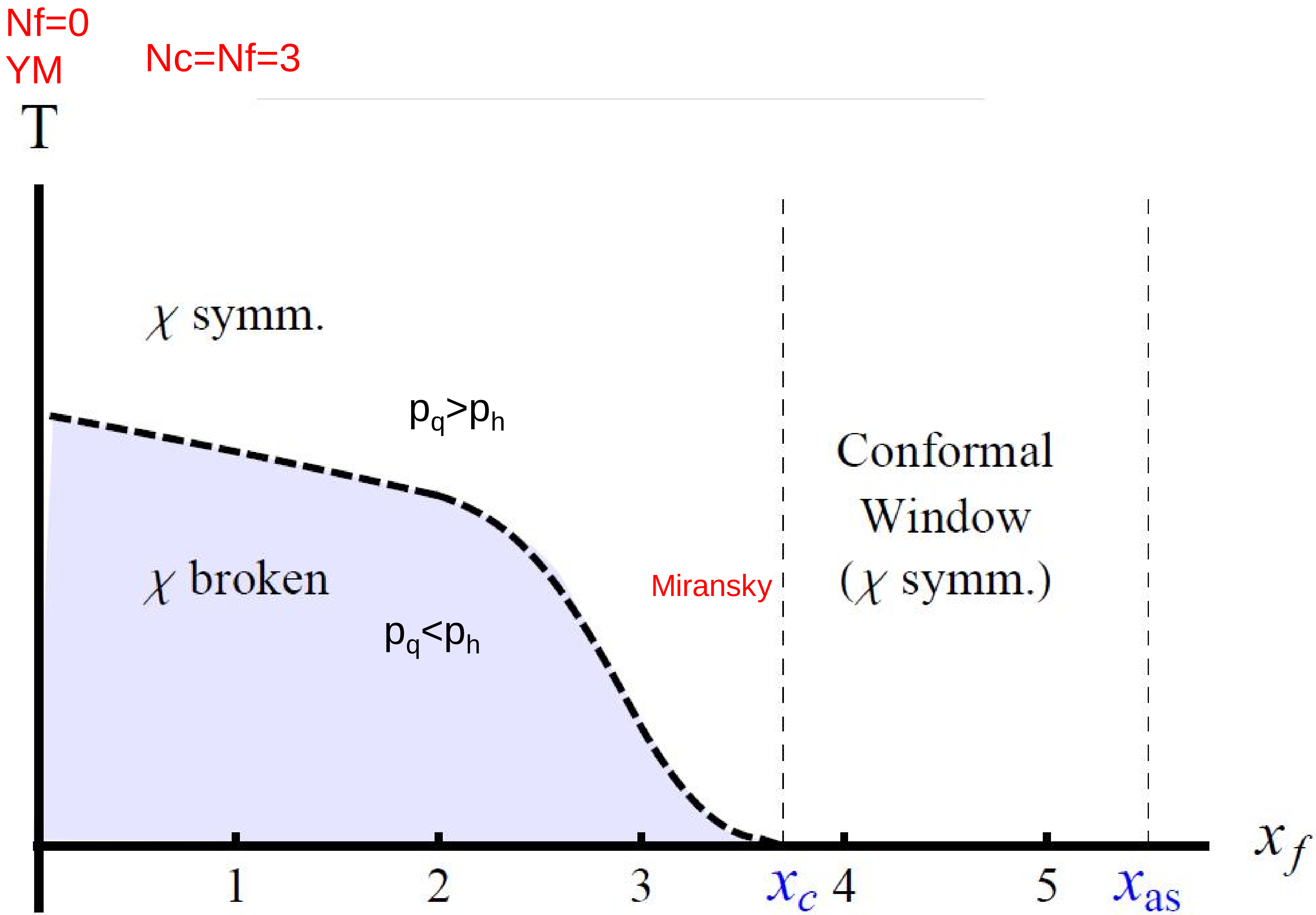
Massless dofs at T=0 N_f^2

Equal at x_f=4

Conformal domain: $x_c \approx 4 < x_f < 5.5$

$T_c(N_f)$:

1210.4516



Universality + ε expansion suggest 1st order!

We want p(T), all T

$$s(T) = p'(T) \qquad \epsilon(T) = Ts(T) - p(T)$$

$$T\frac{d}{dT}\frac{p(T)}{T^4} = \frac{\epsilon - 3p}{T^4}$$

interaction measure

$$\left(T\frac{d}{dT}\right)^2 \frac{p(T)}{T^4} = \frac{(c_s^{-2} - 3)(\epsilon + p) - 4(\epsilon - 3p)}{T^4}$$

sound velocity

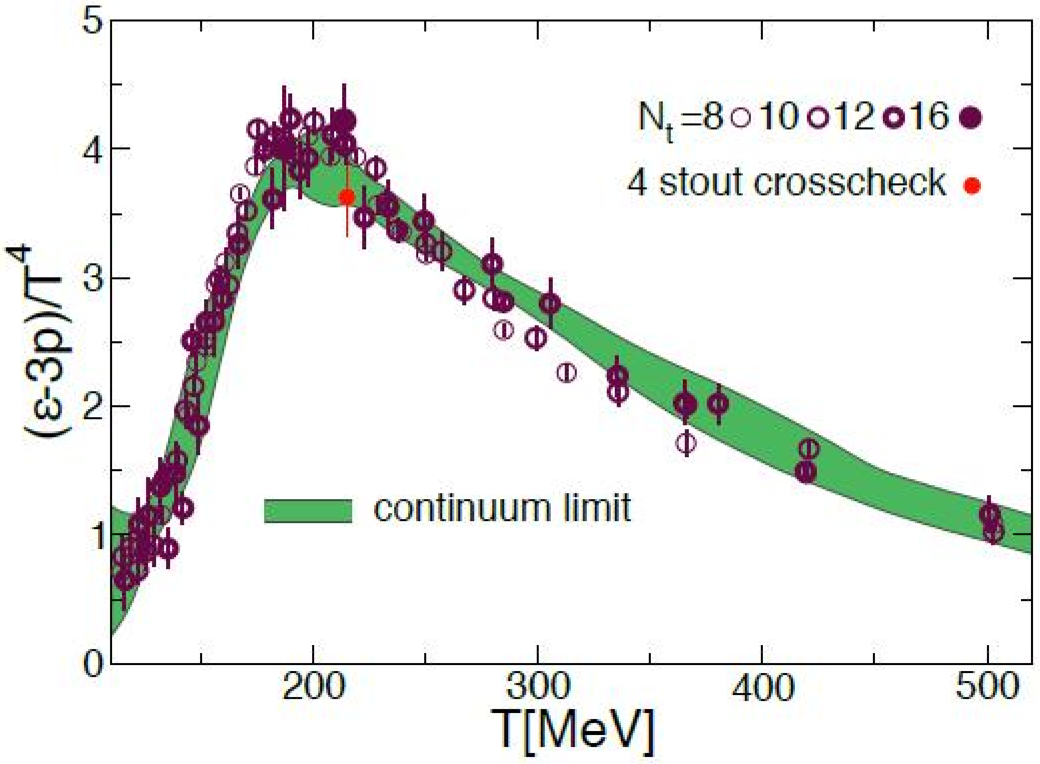
Massless particles, conformal

$$\epsilon = 3p \sim T^4, \qquad c_s^2 = \frac{1}{3}$$

Lattice data for interaction measure:

2013: QCD_{uds}

Wuppertal-Budapest 1309.5258



Hadron gas

Quark-gluon plasma

Here computed from one theory: QCD

How do you connect if you have two different appros of QCD?

