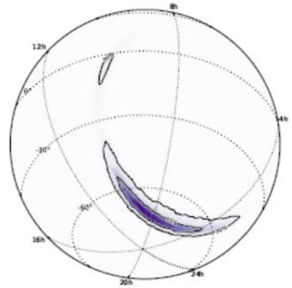
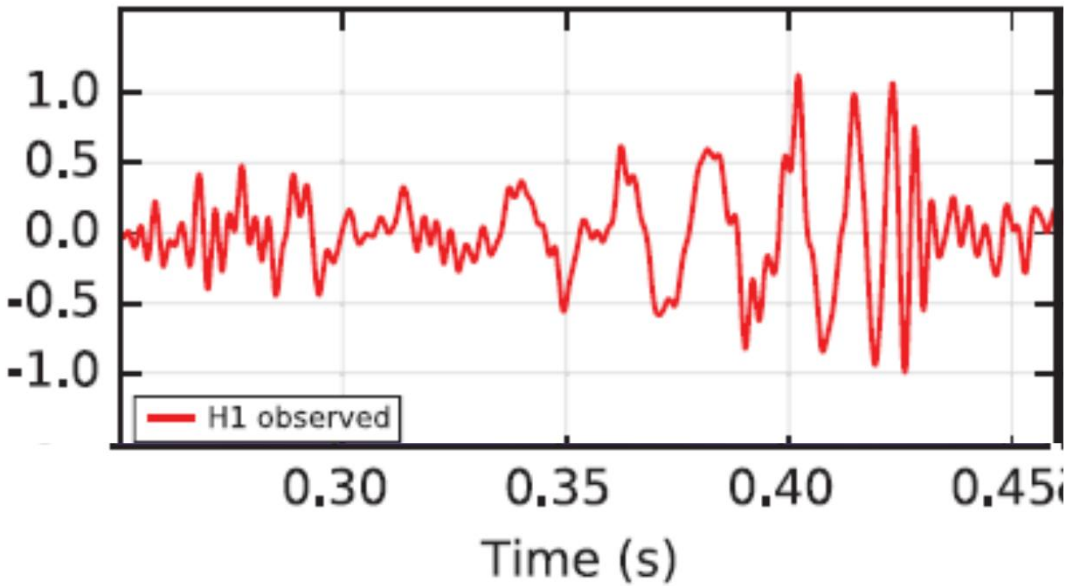
Gravitational radiation and black holes

K. Kajantie HIP 1 March 2016



1. LIGO GW150914:



Two detectors!

A 0.2 s long pulse, f=35...250/s, of gravitational radiation from a 36+29 = 62+3 M_{sun} BH/BH merger at 1.3 Gly = 1.3 x 10^{25} m, z=0.09.

$J \sim 0.7\ GM^2/c$ $\quad W_{GW}(f=25/s) = 10^{-9}$

Striking how much of this was predicted and expected

(Hulse-Taylor 1975). Compare Higgs: it was really a discovery!

The range of parameters was textbook material:

Carroll:

36 + 29

Let's see what this implies for the kind of source we might hope to observe. A paradigmatic example is the coalescence of a black-hole/black-hole binary. For typical parameters we can take both black holes to be 10 solar masses, the binary to be at cosmological distances ~ 100 Mpc, and the components to be separated by ten times their Schwarzschild radii:

410

$$R_S \sim 10^6 \text{ cm}$$
$$R \sim 10^7 \text{ cm}$$
$$r \sim 10^{26} \text{ cm}. \tag{7.196}$$

