Actually mainly
AdS/CFT

AdS/QCD

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Literature: Go to arXiv th or phen. Find Gubser, Son, Starinets, Witten, Yaffe

Lyng-Petersen, hep-th/9902131

1. Background material: classical gravity, AdS, conformal invariance, string theory

2. Finite T equation of state

3. Green's functions, correlators, viscosity

4. Wilson loops (tutorial material)

(or any other)

We want to **solve** QCD, a quantum field theory

$$\mathcal{L}_{\text{QCD}} = \frac{1}{4g^2} \sum_{a=1}^{N_c^2-1} F^a_{\mu\nu} F^a_{\mu\nu} + \sum_{i=1}^{N_f} \bar{\psi}_i [\gamma_\mu D_\mu + m_i] \psi_i$$

$$D \sim \partial + gA \qquad F \sim \partial A + gA^2$$

by replacing it by a **classical** theory

in the limit $N_c \to \infty$

$$3 \gg 1$$

Not as weird an idea you might to think: there was an old dream of a

Master Field

Gopakumar-Gross
hep-th/9411021

Scale $$g = \frac{\tilde{g}}{\sqrt{N_c}}, \quad A = \frac{\sqrt{N_c}}{\tilde{g}}, \quad \psi = \sqrt{N_c}\tilde{\psi}$$

Action: $$N_c \int d^4x \left[\frac{1}{2g^2}\mathrm{tr}F^2 + \bar{\psi}(D+m)\psi\right]$$

For $N_c \longrightarrow \infty$ one semiclassical $\hbar \sim 1/N_c^2$ configuration

$A_\mu(x) = A_\mu^{\mathrm{master}}(0), \ldots$ might dominate

Just like quantum $e^{\frac{i}{\hbar}\int dt L}$ to classical

Now one thinks that one further needs: $\int d^4x \to \int d^4x dz$ + also large g^2N

one (or 2,3,4,..) more dimensions

”classical” becomes classical gravity

Anti de Sitter/Conformal Field Theory, AdS/CFT duality:

The duality $\sim$ equality will be between

Quantum field theory (a special one!) in 4d

Classical gravity in 5d (for $N_c \gg 1$, $g^2 N_c \gg 1$)

Gauge/gravity duality: try to extend to non-conformal theories, QCD

I do not belive that there is a rigorous classical gravity dual of QCD!

Some Gravity

Carroll, Spacetime and geometry

$$ds^2 = -dt^2 + d\mathrm{x}^2 + dz^2$$ Flat space

$$ds^2 = -f(r)dt^2 + r^2 d\Omega^2 + \frac{dr^2}{f(r)}, \quad f(r) = 1 - \frac{r_s}{r}$$ Usual 4d Black Hole, spherical

$$ds^2 = \frac{\mathcal{L}^2}{z^2}\left(-dt^2 + d\mathrm{x}^2 + dz^2\right) \quad \mathcal{L} =$$ AdS radius

5dim AdS space, often $r = \frac{\mathcal{L}^2}{z}$

Warning: configs can be changed by choosing new coordinates, like performing gauge transf in Yang-Mills!
Gauge inv -> Reparam. inv

$$ds^2 = \frac{\mathcal{L}^2}{z^2}\left(-f(z)dt^2 + d\mathrm{x}^2 + \frac{dz^2}{f(z)}\right) \quad f(z) = 1 - \frac{z^4}{z_h^4}$$

5dim AdS BH, flat

Here is one in 10dim space with coordinates

$$x^{\mu}, \quad \overset{\text{periodic}}{\tau}, \quad U > U_{KK}, \quad \Omega_4$$

$$ds^2 = \left(\frac{U}{R}\right)^{3/2} \left(\eta_{\mu\nu} dx^{\mu} dx^{\nu} + f(U) d\tau^2\right) + \left(\frac{R}{U}\right)^{3/2} \left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)$$

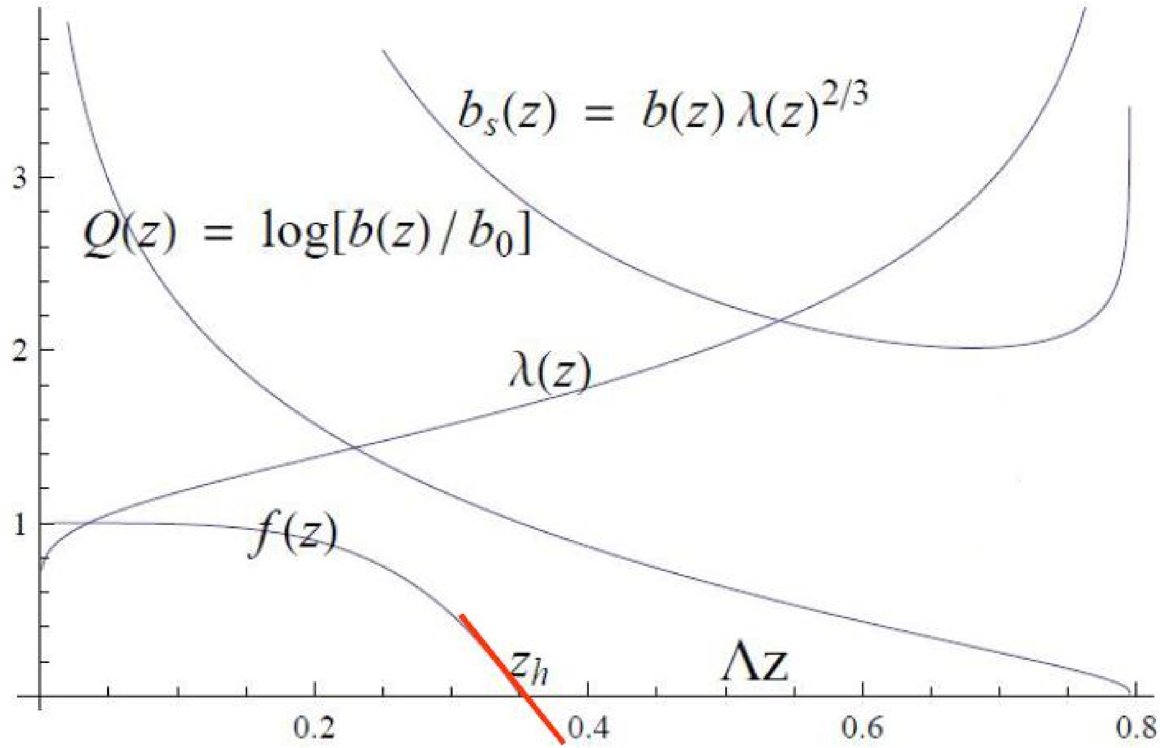
$$f(U) = 1 - \frac{U_{\mathrm{KK}}^3}{U^3}$$

Sakai-Sugimoto model for QCD

And here is one I have been working with: gravity + scalar

$$ds^2 = b^2(z)\left[-f(z)dt^2 + d\mathbf{x}^2 + \frac{dz^2}{f(z)}\right] \qquad \lambda(z) = e^{\phi(z)}$$

flat BH



All z-dependence from
Einstein's equations!