Interaction measure in cosmology:

$$R = g^{\mu\nu} R_{\mu\nu} = -8\pi G T^{\mu}_{\mu} = 8\pi G(\epsilon - 3p) = 8\pi G(\epsilon_{\rm QCD} - 3p_{\rm QCD} + \epsilon_{\rm DM} + 4\epsilon_{\Lambda})$$

At T_{QCD} the QCD sector dominates over DM the curvature of the Universe by the factor

$$\frac{T_{\rm QCD}}{T_0} \cdot \frac{\rho_{\rm rad,0}}{\rho_c} \approx 10^{12} \cdot 10^{-5}$$

$$R \sim \frac{1}{t^2} \frac{\epsilon_{\rm QCD} - 3p_{\rm QCD}}{T^4}$$

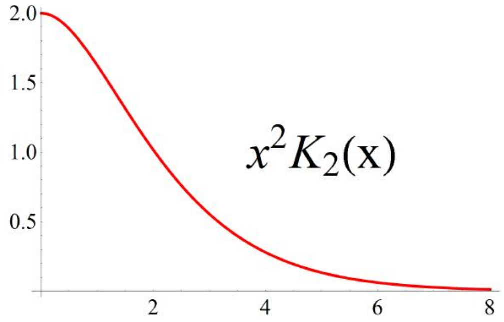
t ~ 10km

Similarly for EW etc

Hadron gas

Sum

$$\frac{p_h(T,m)}{T^4} = \frac{m^2}{2\pi^2 T^2} K_2\left(\frac{m}{T}\right)$$



over all hadrons in Particle Data Tables

m=0:

$$p_{\rm gb} = N_f^2 \frac{\pi^2}{90} T^4$$
$$= N_c^2 T^4 \cdot x_f^2 \frac{\pi^2}{90}$$

or integrate over the Hagedorn spectrum

$$\rho(m,b,a,m_0) = \delta(m) + \frac{\rho_0}{m_0}\left(\frac{m}{m_0}\right)^a e^{bm}\theta(m-m_0)$$

$$\int_{m_0}^{\infty} dm\, \exp[b\,m - \frac{m}{T}]\, m^a \ldots \quad \longrightarrow \quad T \leq T_H \equiv \frac{1}{b}$$

$m_{\rm min}$ from holography: $m_0 \equiv m_{\rm min} = 0.707\Lambda, \quad x_f = 2$

Plasma phase from holography, gauge/gravity duality

$$"\pi = 3"$$

$$S = \frac{1}{16\pi G_5}\int d^5x\,\sqrt{-g}\left[R - \tfrac{4}{3}(\partial_z\phi)^2 + V_g(\phi) - V_f(\phi,\tau)\sqrt{1+(\partial_z\tau)^2}\right]$$

Järvinen-Kiritsis 1112.1261

$$ds^2 = b^2(z)\left[-f(z)dt^2 + d\mathbf{x}^2 + \frac{dz^2}{f(z)}\right] \quad \lambda(z) \quad \tau(z)$$

$$f(z_h) = 0, \quad -f'(z_h) = 4\pi T, \quad s = \frac{A}{4G_5} = \frac{b^3(z_h)}{4G_5}$$

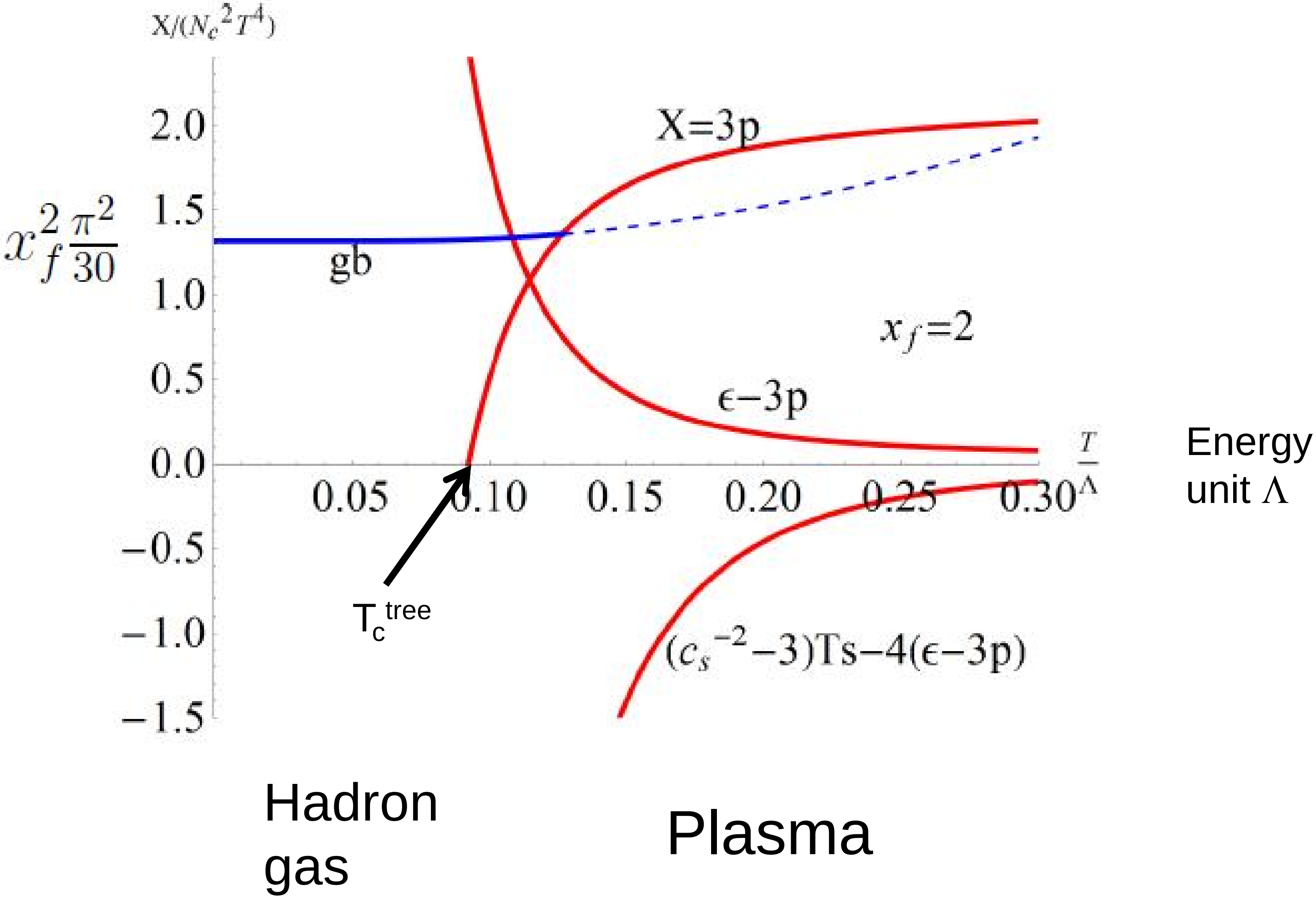
$p_q(T)$ from classical gravity black hole solutions:"tree level"