Such a source is thus characterized by

$$f \sim 10^2 \text{ s}^{-1}, \qquad h \sim 10^{-21}. \tag{7.197}$$

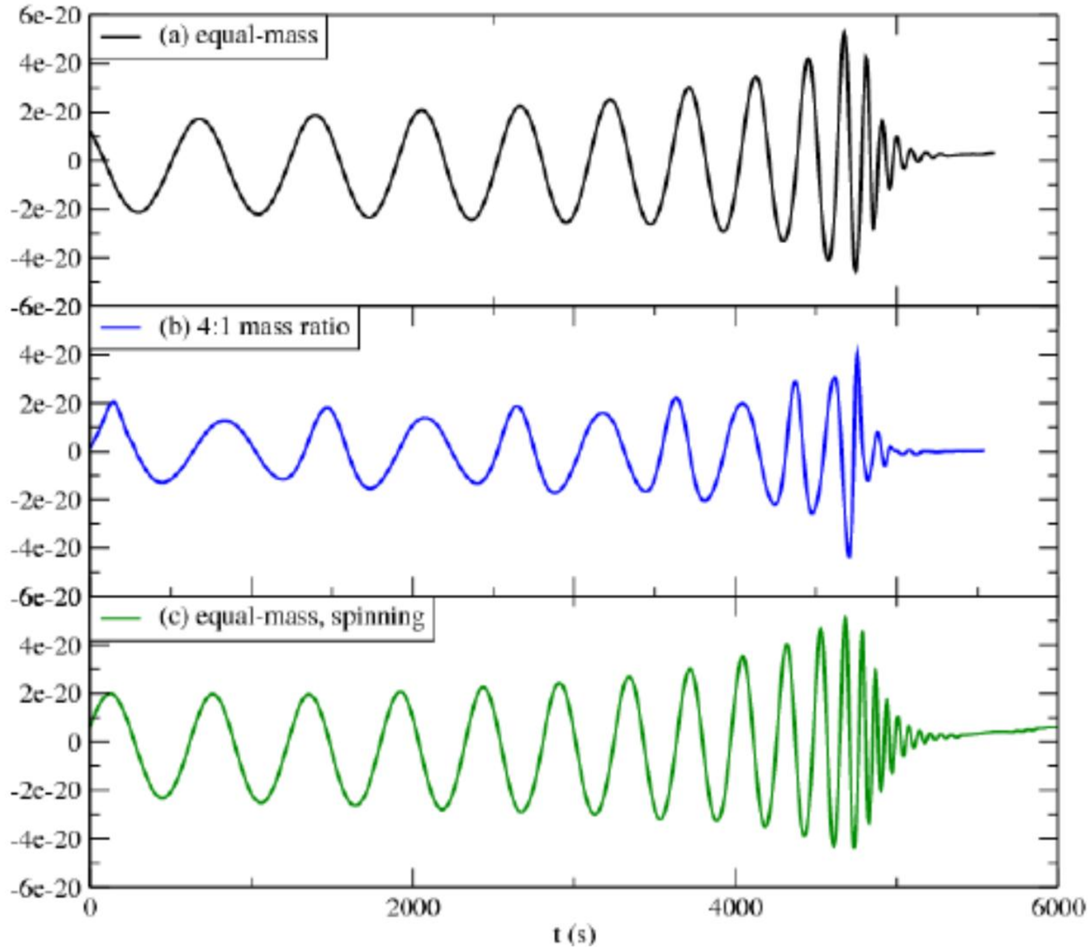
If we are to have any hope of detecting the coalescence of a binary with these parameters, we need to be sensitive to frequencies near 100 Hz and strains of order 10^{-21} or less.

Accessible V is ~ $M^{5/2}$ and one expected to first see BH/BH rather than neutron star/NS

BHs are rarer but n stars have mass < 2 M_{sun}, massive BH signal is stronger

Signal from merger was computed:

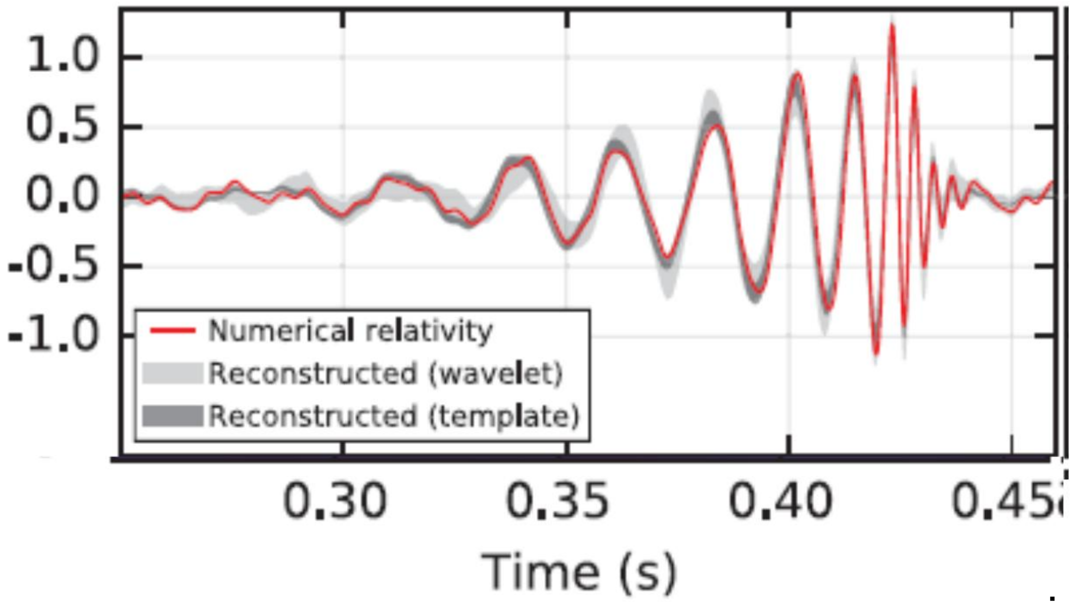
1109.3492



Very rich physics: J_1, J_2, L, J, BH, NS,...

