Hot QCD in the limit of large number of colors and flavors

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Alho - Järvinen - Kajantie - Kiritsis –Tuominen, 1210.4516 16 Oct

Järvinen-Kiritsis 1112.1261

Holographitis is spreading: Jokela, Järvinen, Tahkokallio,
Keränen, Suur-Uski, Alanen, Taanila, Alho, Franti + 4 older

1. Hot QCD

$$e^{p(T)\frac{V}{T}} = \int \mathcal{D}A\mathcal{D}\psi \; e^{-\int d^4x \left[\frac{1}{g^2(\mu)}F^2 + \bar{\psi}(\partial + A)\psi + m(\mu)\bar{\psi}\psi\right]}$$

Solve p(T) for m_q=0 protected by **chiral symmetry**

- Chiral effective theories $\Phi_{ij} = \langle \psi_i \bar{\psi}_j \rangle$
- Lattice Monte Carlo (vast effort!) $V \to \infty,\; a \to 0,\; m_q \to 0$ or $m_u, m_d, m_s, m_c, ..$ physical
- Pert theory: $p/T^4 = 1 + g^2 + g^3 + g^4 \log g + g^4 + g^5 + g^6 \log g + g^6 + ..$
- Holography: >5dim gravity dual, $N_c,\; N_c g^2 \gg 1$

Coupling and mass run, are scheme dependent:

$$\lambda(\mu) \equiv \frac{N_c g^2(\mu)}{8\pi^2} \qquad x_f = \frac{N_f}{N_c}$$

We include **all** $\lambda(\mu)$

$$\mu\frac{d\lambda}{d\mu} = \beta(\lambda) = -\underbrace{\frac{11 - 2x_f}{3}}_{b_0}\lambda^2 - \underbrace{\frac{34 - 13x_f}{6}}_{b_1}\lambda^3 - ..$$

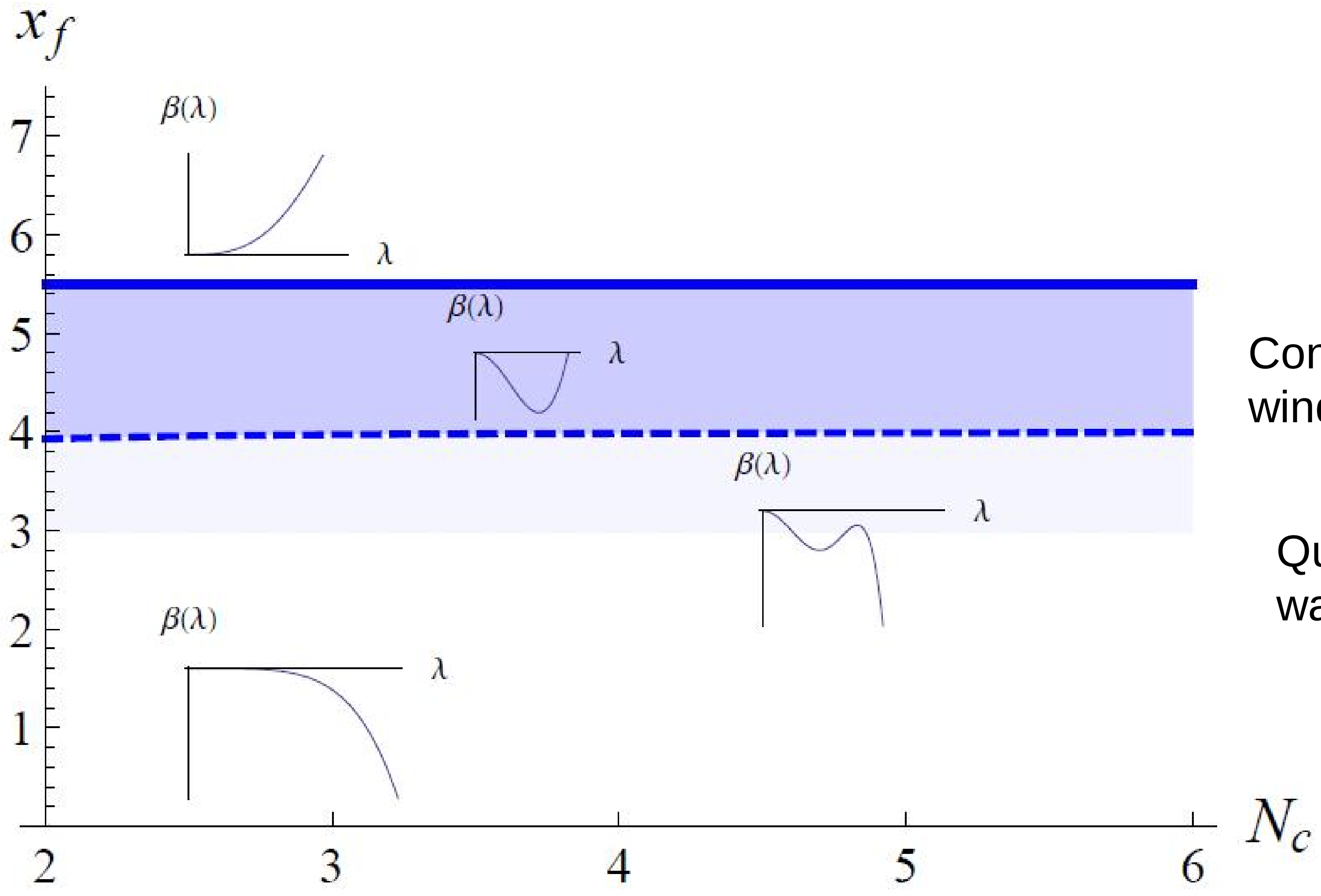
$$\mu\frac{d\log m}{d\mu} = \gamma_m(\lambda) = -\underbrace{\frac{3}{2}}_{\gamma_1}\lambda - ..$$