No classical solns for $p_h(T)$: arises from "1loop or stringy effects"!

Mass spectrum from fluctuations around f(z)=1, T=0 solutions

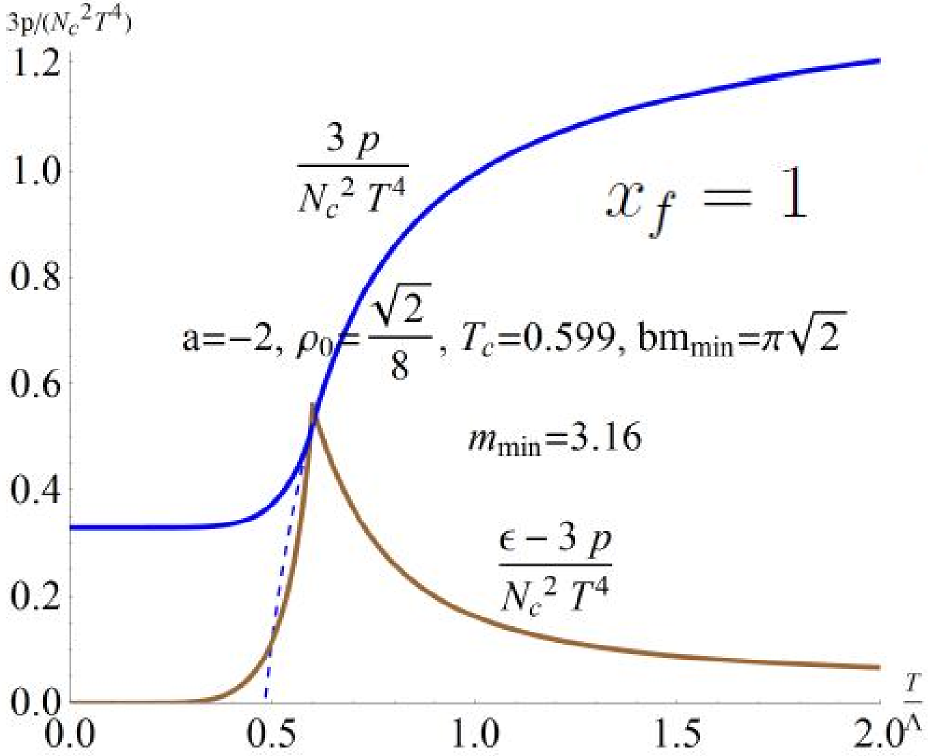
1loop computation!

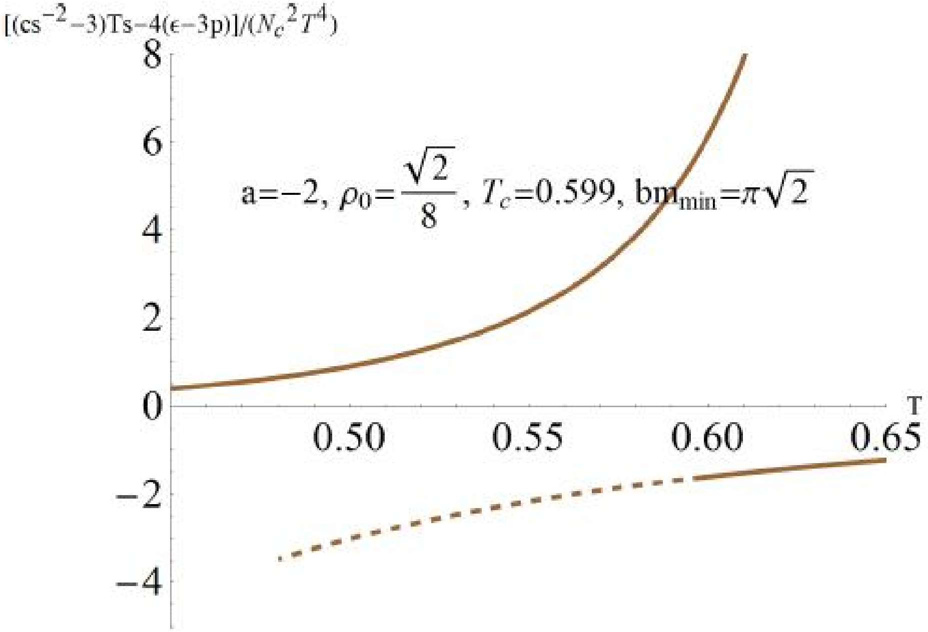
1st order transition $p_h(T_c)=p_q(T_c)$ is trivial: draw line at gb



$m_{\min} = 0.707\Lambda$

2nd order transition $p_h(T_c)=p_q(T_c)$, $p_h'(T_c)=p_q'(T_c)$ is simple:





A cusp in interaction measure

Cusp: 2nd derivatives of opposite sign!

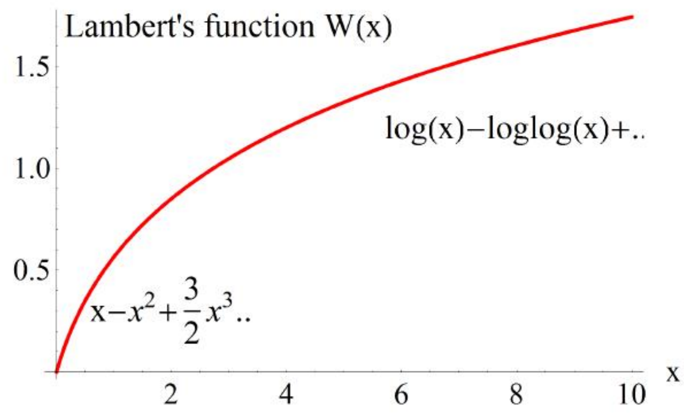
Must be equal for 3rd order!

We found that bending the 2nd derivative of HG down to negative side required including repulsive interactions between hadrons arising from their finite size!