Details of the GW150914 signal gave parameters of merger to remarkable accuracy:

10^{-21}



inspiral

Post-Newtonian

merger

Numerical

ringdown

BH quasinormal modes

QN modes: kick a BH, it starts oscillating with f and damps in time t_{damp}:

$$\frac{1}{f} = 0.8 \cdot 10^{-4} \frac{M}{M_{\mathrm{sun}}} s \qquad \tau_{\mathrm{damp}} = 0.5 \cdot 10^{-4} \frac{M}{M_{\mathrm{sun}}} s = 5.5 \frac{r_s}{c}$$

Helsinki: lots of work on QN modes in gauge-gravity duality

$$c,\ \hbar,\ G,\ G\hbar$$

Astropeople do not need $\hbar$!

c Special relativity SR you need when v/c is non-negligible

$$c = 299792458\,\mathrm{m/s} \quad c^2 \approx 10^{17}$$

$\hbar$ Quantum mechanics QM you need when

$$p \cdot x \sim \hbar = 1.0546 \cdot 10^{-34}\,\mathrm{J\,s} = 6.582 \cdot 10^{-25}\,\mathrm{GeV\,s}$$

G General relativity GR you need for large O(1)

$$\frac{2GM}{c^2 r} \equiv \frac{r_s}{r} \qquad r_s = r_{\mathrm{Schwarzschild}} = 2GM/c^2$$

BH radius

$$\frac{r_s}{r_{\mathrm{earth}}} = 1.3 \cdot 10^{-9} \qquad r_s(\mathrm{Earth}) = 0.88\,\mathrm{cm} \quad r_s(\mathrm{Sun}) = 2950\,\mathrm{m}$$

$$\frac{J_{\mathrm{BH}}^{\mathrm{max}}}{\hbar} = \frac{G}{\hbar c} m^2 = \frac{m^2}{m_{\mathrm{Pl}}^2} = \frac{S_{\mathrm{BH}}}{4\pi} \qquad S = \mathrm{entropy}$$

BH angular momentum

Planck units:

$$\begin{aligned} \text{mass} &= \sqrt{\hbar c/G} = 2.177 \cdot 10^{-8}\,\text{kg} \\ \text{length} &= \sqrt{\hbar G/c^3} = 1.616 \cdot 10^{-35}\,\text{m} \\ \text{time} &= \sqrt{\hbar G/c^5} = 5.391 \cdot 10^{-44}\,\text{s} \\ \text{power} &= c^5/G = 3.63 \cdot 10^{52}\,\text{W} \end{aligned} \quad = 2.0 \cdot 10^5 \, \frac{M_{\text{sun}} c^2}{s}$$

Solar luminosity $L_{\text{sun}} = 3.83 \cdot 10^{26}\,\text{W}$

Planck power (no $\hbar$) is huge since you release Planck energy

$$M_p = \sqrt{\frac{\hbar c}{G}} = 21.8\mu\text{g} = 1.22 \cdot 10^{19}\,\text{GeV}/\text{c}^2 = 543\,\text{kWh}$$

in extremely short Planck time

$$B_{\text{Pl}} = \frac{c^3}{eG} = 2.5 \cdot 10^{54}\,\text{T}$$

More big numbers: Power in GW150914

GW150914 released 3 solar masses of energy in ~ 0.01s:

$$P_{GW150914} = 5 \times 10^{49}\ \mathrm{W}$$

Universe has 2×10^{78} baryons, W_c=0.04, or $10^{78-57} = 10^{21} = 10^{10+11}$ stars

Power of all the stars = $10^{21} \times 4 \times 10^{26}$ = 4×10^{47} W

Official number: power of GW150914 was 50 x that of all the stars!

Flux on earth at a distance 410Mpc = 1.2×10^{25} m:

200 ms pulse with $0.03\,\frac{\mathrm{W}}{\mathrm{m}^2}$ No health hazard, couples weakly!

Mobile: ~ ms pulses 5 ms apart, peak power ~ W

For comparison: supernova luminosity

Collapse of Fe core of a massive star: binding energy of a neutron star

$$0.1 \times 1.4 \cdot M_{\rm sun} c^2 = 2.5 \cdot 10^{46}\,\mathrm{J}$$

released in the gravitational time scale $1/\sqrt{G\rho_{\rm Fe}} \approx 1\,\mathrm{s}$

in the form of 10^{58} neutrinos of energy 3 T ~ 15MeV

The previous close by was SN1987A on 23 Feb 1987, d ~ 10^{21} m
(K. Kajantie Arkhimedes 40, 57-65 (1988))

Flux ~ 10kW/m^2 in the form of 10^{15} neutrinos, 10 observed.