Gürsoy-Kiritsis-Mazzanti-Nitti 0903.2859

Gauge/gravity duality means finding these gravity backgrounds, bulk fields, and computing results for strongly coupled field theories

Top down; start from 10dim string theory, go to supergravity

Bottom up: start from what you want, confinement, chiral symmetry breaking, hadron mass spectrum, asymptotic freedom, anomaly structure and construct the background to give this

Knowing only that duality is true, how would you think the mapping 4d <.-> 5(or more)d goes?

Eqs of classical gravity: Einstein-Hilbert:

$$S[g_{\mu\nu}] = \frac{1}{16\pi G}\int d^d x\sqrt{g}(R + 2\Lambda)$$

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu, \quad g = |\det g_{\mu\nu}| \quad g^{\mu\alpha}g_{\alpha\nu} = \delta^\mu_\nu,$$

$$g_{\mu\nu} \Rightarrow R^\alpha_{\mu\beta\nu},\ R_{\mu\nu} = R^\alpha_{\mu\alpha\nu}, \quad R = g^{\mu\nu}R_{\mu\nu}, \quad \dim R = 1/\text{length}^2$$

EOM: $$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \Lambda g_{\mu\nu} = 0$$

AdS$_d$ $$\Lambda = \frac{(d-1)(d-2)}{\mathcal{L}^2} \Rightarrow R_{\mu\nu} = -\frac{(d-1)}{\mathcal{L}^2}g_{\mu\nu}, \quad R = -\frac{(d-1)d}{\mathcal{L}^2}$$

Why AdS, not dS or sth else?

Deepest reason: symmetry

- the symmetry of 4d gauge theory is conformal symmetry: Lorentz O(1,3) + dilatations + special conformal transformation = O(2,4)

- the symmetry of AdS_5 is also O(2,4):

dS would be O(1,5)

AdS_5 can be represented as the surface

$$-t_1^2 - t_2^2 + x_1^2 + x_2^2 + x_3^2 + x_4^2 = -\mathcal{L}^2$$

in the flat 6 dimensional space with metric

$$ds^2 = -dt_1^2 - dt_2^2 + dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2.$$

Just like a 2d sphere S_2 is the surface $x_1^2 + x_2^2 + x_3^2 = R^2$ in the flat R_3 with metric $ds^2 = dx_1^2 + dx_2^2 + dx_3^2$.

Also: AdS has a boundary; AdS has a scale L

$$ds^2 = R^2\left[\frac{dr^2}{1-r^2} + r^2 d\phi^2\right]$$

Conformal symmetry

What is the invariance group of Maxwell's equations?

1. Lorentz 1892: Lorentz transformations + Translations (6+4 parameters, Poincaré group)

2. Cunningham & Bateman 1909: There is more: dilatations (1 parameter) and "special conformal transformations" (4 parameters)

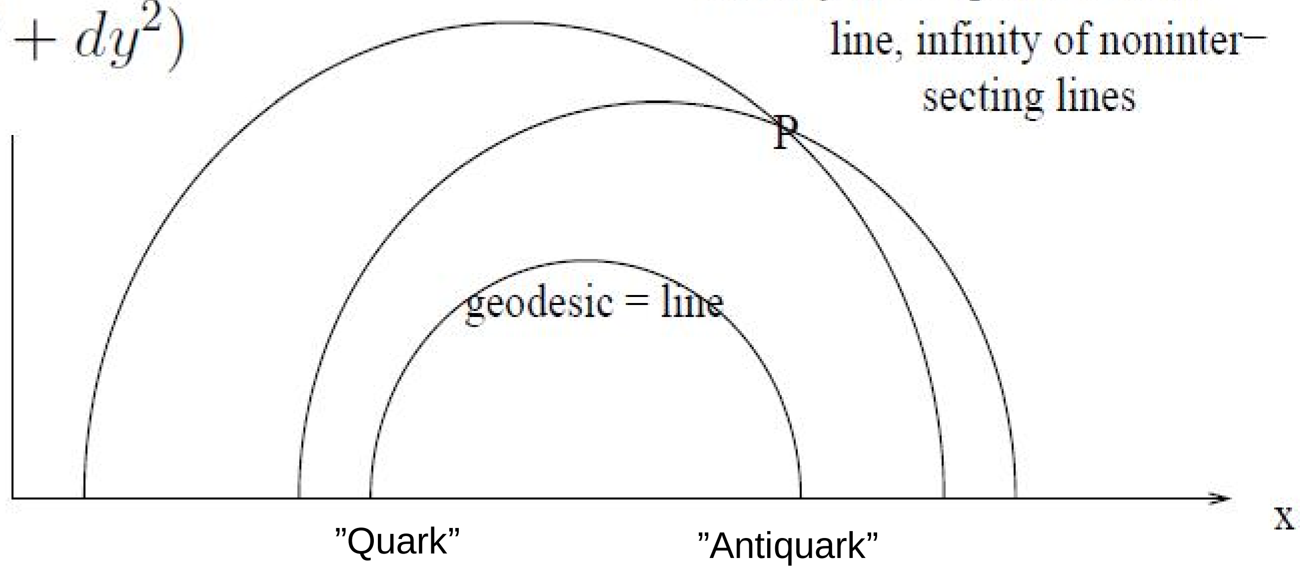
Conformal group O(2,4) (15 parameters)

Running of $g(\mu)$ spoils classical conformal invariance of QCD: a scale Λ_{QCD} is introduced

What is this $\mathcal{L}^2/z^2$ in the AdS metric?