For $x_f > 5.5$ loose asymptotic freedom!

$$\log\mu = \int_{\lambda_0}^{\lambda(\mu)} \frac{d\lambda}{\beta(\lambda)}$$

$$m(\mu) = m_0(\log\mu)^{-\frac{9}{2(11-2x_f)}}$$

Ranges of x_f:



Conformal window

Quasiconformal walking

Simple thermal argument for $x_c=4$:

At low T dominant, easiest-to-excite modes are N_f^2 Goldstone bosons (usual chiral symmetry breaking)

At some Tc these melt into $2N_c^2 + \frac{7}{8} 4 N_f N_c$ gluons and quarks

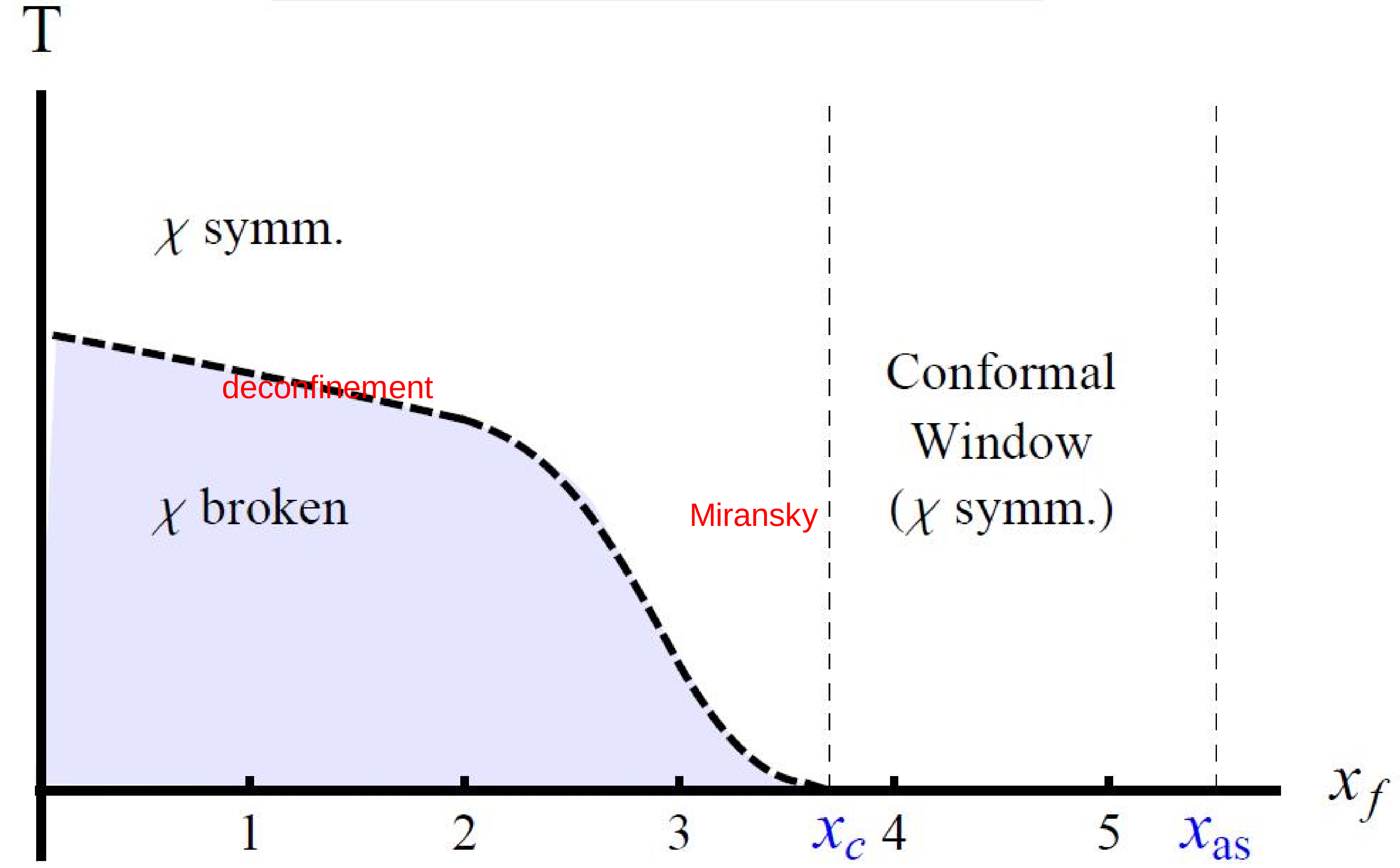
Assume latent heat is ~ difference between these numbers. One enters the conformal regime when latent heat vanishes.

It vanishes at $N_f = 4\ N_c$

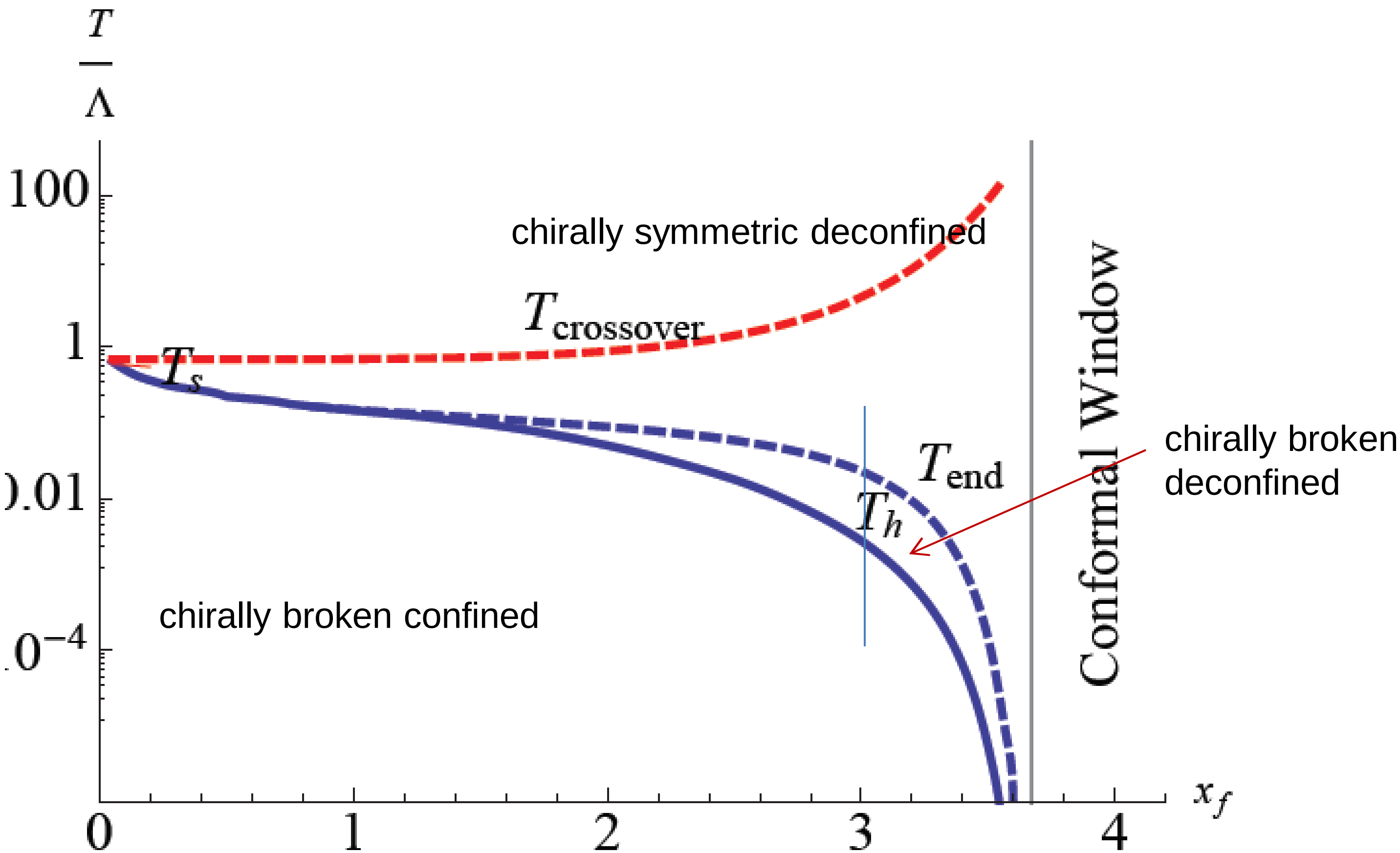
Expected phase diagram:

Nf=0
YM

Nc=Nf=3

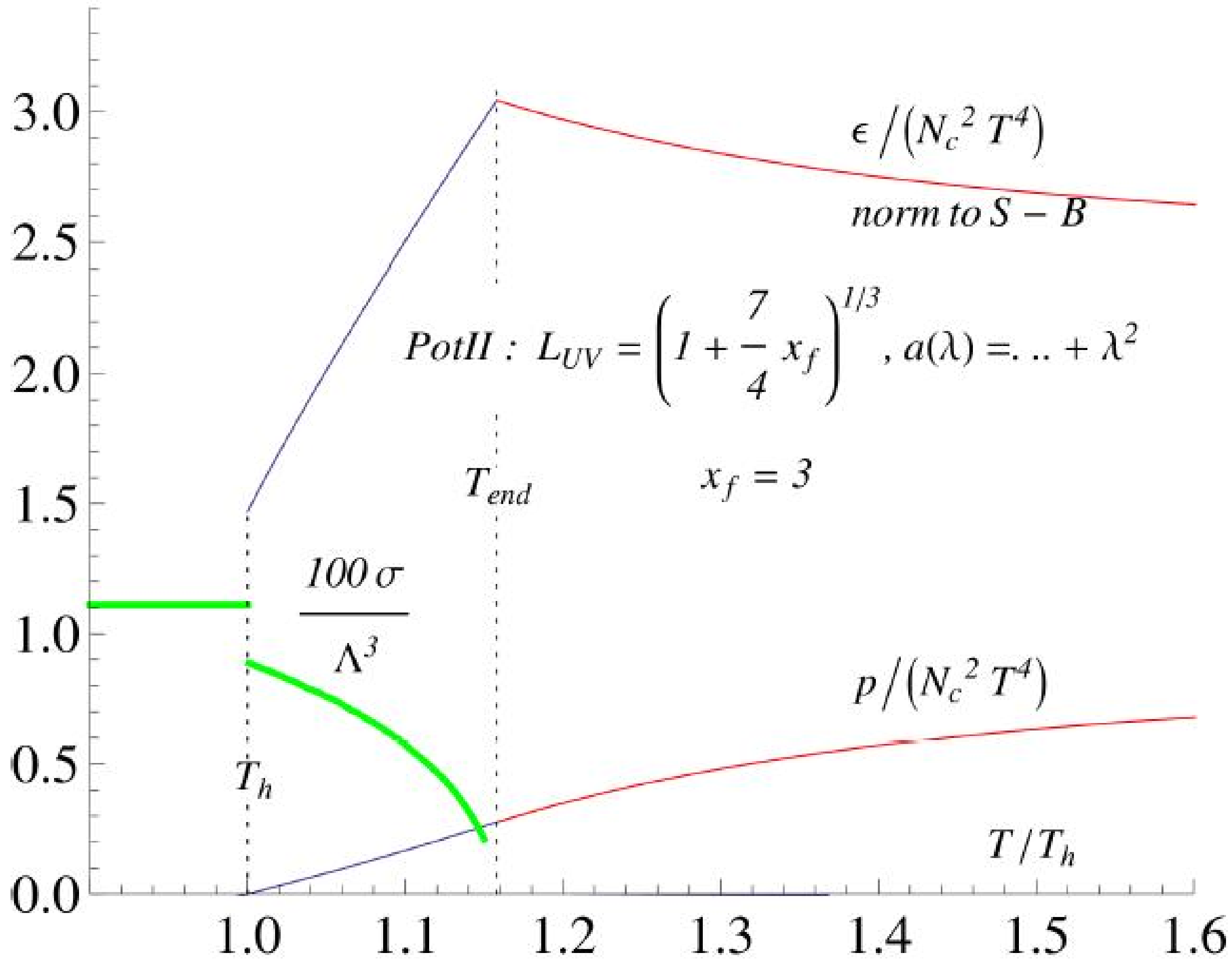


Results of computations using holography