And neutrino detectors are waiting waiting waiting for the next one....

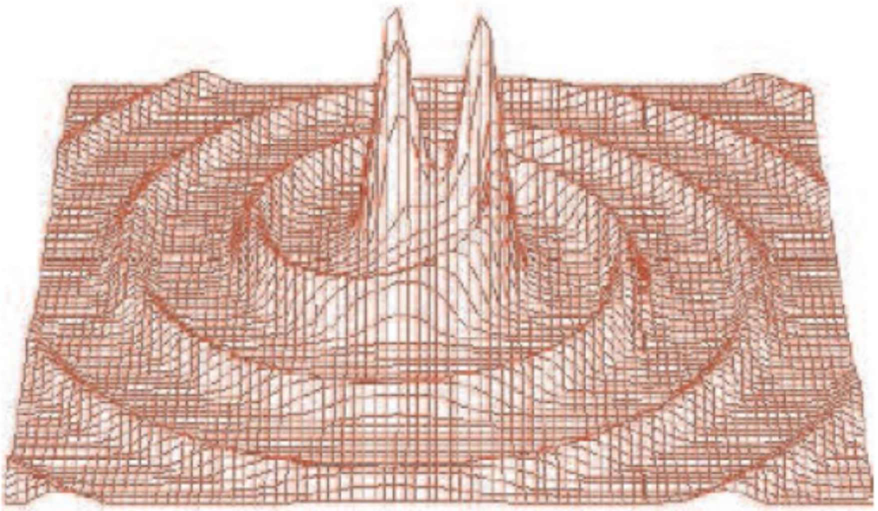
Source of gravitational wave luminosity: black hole merger

Two equal-mass BHs on circular path of diameter 2R radiate with power

$$P = -\frac{2}{5}\,\frac{c^5}{G}\left(\frac{r_s}{2R}\right)^5$$

Carroll (7.193)

$P_{GW150914}$ is obtained for $2R \sim 3r_s$, almost overlap, but of course this formula does not work for a merger, which has to be done numerically



No need to worry about
BH information paradox etc!

A special BH merger (in 10 ka !) OJ287, the Tuorla BH binary

(observed during > 100 y)

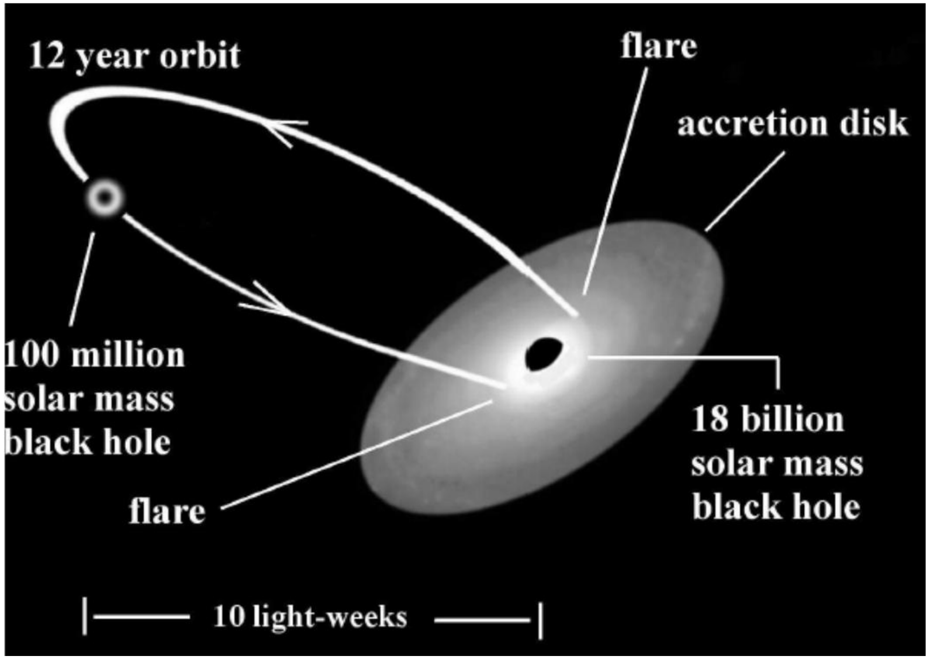
Helsinki: Pihajoki

r_{sbig} = 360 au
r_{ssmall}= 2.6 au
a = 10500 au
e = 0.72

$$P_{\mathrm{OJ287}} = 1.7 \cdot 10^{-11}\, P_{\mathrm{Planck}}$$

Peters PR136,B1224

Merger in 10 ka!