Poincare plane: model of non-Euclidian geometry

$$ds^2 = \frac{1}{y^2}(dx^2 + dy^2)$$



Geodesics:

$$s = \int ds = \int dy \frac{1}{y}\sqrt{1 + x'(y)^2} \equiv \int dy L[x'(y)]$$

$$\Rightarrow \frac{d}{dy}\left[\frac{x'(y)}{y\sqrt{1 + x'^2}}\right] = 0 \Rightarrow (x-a)^2 + y^2 = c^2$$

Black hole temperature

Famous computation:

$$-f(r)dt^2 + \frac{dr^2}{f(r)} \quad \tau = it \quad f(r) = f(r_0) + f_0'(r - r_0) + ..$$

$$\frac{dr^2}{f_0'(r - r_0)} + f_0'(r - r_0)d\tau^2 \quad d\rho = \frac{dr}{\sqrt{f_0'(r - r_0)}} \quad \rho \sim \sqrt{r - r_0}$$

$$d\rho^2 + \rho^2 \left(d\tfrac{1}{2} f_0'\tau\right)^2 = d\rho^2 + \rho^2 d\phi^2$$

Here you have to know that finite T configs are periodic in tau, with period hbar/T

If not periodic, space is conical!!

$$\tfrac{1}{2} f_0'\tau + 2\pi \quad \tau + \frac{4\pi}{f_0'} = \tau + \frac{1}{T}$$

$$T_H = \frac{|f'(r_0)|}{4\pi}$$

$$r_s = \frac{2MG}{c^2}$$

$$1 - \frac{r_s}{r} \Rightarrow T = \frac{\hbar c}{4\pi r_s} \qquad 1 - \frac{z^4}{z_h^4} \Rightarrow \pi T = \frac{1}{z_h}$$

Black hole entropy

$$S = \frac{c^3}{\hbar}\frac{A}{4G}$$

$$ds^2 = -f(r)dt^2 + r^2 d\Omega^2 + \frac{dr^2}{f(r)}, \quad f(r) = 1 - \frac{r_s}{r}$$

$$S = \frac{r_s^2 \cdot 4\pi}{4G} = 4\pi G M^2$$

$$ds^2 = \frac{\mathcal{L}^2}{z^2}\left(-f(z)dt^2 + d\mathrm{x}^2 + \frac{dz^2}{f(z)}\right) \quad f(z) = 1 - \frac{z^4}{z_h^4}$$

$$S = \frac{1}{4G_5}\frac{\mathcal{L}^3}{z_h^3}V_3 = \frac{\mathcal{L}^3}{4\pi G_5}\pi^4 T^3 V_3$$

Where duality really works:

N=4 SuSy Y-M in 4d is the same as string theory on $\mathrm{AdS}_5 \times \mathrm{S}^5$

N=1 SuSy: $$S[A_\mu,\lambda] = \int d^dx \left[-\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} - \frac{1}{2} \bar{\lambda}^a i \Gamma^\mu \left(D_\mu \lambda\right)^a \right]$$

$\mathcal{N} = 4$ SuSy (1 vector, 4 fermions, 6 scalars, all adjoint)

$$S[A^a_\mu, \phi^a_i, \psi^a, \bar{\psi}^a] = \frac{1}{2g^2} \int d^4x \left\{ \frac{1}{2} F^{a\,2}_{\mu\nu} + (\partial_\mu \phi^a_i + f_{abc} A^b_\mu \phi^c_i)^2 + \bar{\psi}^a i \gamma^\mu (\partial_\mu \psi^a + f_{abc} A^b_\mu \psi^c) \right.$$
$$\left. + i f_{abc} \bar{\psi}^a \Gamma^i \phi^b_i \psi^c - \sum_{i<j} f_{abc} f_{ade} \phi^b_i \phi^c_j \phi^d_i \phi^e_j + \partial_\mu \bar{c}^a (\partial_\mu c^a + f_{abc} A^b_\mu c^c) + \xi (\partial_\mu A^a_\mu)^2 \right\}$$

Beta function $$\mu \frac{\partial g(\mu)}{\partial \mu} = \beta(g) = 0$$

No scale generated by regularisation, no Λ_{QCD}
g is a number !

Conformal field theory

Maybe this theory is fully integrable? The harmonic oscillator of relat QFT!

Forefront of research today.

A few lines on string theory:

String $X^\mu(\tau,\sigma)$ moving in a space with metric $ds^2 = G_{\mu\nu}dx^\mu dx^\nu$

$$S = -T\int d\tau d\sigma\sqrt{-\mathrm{det}h_{ab}}, \qquad h_{ab} = G_{\mu\nu}\frac{\partial X^\mu}{\partial\sigma^a}\frac{\partial X^\nu}{\partial\sigma^b}, \qquad \sigma^a = (\tau,\sigma)$$

$$T = \frac{1}{2\pi\alpha'} = \mathbf{Tension}.$$

IIB string theory on $G_{\mu\nu} \leftrightarrow \mathrm{AdS}_5\times\mathrm{S}^5$

$$-\frac{T}{2}\int d^2\sigma\sqrt{-\mathrm{det}h_{ab}}\left[h^{ab}G_{\mu\nu}(X)\partial_a X^\mu\partial_b X^\nu + \epsilon^{ab}B_{\mu\nu}(X)\partial_a X^\mu\partial_b X^\nu + \ldots\right.$$
$$\left.-G_{\mu\nu}(X)e^\alpha_a\bar\psi^\mu i\rho^a\partial_\alpha\psi^\nu + \ldots\right] \qquad X^\mu(\sigma^1,\sigma^2)$$

This theory should be the same as $\mathcal{N} = 4$ SuSy Y-M

Role of S^5 Invariance under SU(4) transf of the 4 SuSy generators
SU(4) = O(6); S^5 is invariant under O(6)! Also 6 scalars