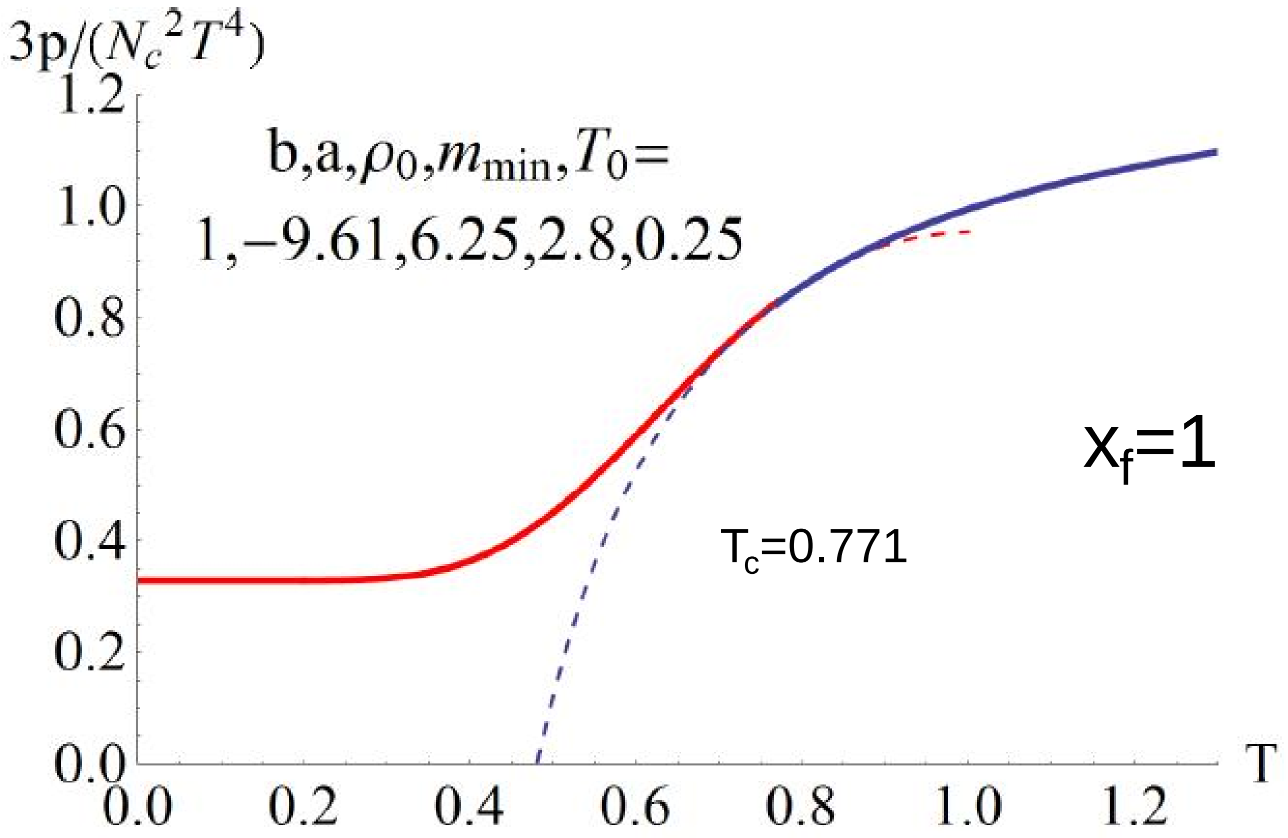
Include in the textbook derivation of BE distribution

$$V \to V - v_0 N, \quad v_0 \equiv \frac{1}{T_0^3}$$

$$\frac{p_{\rm h}}{N_c^2 T^4} = \frac{\pi^2}{90} x_f^2 + \frac{\rho_0}{m_0} x_f^2 \int_{m_0}^{\infty} dm \, \frac{m^a}{m_0^a} \, e^{bm} \, \frac{T_0^3}{T^3} \, W\left(\frac{m^2 T}{2\pi^2 T_0^3} K_2\left(\frac{m}{T}\right)\right)$$



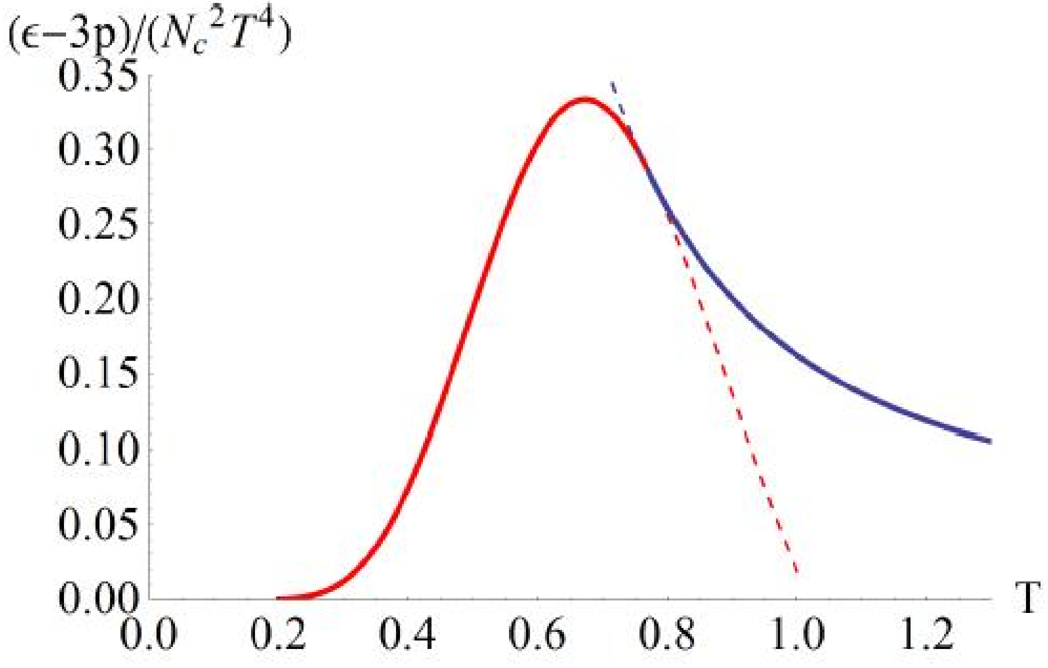
3rd order transition $p_h(T_c)=p_q(T_c)$, $p_h'(T_c)=p_q'(T_c)$, $p_h''(T_c)=p_q''(T_c)$:



$$\rho(m,b,a,m_0) \approx \delta(m) + \frac{6}{m_0}\left(\frac{m}{m_0}\right)^{-9} \exp\left(3\frac{m}{m_0}\right)\theta(m-m_0)$$