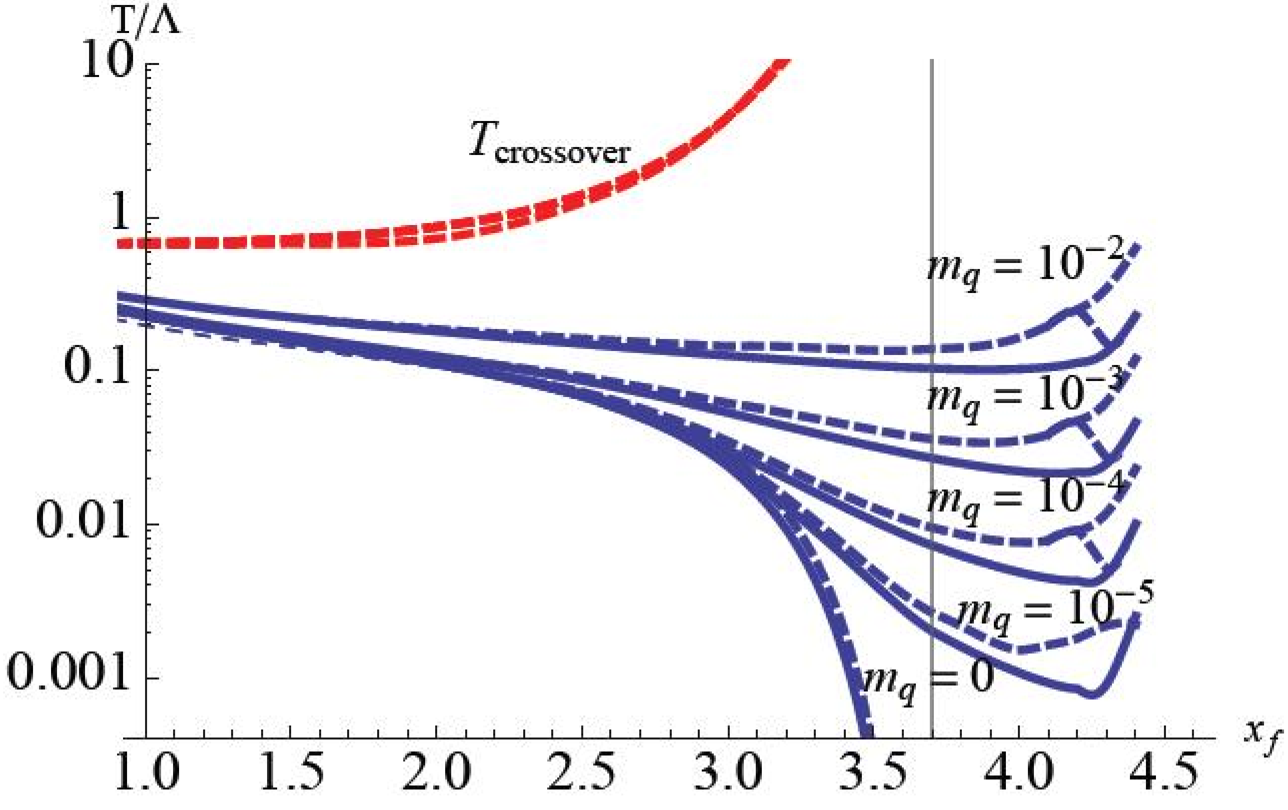
$T_{\text{chiral}} > T_{\text{deconfined}}$