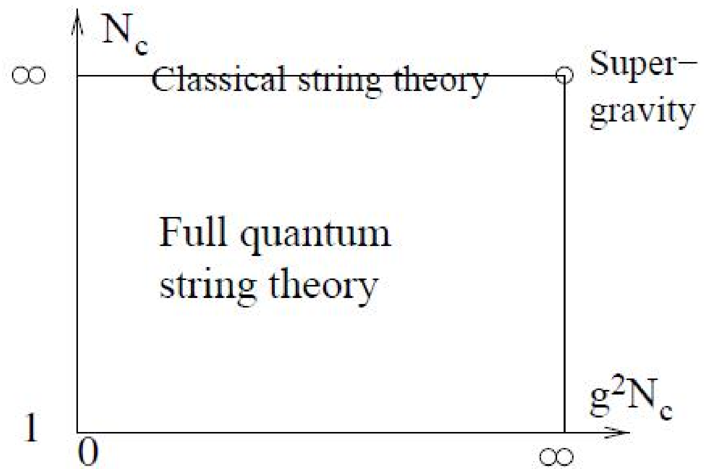
Simplifications:

$T = \frac{1}{2\pi\alpha'} = \textbf{Tension}.$

When tension grows, strings shrink to points, theory becomes supergravity

$$\mathcal{L}^2 = \sqrt{g^2 N_c}\alpha' \qquad \lambda \equiv g^2 N_c \gg 1$$

When further $N_c \gg 1$ we get classical supergravity, no string loops



This is the game we now play

How do we get 4d physics out of this 5d framework?

- Finite T equilibrium state

  Derive the famous 3/4

- shear viscosity: small deviation from equilibrium
  Formally: Green's functions G(omega, k)

  Derive the most famous prediction of string theory (?)
  viscosity/entropy density

- Wilson loops, Quark-Antiquark potential

Finite T equation of state

$$ds^2 = \frac{\mathcal{L}^2}{z^2}\left(-f(z)dt^2 + d\mathbf{x}^2 + \frac{dz^2}{f(z)}\right) \quad f(z) = 1 - \frac{z^4}{z_h^4}$$

$$S = \frac{1}{4G_5}\frac{\mathcal{L}^3}{z_h^3}V_3 = \frac{\mathcal{L}^3}{4\pi G_5}\pi^4 T^3 V_3$$

$$\frac{\mathcal{L}^3}{4\pi G_5} = \frac{N_c^2}{2\pi^2}$$

From string theory – a genuine nontrivial computation:

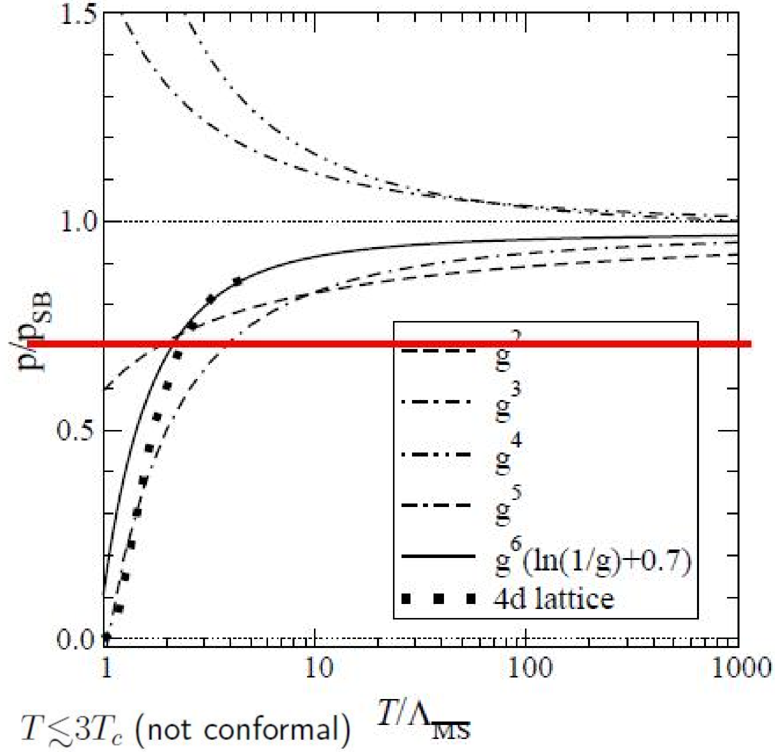
$$s = \frac{N_c^2\pi^2}{2}T^3, \quad p = \frac{N_c^2\pi^2}{8}T^4$$

$$p = \frac{3}{4}p_{\text{ideal}}$$

Ideal gas, six scalars, one vector, four fermions:

$$p(T) = (g_B + \frac{7}{8}g_F)\frac{\pi^2}{90}T^4 = (8+7)d_A\frac{\pi^2}{90}T^4 = \frac{\pi^2(N_c^2-1)}{6}T^4$$

$\mathcal{N} = 4$ SYM prediction "compared with hot QCD":



T>>Tc: weakly coupled

3Tc < T < ...
sQGP, RHIC, LHC

$T \lesssim 3T_c$ (not conformal)

Correction:

$$S[g_{\mu\nu}] = \frac{1}{16\pi G}\int d^{10}x\sqrt{g}[R - \frac{1}{2}(\nabla\Phi)^2 - \frac{1}{4\cdot 5!}F_5^2 + .. + \frac{\zeta(3)\alpha'^3}{8}e^{-\frac{3}{2}\Phi}W + ...]$$

$$p(T) = \frac{\pi^2 N_c^2}{6}T^4\left[\frac{3}{4} + \frac{45\zeta(3)}{32}\underset{\left(\frac{1.4}{g^2 N_c}\right)^{3/2}}{\frac{1}{\lambda^{3/2}}} + \ldots\right]$$

That was equilibrium; what about thermalisation?

For general enlightenment!