$$m_0 = (2.8,\ 0.71,\ 0.08) \quad x_f = (1,\ 2,\ 2.5)$$

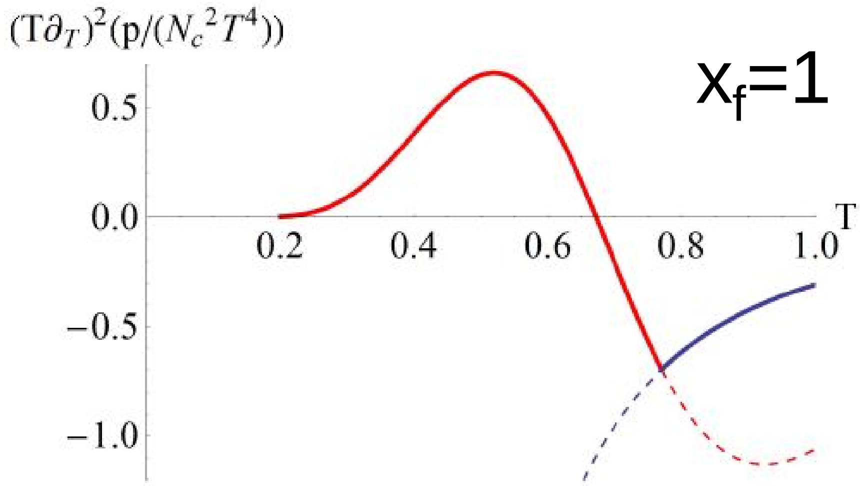
$p_h'(T_c)=p_q'(T_c)$



$p_h''(T_c)=p_q''(T_c)$

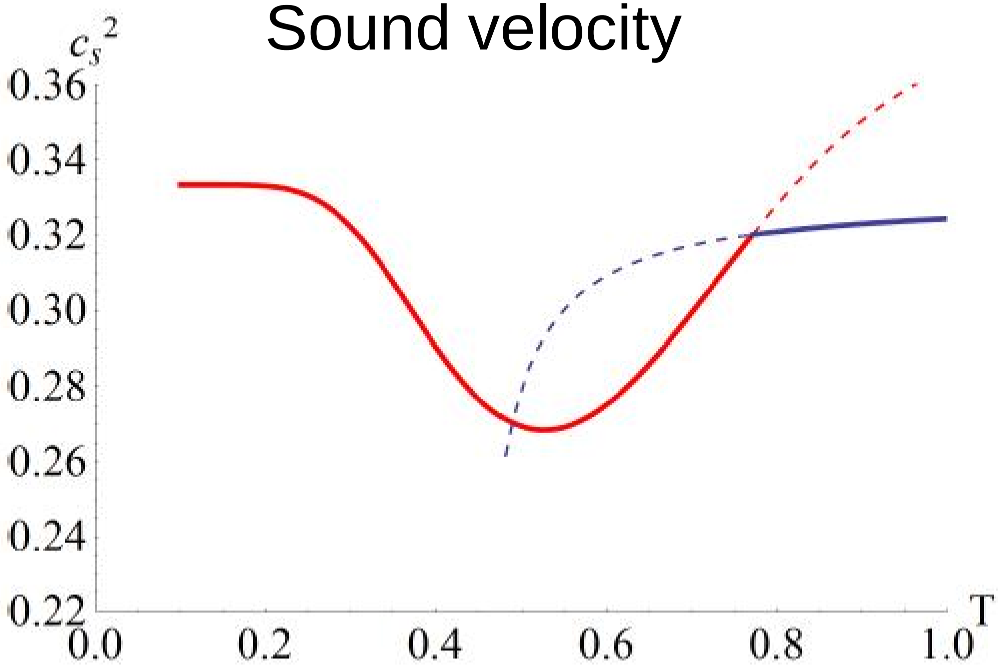
$p_h'''(T_c)<0,\ p_q'''(T_c)>0$

Third order!

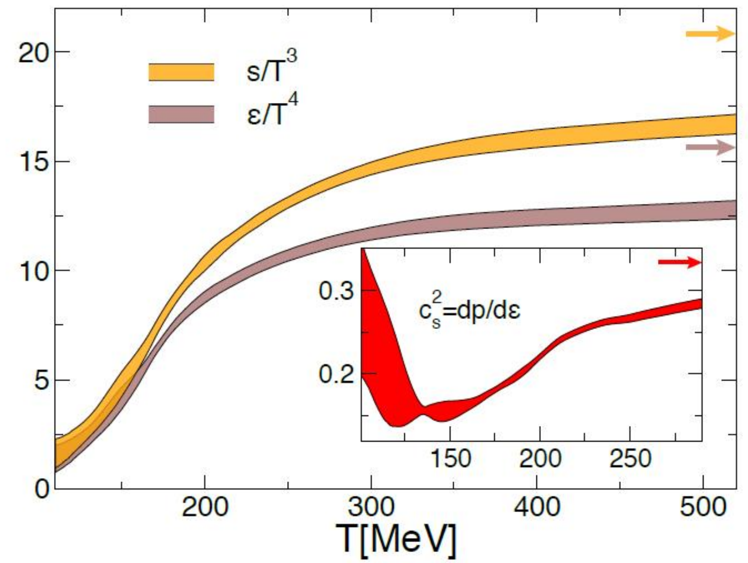


$x_f=1$

Sound velocity



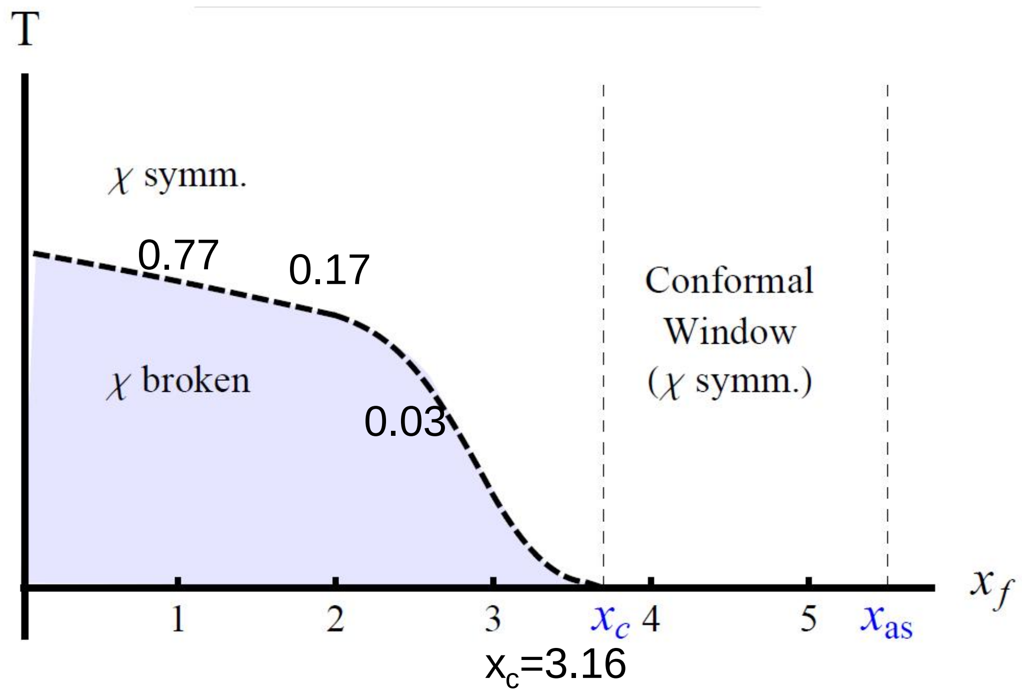
Physical QCD:



$x_f = N_f/N_c$ dependence

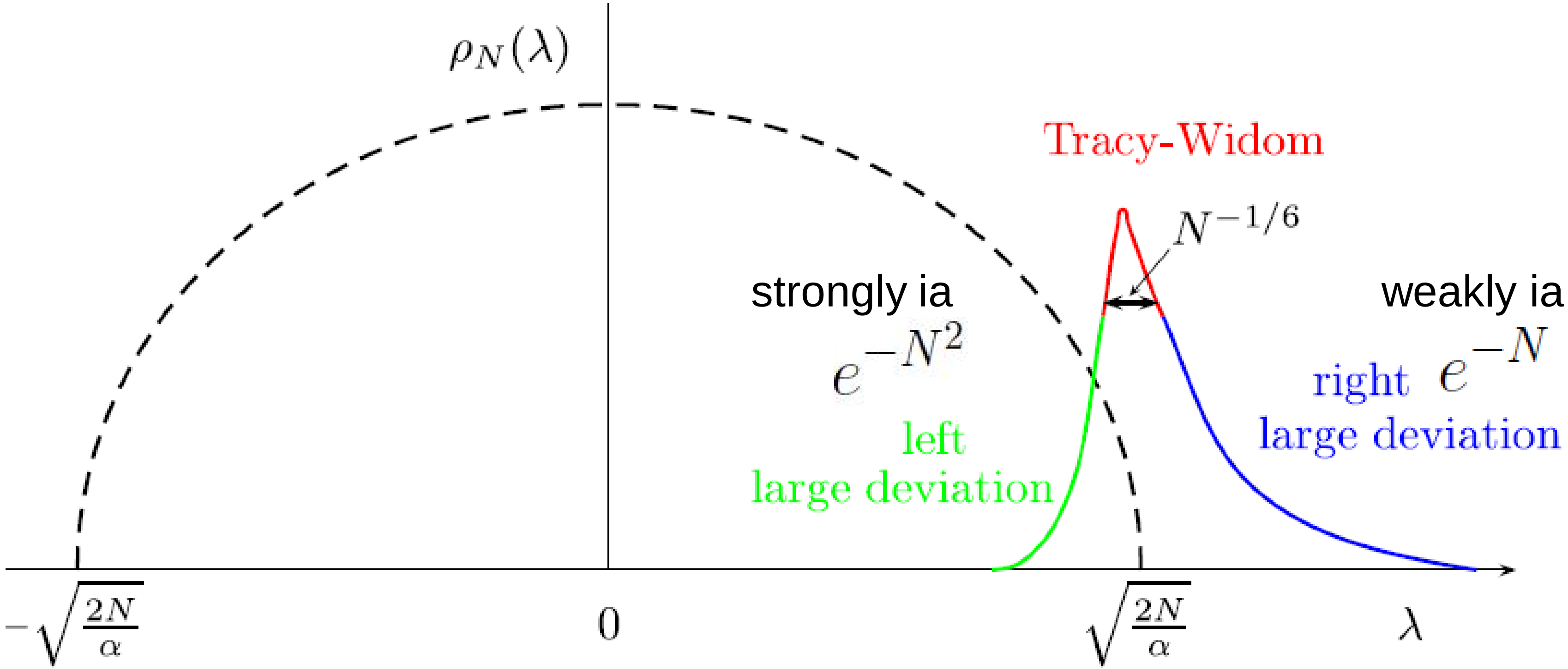
scales with m_{min}: $$T_c = (0.27 \pm 0.03)m_{\rm min}$$

$$m_0 = (2.8,\ 0.71,\ 0.08) \quad x_f = (1,\ 2,\ 2.5)$$



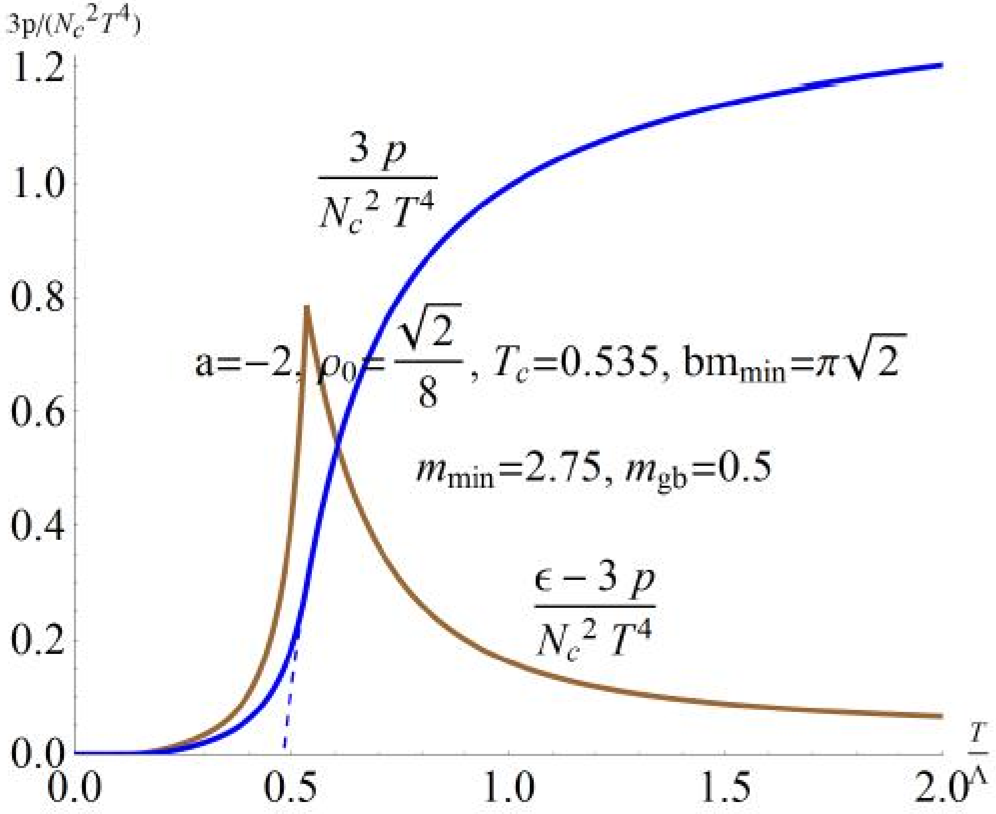
Another 3rd order PT, Tracy-Widom universality class, Gross-Witten-Wadia:

$$\mathcal{D}M e^{\mathrm{Tr}(M+M^\dagger)} \sim e^{\frac{1}{g^2}\sum_1^N(\lambda_i+\lambda_i^*)+\sum_{i<j}|\lambda_i-\lambda_j|^2}$$