Valtonen et al, 0809.1280:

precision of this test is two days out of 10, that is, 20%. The next major periodic outburst is expected in early January 2016, by which time there may be methods to measure the gravitational waves directly.

was in Dec 2015!

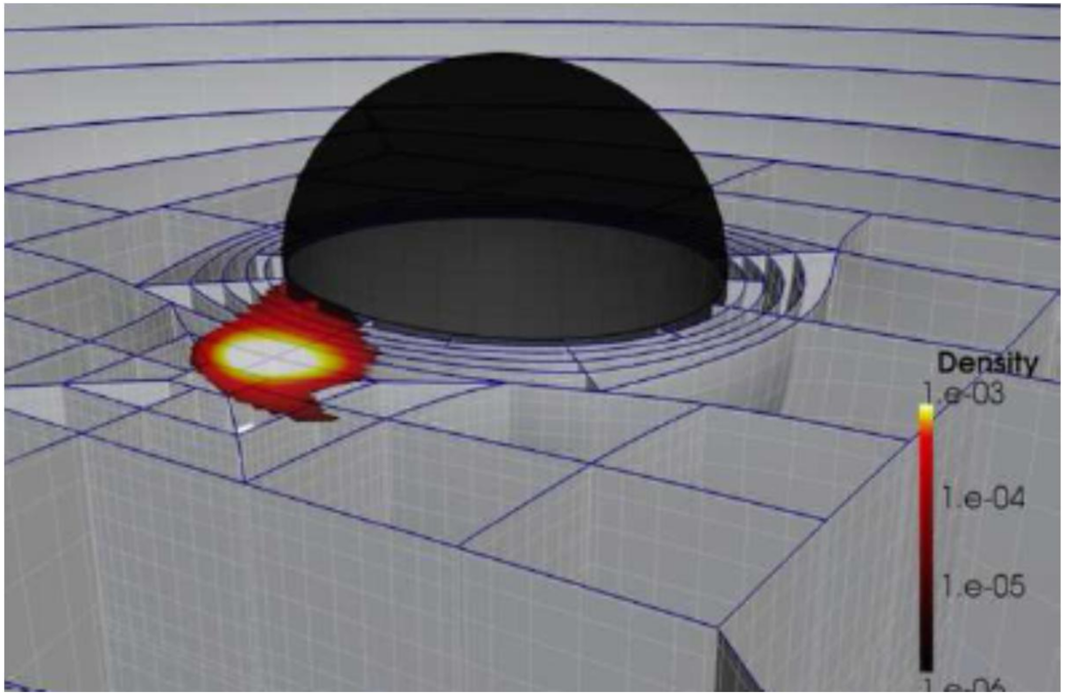
The above scenario is not yet universally accepted!

Laser interferometry of GR makes totally new measurements possible.

Example: measure radii of n stars to 10% accuracy by NS disruption by BHs, radiation pattern changes abruptly at the moment of disruption:

Helsinki: Vuorinen needs this

1212.4810



J_{BH} must be big!

Another new effect for the future: memory effect.

At present one only measures the oscillation of the reflectors of end caps of the interferometer.

After the pulse has passed, the positions of the objects have changed in an ultimately measurable and calculable way.

2. How do you compute intensity of grav rad?

Ask Einstein! Coordinates

$$(t, x, y, z) \quad (t, r, \theta, \phi) \quad (u = t - r, r, \theta, \phi) \quad (x^0, x^1, x^2, x^3)$$

Gravitons have $t \pm r$

Metric

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 = -dt^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

$$= -du^2 - 2dudr + r^2 d\Omega^2 \quad \text{or in general} \quad = g_{\mu\nu}(x) dx^\mu dx^\nu$$

The metric $g_{\mu\nu}$ is determined by matter energy-mom tensor via Einstein:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

2 2+0 -2 + 4

Ricci and scalar curvature are calculable from $g_{\mu\nu}$

Schwarzschild metric

Solve $R_{\mu\nu} = 0$ (no mass, no time dep!) in coordinates $x^\mu = (t, r, \theta, \phi)$

$$g_{\mu\nu}(x) = \begin{pmatrix} -(1-\frac{r_s}{r}) & 0 & 0 & 0 \\ 0 & \frac{1}{1-r_s/r} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix}$$

Carroll, Ch 5

$$d\tau^2 = (1-\frac{r_s}{r})dt^2 - \frac{1}{1-r_s/r}dr^2 - r^2\, d\Omega^2$$

Computing radial geodetic motion and comparing with Newton at large r (weak field) you fix $r_s=2GM/c^2$

Black hole with horizon at $r=r_s$

$$R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta} = \frac{12 r_s^2}{r^4}$$