Quasinormal modes of a BH

Berti-Cardosi-Starinets 0905.2975

What happens if you hit a 4d Schwarzschild BH?

$$\omega = \frac{0.1105 - 0.1049i}{r_s/c}$$

no $\hbar$

inherently strongly damped, wave falls beyond horizon

Natural time unit, time it takes light to cross the Ss radius. For Sun 0.01ms

Suggestive words in gauge/gravity duality: the dual of production of quark-gluon plasma in a HI collision is production of an AdS BH. Its oscillations are strongly damped quasinormal ones: system thermalises rapidly

$$\omega = (c_1 - ic_2)\pi T$$

Has not yet become a workable model

Can also formulate an explicit relation

$$Z_{\mathrm{CFT}} = e^{p(T)V/T} = e^{-S_{\mathrm{grav}}}$$

Evaluate:

$$S = \frac{1}{16\pi G_{d+1}} \left\{ \int d^{d+1}x \sqrt{-g}\, \frac{-2d}{\mathcal{L}^2} - \int d^d x \sqrt{-\gamma} \left[2K + \frac{2d-2}{\mathcal{L}} + \frac{\mathcal{L}}{d-2} R(\gamma) \right]_{z=\epsilon} \right\}$$

$\gamma_{\mu\nu}$ = induced metric on the surface $z = \epsilon$, K = its extrinsic curvature.

$$\frac{-1}{2\pi G_5 \mathcal{L}^2} \int_0^{\beta} d\tau d^3x \int_{\epsilon}^{z_0} dz \sqrt{-g(z_0)}$$ + counter terms

V/T

$$p = \frac{N_c^2 \pi^2}{8} T^4$$ again

Green's fns, viscosity

Reminder: Reynolds number

$$Re = \rho LV/\eta.$$

Puzzle of "small" η: Solutions of Navier-Stokes flow equations (η included) do not go to those of Euler flow ($\eta = 0$) when η is "small"; get turbulence for large Re.

Weak coupling kinetic theory:

$$\eta = p\tau_c \sim \frac{T^4}{nv\sigma} \sim \frac{T^3}{g^4}, \quad \text{parametrically large}$$

but

$$\frac{\eta}{s} \approx \frac{p\tau_c}{s} \approx \frac{T^4\tau_c}{T^3} = T\tau_c \gtrsim \hbar \qquad \text{uncertainty principle}$$

Experimental fact: QCD matter observed in heavy ion collisions at RHIC/BNL has T up to $5T_c$ (strongly coupled!!) and flows nearly ideally.

Seems paradoxical: weakly coupled fluid has a "large" viscosity!

Bjorken flow: $v(t,x) = x/t$,

$$T(\tau) = \left(T_i + \frac{1}{6\pi\tau_i}\right)\left(\frac{\tau_i}{\tau}\right)^{1/3} - \frac{1}{6\pi\tau}.$$

Strong coupling result:

$$\eta = \frac{\pi}{8} N_c^2 T^3 \left[1 + \frac{75\zeta(3)}{4\lambda^{3/2}} + \ldots\right] \qquad 1 + \left(\frac{8.0}{\lambda}\right)^{3/2}$$

$$\frac{\eta}{s} = \frac{\hbar}{4\pi} \left[1 + \frac{135\zeta(3)}{8\lambda^{3/2}} + \ldots\right] \qquad 1 + \left(\frac{7.4}{\lambda}\right)^{3/2}$$

From the correlator:

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt d^3x \, e^{i\omega t} \langle T_{12}(x) T_{12}(0) \rangle \qquad \int d^4x \, T_1^2(x) g_2^1(x, z=0)$$

behaves like
a scalar source!

Lower limit for all physical systems:

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi}$$

$=$ holds for systems having a gravity dual.

Air ($\eta \sim 10^{-5}$, $s = S/V \sim N/V \sim 1\mathrm{kg}/m_p/\mathrm{m}^3 \sim 10^{27}/\mathrm{m}^3$):

$$\frac{\eta}{s} \approx \frac{10^{-5}}{10^{27}} \gg \frac{10^{-35}}{4\pi}$$

Correlators, Green's functions, spectral functions can be computed in g/g duality!

Master formula for 4d gauge quantum field theory $\Leftrightarrow$ 5d classical gravity:

Gubser-Klebanov-Polyakov hep-th/9803023 4184 citations

$$\langle \exp\left[\int d^4x\, O(x)\phi(x,0)\right]\rangle_{\mathrm{FT}} = \exp\left\{-\int d^4x\, \int_0^{z_0} dz\, \mathcal{L}_{\mathrm{class}}[\phi(x,z)]\right\}$$

$$x^\mu = (t, x^1, x^2, x^3) \qquad\qquad x^M = (t, x^1, x^2, x^3, z)$$

LHS: All there is in the field theory, all operator expectation values:

T=0 or finite T!

$$\text{e.g.,}\quad \frac{\delta^2 \mathrm{LHS}}{\delta\phi(x,0)\delta\phi(y,0)} = \langle O(x)O(y)\rangle_{\mathrm{FT}}$$

RHS: Find the field, current $\phi(x)$ to which the operator $\mathcal{O}$ couples ($\mathcal{O} = F^{a\,2}_{\mu\nu} \Rightarrow \phi(x)$, $\mathcal{O} = T_{\mu\nu} \Rightarrow \phi = g_{\mu\nu}$, etc). Then solve classical 5d gravity EOM for $\phi(x,z)$ with proper BC and compute the LHS. Approximation works when the coupling of LHS is large, non-perturbative!

Key issue: holography

Dofs can match since number of dofs for gravity $\sim$ area, not volume.

Collection of formulas for Green's functions

$A(t) = e^{iHt}A(0)e^{-iHt}$ and $B(t)$ are two operators, $\langle O \rangle = Z^{-1}\mathrm{Tr}e^{-\beta H}O$.

$$J_1(\omega) = \int_{-\infty}^{\infty} dt\, e^{i\omega t}\langle A(t)B(0)\rangle \qquad J_2(\omega) = \int_{-\infty}^{\infty} dt\, e^{i\omega t}\langle B(0)A(t)\rangle = e^{-\beta\omega}J_1(\omega).$$

$$G_R(t) = \langle\, i\,[A(t), B(0)]\,\theta(t)\,\rangle \quad G_R(\omega) = \int_{-\infty}^{\infty} \frac{d\omega'}{\pi}\frac{\rho(\omega')}{\omega' - \omega - i\epsilon} = G_A^*(\omega).$$

$$\rho(\omega) = \frac{1}{2}(1-e^{-\beta\omega})J_1(\omega) = \mathrm{Im}G_R(\omega) = \frac{1}{2}\sum_{m,n} 2\pi\delta(\omega+E_n-E_m)\langle n|A(0)|m\rangle\langle m|B(0)|n\rangle(e^{-\beta E_n}-e^{-\beta E_m})$$