Along $x_f = 3$:



3.0
2.5
2.0
1.5
1.0
0.5
0.0
1.0
1.1
1.2
1.3
1.4
1.5
1.6
$\epsilon / (N_c^2 T^4)$
norm to S − B
$PotII : L_{UV} = \left(1 + \frac{7}{4} x_f\right)^{1/3}, a(\lambda) =. .. + \lambda^2$
T_{end}
$x_f = 3$
$\frac{100\,\sigma}{\Lambda^3}$
$p / (N_c^2 T^4)$
T_h
T / T_h

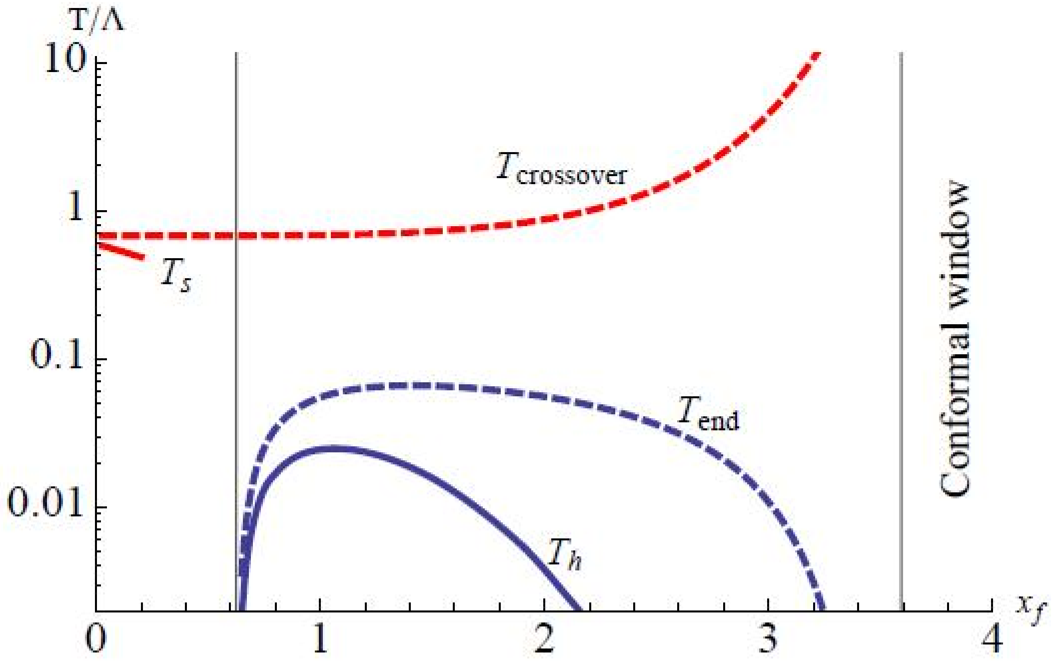
Quark mass has drastic effects;