For N=infty a 3rd order PT between left and right! GWW1980

Goldstone bosons at T=0 disappear with finite m_q:



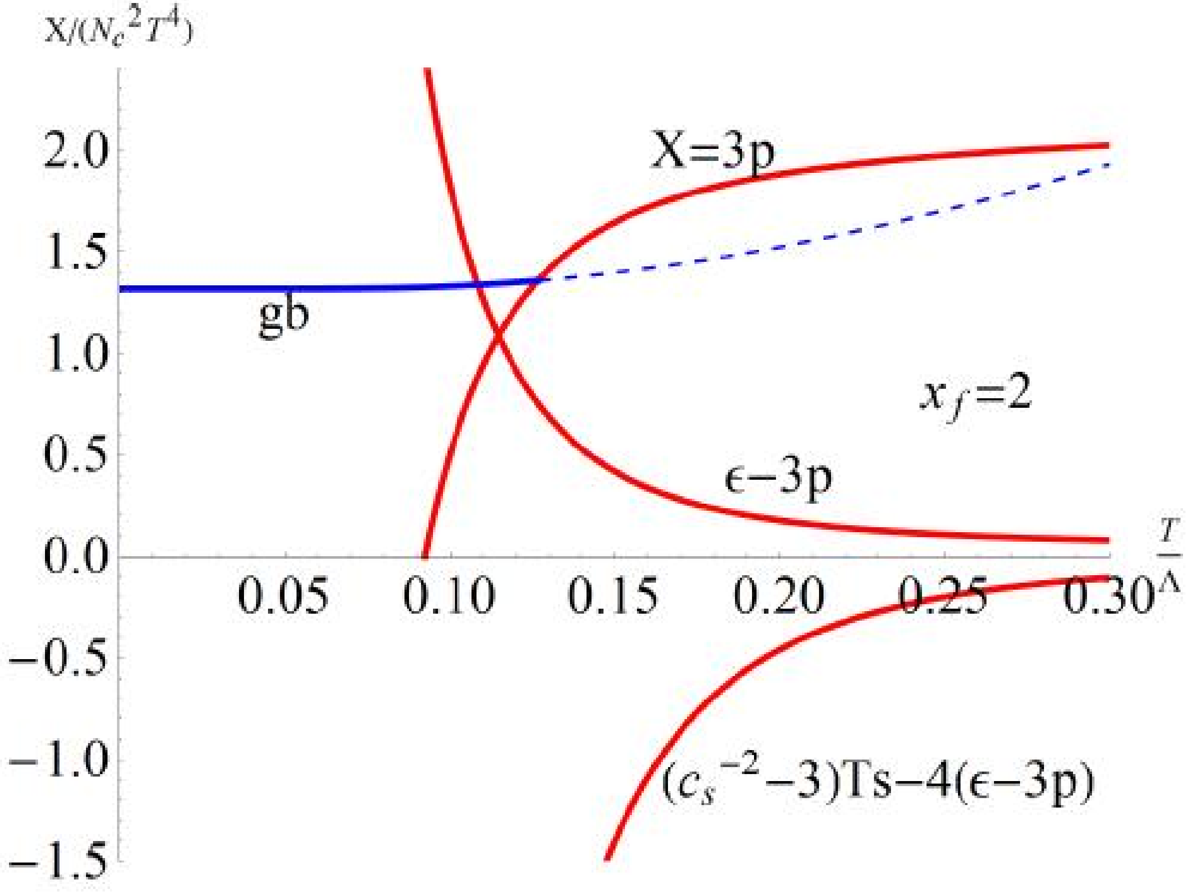
Concluding questions:

- What is the chiral transition really: 1st, 2nd,3rd,..,continuous?

- Can lattice MC in practice give the answer: $V \to \infty, \quad a \to 0, \quad m_q \to 0$?

- What do you get when μ is included? Theoretical problem: what are the potentials in the action? Practical problem: how to organise the very demanding computation