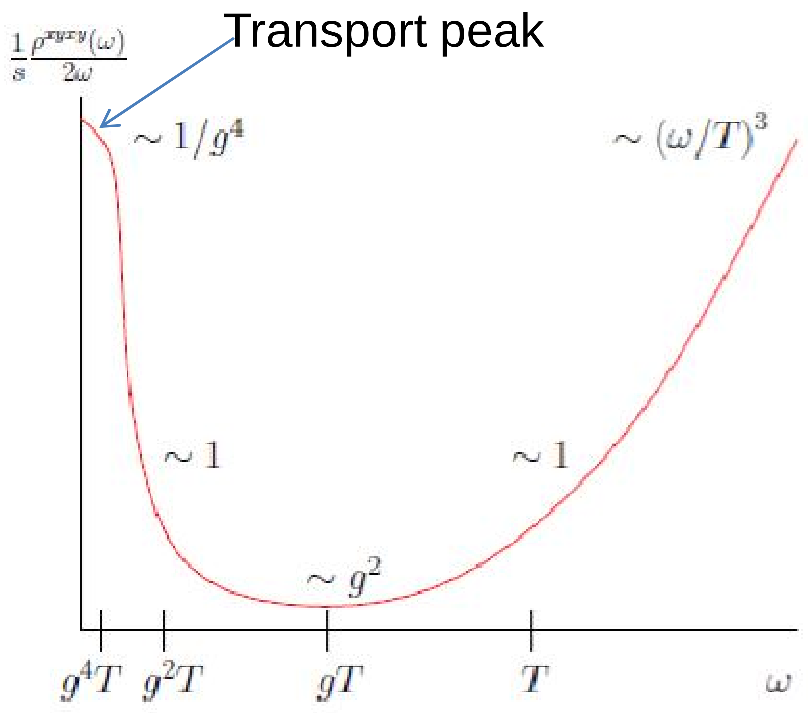
$$G_\beta(\omega_n) = G_R(\omega + i\epsilon \to i\omega_n \equiv i2\pi nT) = \int_0^\beta d\tau\, e^{i\omega_n\tau}G_\beta(\tau),$$

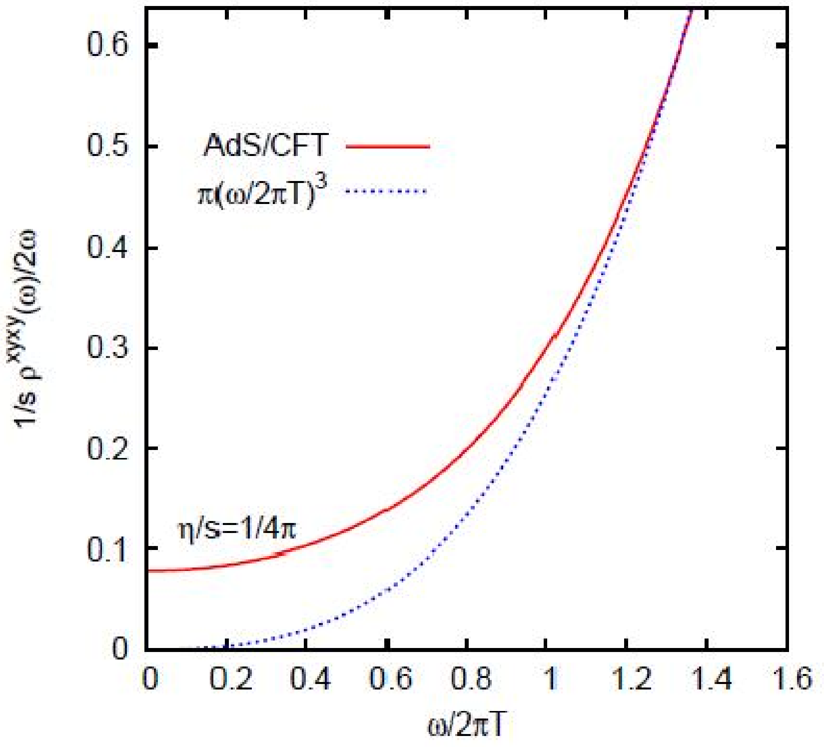
$$G_\beta(\tau) = T\sum_{n=-\infty}^{\infty} e^{-i\omega_n\tau}G_\beta(\omega_n) = \int_{-\infty}^{\infty}\frac{d\omega}{\pi}\rho(\omega)\frac{\exp(-\omega\tau)}{1-\exp(-\beta\omega)} = \int_0^{\infty}\frac{d\omega}{\pi}\rho(\omega)\frac{\cosh(\frac{1}{2}\beta - \tau)\omega}{\sinh\frac{1}{2}\beta\omega}$$

It is really $\rho(\omega, k; T) = \mathrm{Im}G_R(\omega, k; T)$ we want

Small-ω structure of $\rho(\omega)$ is complicated in weak coupling (many different scales), simpler in strong coupling (T is the only scale) Aarts-Resco hep-lat/0110145

Meyer 0907.4095





Schafer-Teaney 0904.3107

Generating functional of all correlators $\langle \mathcal{O}(x_1)\mathcal{O}(x_2)....\rangle$:

$$\frac{1}{Z(0)}\int \mathcal{D}\psi\ \exp\left[iS[\psi(x)]+i\int d^4x\,\phi_0(x)\mathcal{O}(x)\right] \equiv \langle e^{i\int d^4x\phi_0\mathcal{O}}\rangle$$
$$= \exp\{iS_{\text{grav}}[\phi(x,z),\phi(x,z\to 0)=\phi_0(x)\}$$

Boundary sources, currents, become bulk fields:

-4 0 4

$$\int d^4x\,\phi\,\tfrac{1}{4}\,F^2_{\mu\nu}$$

bulk scalar; glueball masses

--4 1 3

Global symm on bdry becomes gauge symm in bulk!