Observationally mostly J_{BH} close to maximum GM^2/c: Kerr metric

Weak fields: Quantum Field Theory of gravitons

Linearize action around Minkowski space: $(-+++)$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Carroll 7.1, Weinberg 10.1

$$\int d^4x\, h^{\mu\nu}[-\partial^2\eta_{\mu\alpha}\eta_{\nu\beta}+\partial_\mu\partial_\alpha\eta_{\nu\beta}+\cdots+\eta_{\mu\nu}(\partial^2\eta_{\alpha\beta}-\partial_\alpha\partial_\beta)]\, h_{\alpha\beta}+\mathcal{O}(h^3)+\mathcal{O}(h^4)$$

Inverse graviton propagator

Very complicated 3- and 4-graviton vertices

Proceed as in perturbative QFT: finite to 1loop, divergent to 2 loops

Need new term $\sqrt{-g}R^{\mu\nu}{}_{\alpha\beta}R^{\alpha\beta}{}_{\gamma\delta}R^{\gamma\delta}{}_{\mu\nu}$ to cancel divergence: unrenormalisable

Goroff-Sagnotti NPB266(1986)709

Tree level gg->gg: gigantic computation, very compact result! Elvang, Scattering amplitudes, 1209.1607

Linearised equation of motion

$$R_{\mu\nu} - \tfrac{1}{2} R g_{\mu\nu} = -\frac{1}{2}(-\partial_t^2 + \nabla^2)(h_{\mu\nu} - \tfrac{1}{2} h \eta_{\mu\nu}) = 8\pi G T_{\mu\nu}$$

Just like in ED $\square A_\mu = J_\mu$ (skip gauge choice issues – crucial)

Fundamental equation of gravitational waves

$$(\partial_t^2 - \nabla^2) h_{ij} = 16\pi G\, T_{ij}$$

Just a wave equation solved with Green's functions, physics sits in T_{ij}

In Helsinki: Hindmarsh, Rummukainen, Weir (cosmological phase transitions)
Laine (cosmic matter at $T > T_{EW} = 100$ GeV), Hindmarsh (cosmic strings)

Simplest: $T_{\mu\nu}$ = 0

$$g_{\mu\nu} = \eta_{\mu\nu} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \epsilon_{11} & \epsilon_{12} & 0 \\ 0 & \epsilon_{12} & -\epsilon_{11} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{i\omega(t-z)} \quad p = (\omega, 0, 0, \omega)$$

Transverse traceless gauge

Very physical: if 2 masses separated by x^i, this changes by $\frac{1}{2} h_{ij} x^j$

Dx measured in LIGO with laser interferometry – l does not change

Amplitude of the gravitational wave, magnitude of e_{ij}=?

Want ~ G since this is gravity and ~ 1/r since this is radiation plus mass and time dependent stuff making h dimless

$$\ddot{I} \sim M\, L^2\, \Omega^2$$

$$\frac{r_s}{r} \sim \frac{GM}{c^2 r} \qquad \frac{G}{c^2 r}\frac{\dot{M}L}{c} \qquad \frac{G}{c^2 r}\frac{\ddot{M}L^2}{c^2} \equiv \frac{G\ddot{I}}{c^4\, r}$$

Mass cannot oscillate

Dipole moment cannot oscillate

2nd time derivative of quadrupole mom is fine!

Equal masses at a distance 2L rotating with frequency W

$$h_{ij} = \frac{r_s}{r}\frac{r_s}{2L}\begin{pmatrix} -\cos 2\Omega u & -\sin 2\Omega u & 0 \\ -\sin 2\Omega u & \cos 2\Omega u & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

u=t-z

Rs ~ 10^5 m, r ~ Ga ~ 10^{25} m, r_s/L ~ 10^{-1} get h ~ 10^{-21}

3. Gravitational radiation and inflation

Helsinki: Huge cosmo group

On 17 March 2014 Bicep2 reported 6 s evidence for ”r=0.20”, effectively grav rad. Disappeared into dust. What is this gravitational radiation?

Inflation has 4 scalar (4 dofs), 2 vector (4 dofs) and a tensor (2 grav rad dofs) perturbation around the RW metric $g_{\mu\nu} = \mathrm{Diag}\left(-1, a^2(t), a^2(t), a^2(t)\right)$ very very early. These evolve also beyond the horizon and seed the inhomogeneities observed in CMB.

The observation of the effect of TT modes on the spectral index is a very indirect and thoroughly processed observation of gravitational waves

GW150914 was direct

4. $\hbar$ physics

Ligo has initiated a new path in BH physics and astrophysics in general

Problems remain. BHs only depend on M and J (and Q). If a drop a ton of matter to a BH its mass increases by a ton, the same happens if I drop a ton of antimatter.