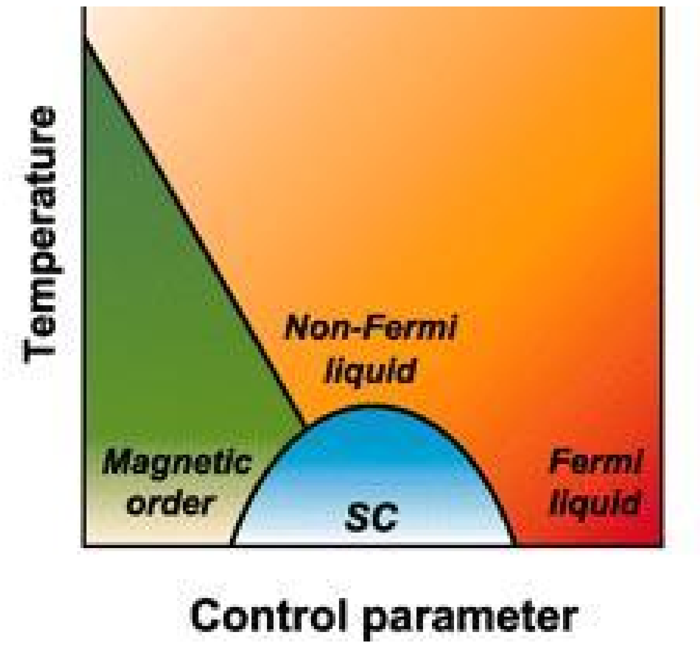


Conformal
window
disappears

Weirder (for QCD) phase diags are also produced:

But compare "superconducting dome"





2. Building blocks of the gravity dual

Metric functions b(z), f(z):

$$ds^2 = b^2(z)\left[-f(z)dt^2 + d\mathbf{x}^2 + \frac{dz^2}{f(z)}\right]$$

If $b(z) = \frac{\mathcal{L}}{z}, \quad f(z) = 1 - \frac{z^4}{z_h^4}$

AdS_5 black hole dual to
hot N=4 SYM = conformal

Dilaton λ(z): $\beta(\lambda) = b\frac{d\lambda}{db} \quad \lambda(z) = \frac{1}{b_0 \log(1/\Lambda z)} + \ldots$

breaks
conformality

leading UV running

Tachyon τ(z):

$$\tau(z) = m\,(\log\frac{1}{\Lambda z})^{-\frac{3}{2b_0}}\,z + \langle\bar{q}q\rangle\,(\log\frac{1}{\Lambda z})^{\frac{3}{2b_0}}\,z^3 + ..$$