$$\int d^4x\,A^a_\mu J^{a\mu}$$

bulk vector: hadron spectrum, conductivities, superfluidity, -conductivity,

$$\int d^4x\,h_{\mu\nu}T^{\mu\nu}$$

bulk tensor= fluctuation of background metric: viscosity

Fundamental computation: evaluating gravity action:

in excruciating detail

$$\overset{-3}{2*16\pi} \overset{0}{G_5 S_{\text{grav}}[\phi]} = \int \overset{-5}{d^5x} \sqrt{-g}\, \overset{1}{g^{\mu\nu}} \overset{0}{\partial_\mu\phi} \overset{1}{\partial_\nu} \overset{0}{\phi}$$

massless scalar in bulk

one partial integration:

for upper limit, see Gubser 0806.0407

$$= \int d^5x \sqrt{-g} \left[\frac{-1}{\sqrt{-g}} \partial_\mu \sqrt{-g} g^{\mu\nu} \partial_\nu \phi \right] \phi + \int d^5x\, \partial_\mu \left[\sqrt{-g} g^{\mu\nu} \partial_\nu \phi \cdot \phi \right]$$

$$-\Box\phi = 0$$

only surface term remains:

$$\int d^4x \sqrt{-g}\, g^{zz} \partial_z \phi(x,z) \cdot \phi(x,z)|_{z_{\text{bdry}}}$$

eff'ly back to 4d !

$$\ddot{\phi} - \frac{3+z^4}{z(1-z^4)} \dot{\phi} + \left[\frac{\omega^2}{(1-z^4)^2} - \frac{k^2}{1-z^4} \right] \phi = 0$$

$$\phi \equiv \phi(z,\omega,k)$$

$$\phi = z^p \quad p(p-1) - 3p = 0 \quad p = 0, 4$$

$$\ddot{\phi} + P\dot{\phi} + Q\phi = 0$$

Here is a piece of Mathematica code to integrate the equation
from 1-epsh (cannot start exactly at 1)
to eps (cannot go exactly to 0):

```
sol[om_,k_]:=NDSolve[{f''[z]-(3+z^4)/(z (1-z^4))f'[z]+(om^2/(1-z^4)^2-k^2/(1-z^4)) f[z]==0,
f[1-epsh]==fin[1-epsh,om,k],
f'[1-epsh]==finpr[1-epsh,om,k]},f,{z,eps,1-epsh}];
fh[z_,om_?NumericQ,k_?NumericQ]:=f[z]/.sol[om,k][[1]];
fhpr[z_,om_?NumericQ,k_?NumericQ]:=f'[z]/.sol[om,k][[1]]
```

Desperate enterprise if you do not know the correct syntax, when
you know, this is extremely fast and accurate

$$ds^2 = \frac{\mathcal{L}^2}{z^2}\left(-f(z)dt^2 + d\mathbf{x}^2 + \frac{dz^2}{f(z)}\right) \quad f(z) = 1 - \frac{z^4}{z_h^4}$$

$$\sqrt{-g} = \frac{\mathcal{L}^5}{z^5} \qquad g^{zz} = \frac{z^2}{\mathcal{L}^2}f(z)$$

Fourier: $\phi(x,z) \longrightarrow \phi_0(k)\phi(z,k) \quad \phi(k,0) = 1$

$$G(k) = \frac{\delta S}{\delta\phi_0(k)\delta\phi_0(-k)} = \frac{\mathcal{L}^3}{16\pi G_5}\frac{1}{z^3}\partial_z\phi(z,-k)\phi(z,k)$$

$$z \longrightarrow 0$$

Due to the $1/z^3$ have to work carefully: fun math from the theory of

$$\ddot{\phi} + P\dot{\phi} + Q\phi = 0$$

$$\ddot{\phi} + P\dot{\phi} + Q\phi = 0$$

1. Two independent solutions if the Wronskian is nonzero:

$$W(\phi_1, \phi_2) = \phi_1 \phi_2' - \phi_2 \phi_1' = W_0 \frac{z^3}{1-z^4} \quad W'/W = -P$$

2. Around the horizon, z = 1, the indicial equation has two solns, the one with + is physical, contains only infalling wave:

$$(1-z)^{\pm i\omega/4} \qquad \phi_h(z) = (1-z)^{\pm i\omega/4}(1 + c_1(1-z) + \dots)$$

3. Around z=0 the two solns are

$$\phi_n = z^4(1 + c_1 z^2 + \dots) \qquad \phi_u = 1 + C_2 z^2 + \dots. + C \log(z) \phi_n$$

4. Write

"source" "vev"

$$\phi_h(z,k) = A(k)\phi_u(z) + B(k)\phi_n(z)$$

0 0 0 4 -4

Re part divergent!

$$\frac{\omega^2 - k^2}{2z^2} - \frac{(\omega^2 - k^2)^2}{16}(4\log z - 1) + 4 \cdot \frac{B(K)}{A(K)} + \mathcal{O}(z^2)$$

insert to

$$G(k) = \frac{\delta S}{\delta\phi_0(k)\delta\phi_0(-k)} = \frac{\mathcal{L}^3}{16\pi G_5}\frac{1}{z^3}\partial_z\phi(z,-k)\phi(z,k)$$

normalise by dividing by A(k)A(-k) and the the grand result

Kovtun-Starinets hep-th/0506184

$$\mathrm{Im}G(k) = \frac{\mathcal{L}^3}{4\pi G_5}\frac{B(k)}{A(k)}$$

Spectral function of an operator coupling to a scalar source
in a strongly coupled field theory!

Similarly for vector sources starting from

$$\int d^5x\sqrt{-g}\,\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}F_{\alpha\beta}F_{\mu\nu}$$

EOM is 5d Maxwell
in curved space

Herzog-Kovtun-Son 0809.4870

For viscosity we need only the limit $k \to 0,\ \omega \to 0$

It is not surprising that

$$\ddot{\phi} - \frac{3+z^4}{z(1-z^4)}\dot{\phi} + \left[\frac{\omega^2}{(1-z^4)^2} - \frac{k^2}{1-z^4}\right]\phi = 0$$

can in this limit be solved exactly:

$$\phi(z,\omega) = (1-z)^{-i\omega/4}\left[1 - \tfrac{1}{4}\, i\omega \log \tfrac{1+z+z^2+z^3}{4} + \mathcal{O}(\omega^2, k^2)\right]$$

$$\partial_z \phi(z,\omega) = i\omega \cdot z^3 + \ldots$$

convert to phys units

$$\frac{\rho(\omega)}{\omega} = \frac{N_c^2}{2\pi^2} \cdot \frac{1}{4} \cdot (\pi T)^3 = \eta = \frac{\pi N_c^2}{8} T^3 = \frac{s}{4\pi}$$

Grand Finale!!