A BH evaporates into photons (and gravitons) in the time

$$t_{\text{ev}} = 20 \cdot 256\pi \frac{M_0^3}{M_p^3}\, t_p \sim 10^{-16} s\, \frac{M_0^3}{\text{kg}^3}$$

How is the difference between matter and antimatter dropped into the BH built in the final state massless particles? Or is the info lost?

I will not talk about this, just show a beautiful pic:

BH info paradox:

How do you unitarily evolve a quantum state over the formation and decay of the BH:

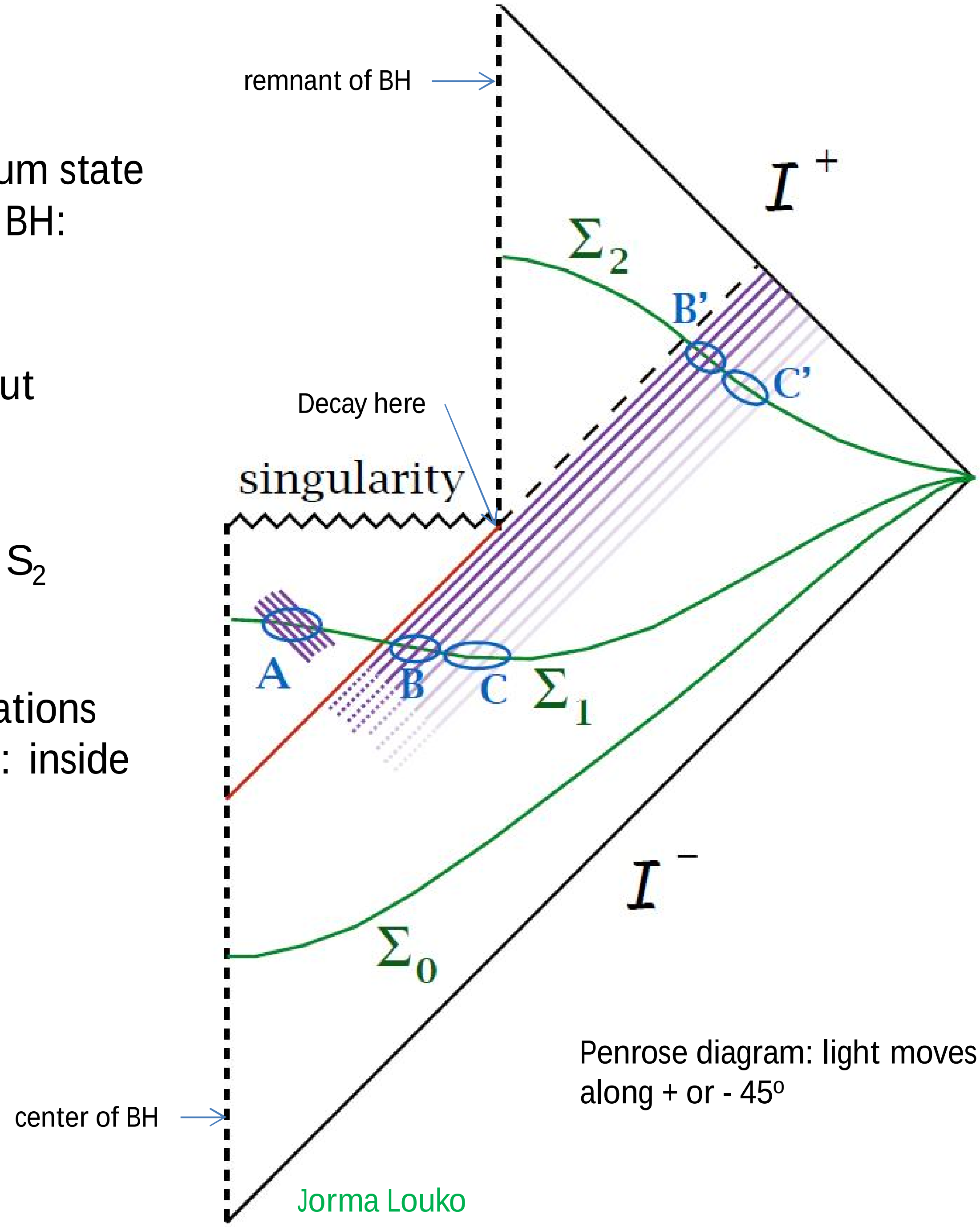
$$\psi(\Sigma_2) = U\,\psi(\Sigma_0)$$

Seemed paradoxical: pure state in, out comes radiation.

Resolution: radiation entangled on S_2

A consequence on S_1 is that correlations in and out of BH have to be broken: inside is basically inaccessible. Firewall!