Overflow



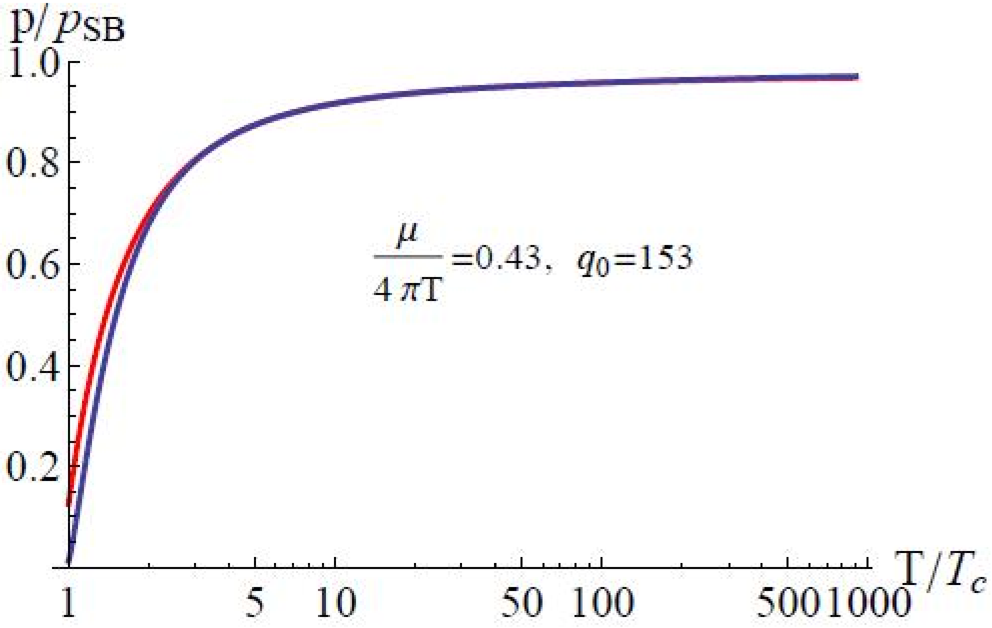
Hadron gas

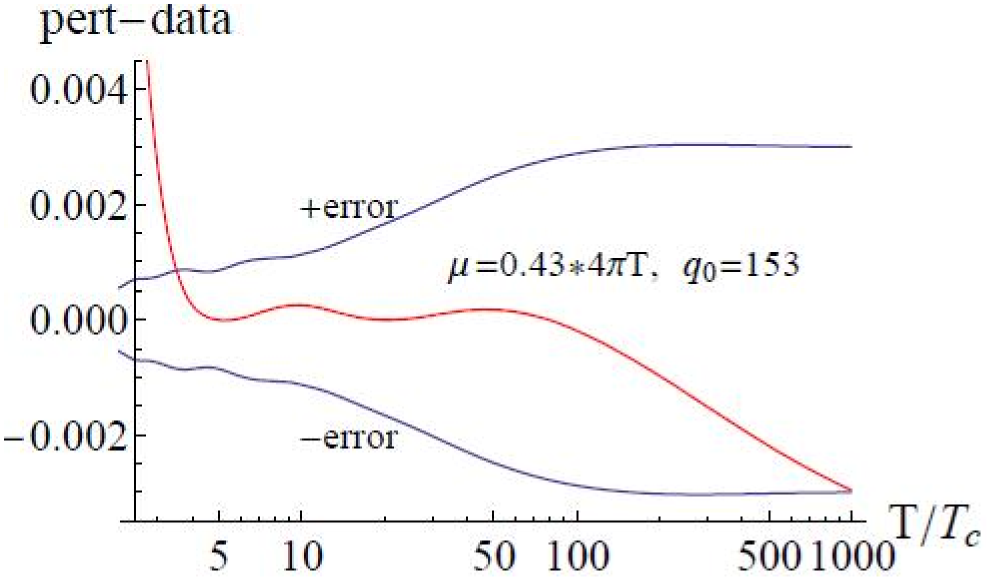
Plasma

Energy Unit Λ

Pure SU(3)

Budapest-Wuppertal 1204.6184





Continuum data! (blue)

Pert (red)

pert < lattice data

Perhaps lattice data
should be corrected
by a tiny amount down

$$c_{\rm SB} + c_2 g^2 + c_3 g^3 + (c_4' \log g + c_4) g^4 + c_5 g^5 + (c_6' \log g + c_6) g^6 + c_7 g^7 + ..$$

c_2 Shuryak 78, c_3 Kapusta 79, c_4' Toimela 83, c_4 Arnold-Zhai 94,
c_5 Zhai-Kastening, Braaten-Nieto 95, c_6' Kajantie-Laine-Rummukainen-Schröder 03

$$p/p_{\rm SB} =$$

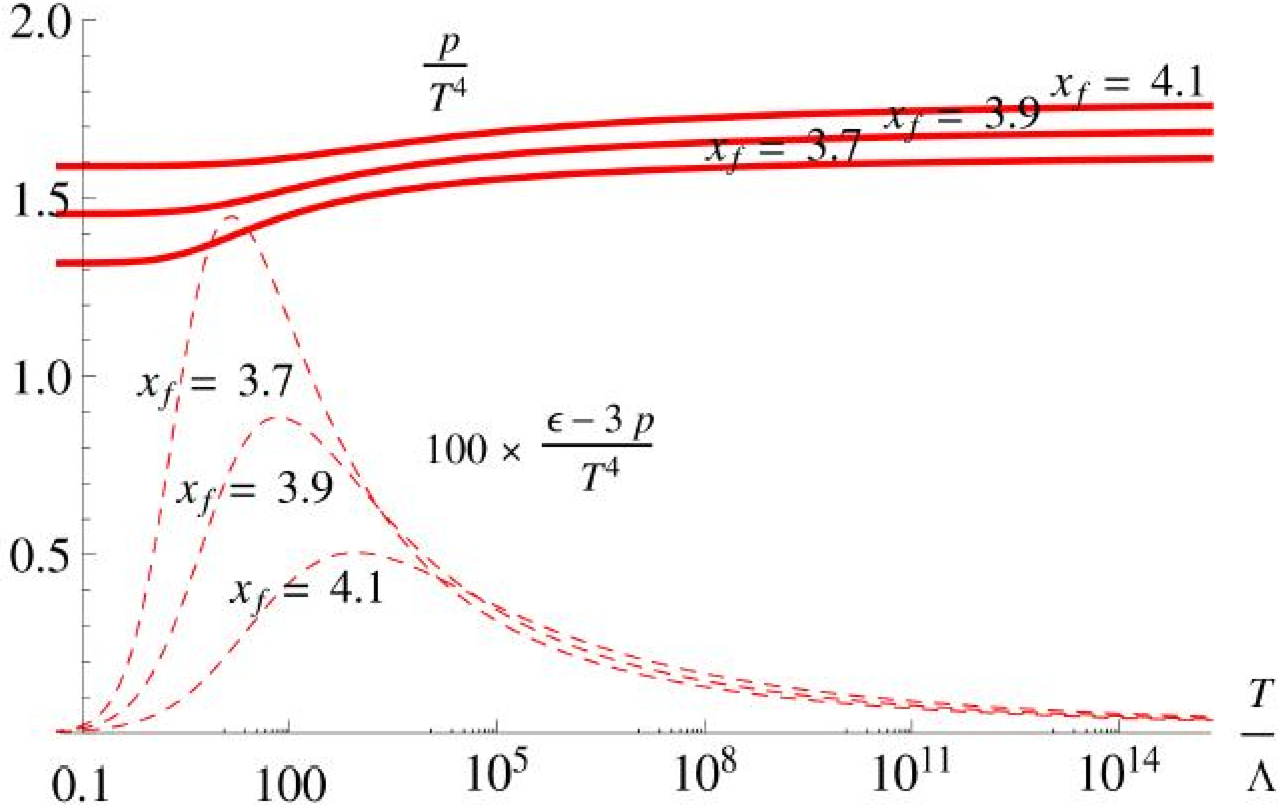
$$1 - \tfrac{5}{2}\lambda(\bar\mu) + \tfrac{20}{\sqrt{3}}\lambda^{3/2} + \left[30\log(\tfrac{2}{3}\lambda) + p_2 b_0 \log\tfrac{\bar\mu}{4\pi T} + 99.0784\right]\lambda^2$$

$$+[\tfrac{3}{2}p_3 b_0 \log\tfrac{\bar\mu}{4\pi T} - 227.746]\lambda^{5/2} +$$

$$\Big\{(-42.8187 + 60 b_0 \log\tfrac{\bar\mu}{4\pi T})\log(\tfrac{2}{3}\lambda) - 140.915\log\lambda + p_2 b_0^2 \log^2\tfrac{\bar\mu}{4\pi T}$$

$$+(p_2 b_1 + p_4 b_0 + 2 b_0\, 99.0784)\log\tfrac{\bar\mu}{4\pi T} + q_0\Big\}\lambda^3 + \mathcal{O}(\lambda^{7/2})$$

$$\frac{1}{\lambda(\mu)} = b_0 \log\frac{\mu}{\Lambda} + \frac{b_1}{b_0}\log(\log\frac{\mu}{\Lambda})$$



Normalised to
SB at T=infty