If the source has a dimension and ϕ is dimless, dim must be carried by z:

$$\int \overset{-4}{d^4x}\, \overset{1}{m_q}\, \overset{3}{\bar{q}q} \qquad \int \overset{-4}{d^4x}\, \overset{4-\Delta}{\phi_0}\, \overset{\Delta}{\mathcal{O}}$$

$$\phi(z) = m_q z + \langle \bar{q}q \rangle\, z^3 + \ldots$$
$$= z^{4-\Delta}\phi_0 + \langle \mathcal{O} \rangle z^{\Delta} + \ldots$$

Klebanov-
Witten hep-th/
9905104 443 cit

Bulk field encodes both source and vev!

Can be enforced by giving mass to the scalar field. Indicial eq:

$$\Delta(\Delta - 4) = (m\mathcal{L})^2$$

Glueball masses

Measured in 4d by $$\langle 0|F^2(\tau)F^2(0)|0\rangle \sim e^{-m_g\tau}$$

are given by poles of G(k), calculable in AdS

In the previous background there are no poles, theory is conformal, no massive particles

Need a confining background!

AdS/QCD

The background

$$ds^2 = \frac{\mathcal{L}^2}{z^2}\left(-dt^2 + d\mathbf{x}^2 + dz^2\right) \quad \mathcal{L} =$$ AdS radius

has no scale. Simplest ways to introduce one:

Hard wall model: $z \leq z_0$ sort of bag model

Soft wall model: multiply metric by e^{-cz^2} not a solution of Einstein

Should do much better!

arXiv find Kiritsis
> 400 pages of papers
on this model

weak source field

big, inherent, make
this dynamical!!

In real QCD: $$\int d^4x\,\phi_0(x) F^2 \Rightarrow \int d^4x \frac{1}{g^2(\mu)} F^2$$

Identify $g^2 \equiv \lambda = e^{\phi} \quad \mu \to \frac{1}{z}$

$$S = \frac{1}{16\pi G_5}\left\{\int d^5x\,\sqrt{-g}\,\left[R - \tfrac{4}{3}\,(\partial_\mu\phi)^2 + V(\phi)\right]\right.$$

$$ds^2 = b^2(z)\left[-f(z)dt^2 + d\mathbf{x}^2 + \frac{dz^2}{f(z)}\right] \qquad \lambda(z) = e^{\phi(z)}$$

flat BH

b,f,λ are determined from Einstein.

Can reproduce SU(N) glueball masses,
thermodynamics

Tune V so that get
- confinement
- asymptotic freedom

$$V(\phi) = \frac{12}{\mathcal{L}^2}\left\{1 + V_0\lambda + V_1\lambda^{4/3}[\log(1+V_3\lambda^2)]^{1/2}\right\}$$

Einstein eqs are:

$$6\frac{\dot{b}^2}{b^2} + 3\frac{\ddot{b}}{b} + 3\frac{\dot{b}}{b}\frac{\dot{f}}{f} = \frac{b^2}{f}V(\phi)$$

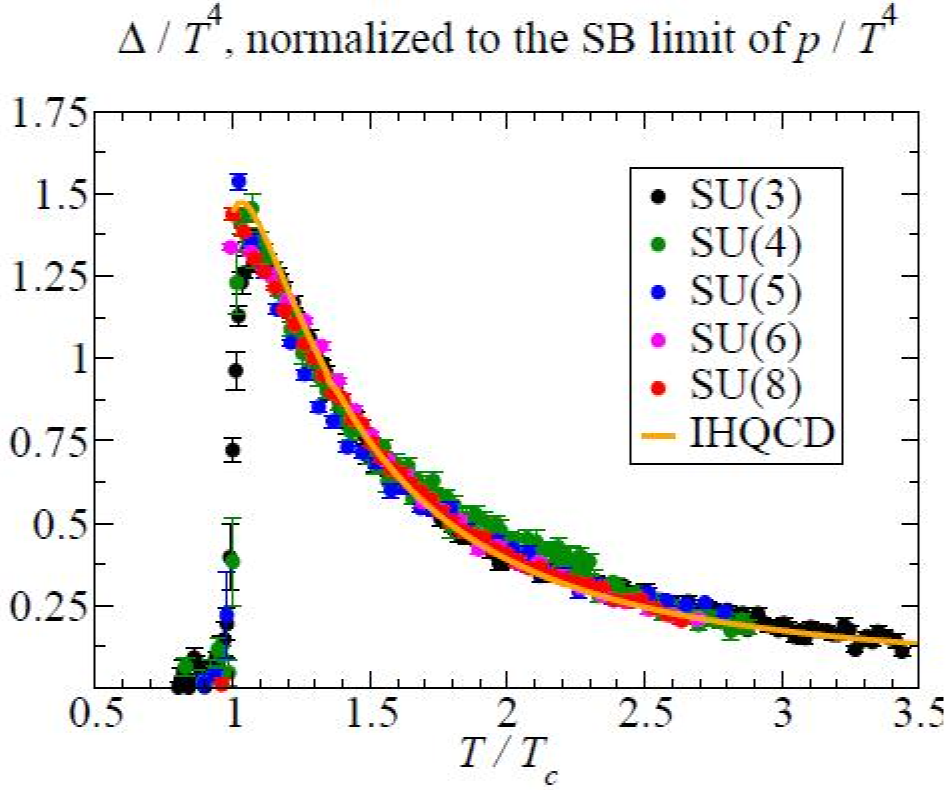
$$6\frac{\dot{b}^2}{b^2} - 3\frac{\ddot{b}}{b} = \frac{4}{3}\,\dot{\phi}^2,$$

$$\frac{\ddot{f}}{\dot{f}} + 3\frac{\dot{b}}{b} = 0,$$

Relation to beta function

$$\beta(\lambda) = b\frac{d\lambda}{db}$$

For SU(N) thermo obtain fits like (yellow line): Panero 0912.2448



Wilson loops