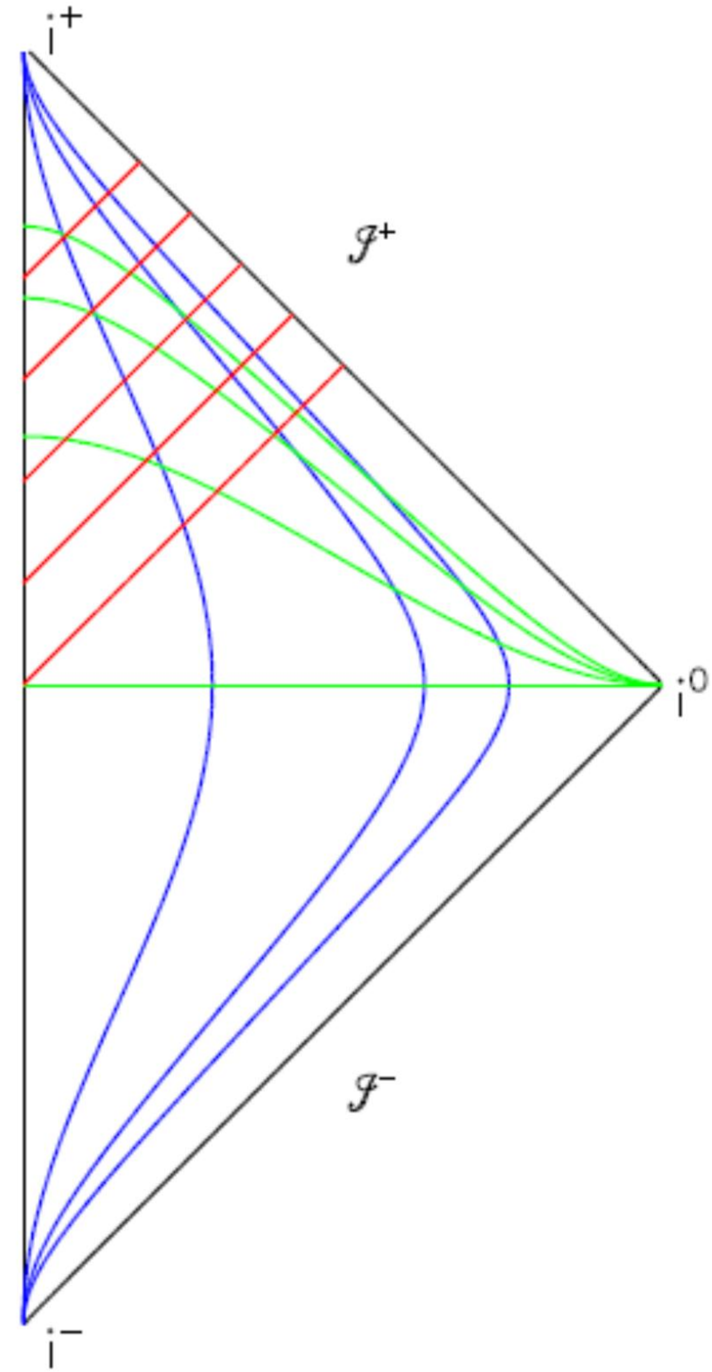
These are just words!
Do not respect them!



Conclusions

New era in observational astronomy starts: remarkably detailed new data on BH/BH, BH/NS, NS/NS mergers will be coming

But path to understanding quantum gravity is as unknown as always



i+
ℐ+
i0
ℐ−
i−

Hawking radiation

BHs have temperature $T_H = \frac{\hbar c}{4\pi r_s} \sim \frac{10^{23}\,\mathrm{kg}}{M}\,\mathrm{K}$ entropy $S = \frac{c^3}{\hbar}\frac{A}{4G} \equiv \frac{A}{4L_P^2}$

BH thermo: $E = Mc^2 \quad dE = T_H\, dS$ Carlip 1410.1486

Prototype derivation: Accelerated mirror in 1+1d Birrell-Davies sect 4.4 Carroll sect 9.5

Traschen gr-qc/0010055

BHs are unstable $\frac{dM}{dt} \sim -r_s^2 T_H^4 = -\frac{\hbar c^4}{60\cdot 265\pi}\frac{1}{G^2M^2}$ $M(t) = M_0(1-\frac{t}{t_{\rm ev}})^{1/3}$

Here decay to photons

$$t_{\rm ev} = 20\cdot 256\pi \frac{M_0^3}{M_p^3}\,t_p \sim 10^{-16} s\,\frac{M_0^3}{\mathrm{kg}^3}$$

In nature $M_{BH} > M_{sun}$?
Milky Way BH is BIG: $M \sim 4\cdot 10^6\, M_{\rm sun} \quad r_s = 12\cdot 10^6\,\mathrm{km} \approx \frac{1}{10}\,\mathrm{au}$