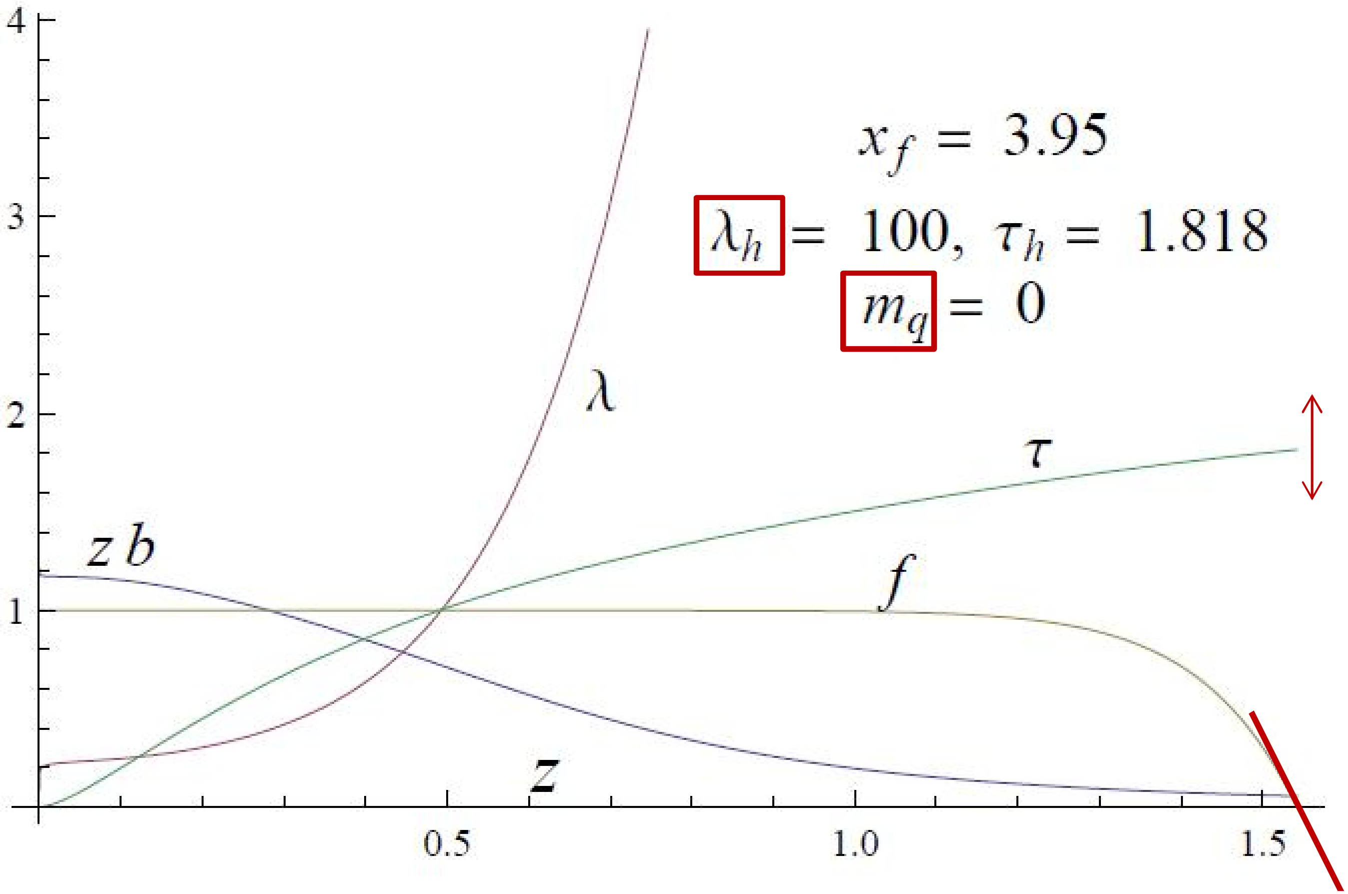
leading UV running of m

The four functions $b(z)$, $f(z)$, $\lambda(z)$, $\tau(z)$ are obtained as solutions of Einstein's equations from an action tuned so that the required UV behavior + confinement at large z is obtained

As in lattice Monte Carlo, particularly time consuming is fixing $m_q = 0$. One has to choose $\tau(z_h)$ properly:

Phases are black hole solutions. Solutions with the smallest Einstein action are the equilibrium ones.

Typical bulk field configuration:



Tune $\tau(z_h)$ so that near $z=0$ $\tau(z) \sim z^3$

$$4\pi T = -f'(z_h)$$

Here is the gravity action:

$$S = \frac{1}{16\pi G_5}\int d^5x\,\sqrt{-g}\,\mathcal{L} = S[g_{\mu\nu},\lambda,\tau]$$

$$\mathcal{L} = R + \left[-\tfrac{4}{3}\,g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V_g(\lambda)\right]$$

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$$-\,x_f V_f(\lambda)e^{-\mu^2\tau^2}\sqrt{1+g^{zz}(1+\lambda(z))^{-4/3}\tau'(z)^2}$$

Matched to β function near λ=0

remnant of

$$e^{-\phi}R + .. \to R + ..$$

$$V_g(\lambda) = \frac{12}{\mathcal{L}_0^2}\left[1+\frac{88\lambda}{27}+\frac{4619\lambda^2}{729}\frac{\sqrt{1+\ln(1+\lambda)}}{(1+\lambda)^{2/3}}\right]$$

confinement at large λ

$$\mathrm{EOM}:\ \frac{\delta S}{\delta g^{\mu\nu}}=0,\quad \frac{\delta S}{\delta\lambda}=0,\quad \frac{\delta S}{\delta\tau}=0$$

Tachyon action is particularly interesting; **string theory** enters

When string tension grows, strings become points

Closed string theory, containing gravity, becomes supergravity

Open string theory, containing gauge theory, goes to DBI:

Dirac-Born-Infeld

$$\mathcal{L}_{\rm DBI} = -\frac{1}{\ell^4}\sqrt{-\det(g_{\mu\nu} + D_\mu\tau D_\nu\tau + \ell^2 F_{\mu\nu})}$$

$$\underset{\tau\equiv 0}{=} -\frac{1}{\ell^4}\sqrt{1-\ell^4(E^2-B^2)-\ell^8(E\cdot B)^2} = \frac{1}{2}(E^2-B^2)+\frac{1}{2}\ell^4(E\cdot B)^2+\ldots$$

$$\ell^2 = 1/T = 2\pi\alpha'$$

Thermo is now computed as follows:

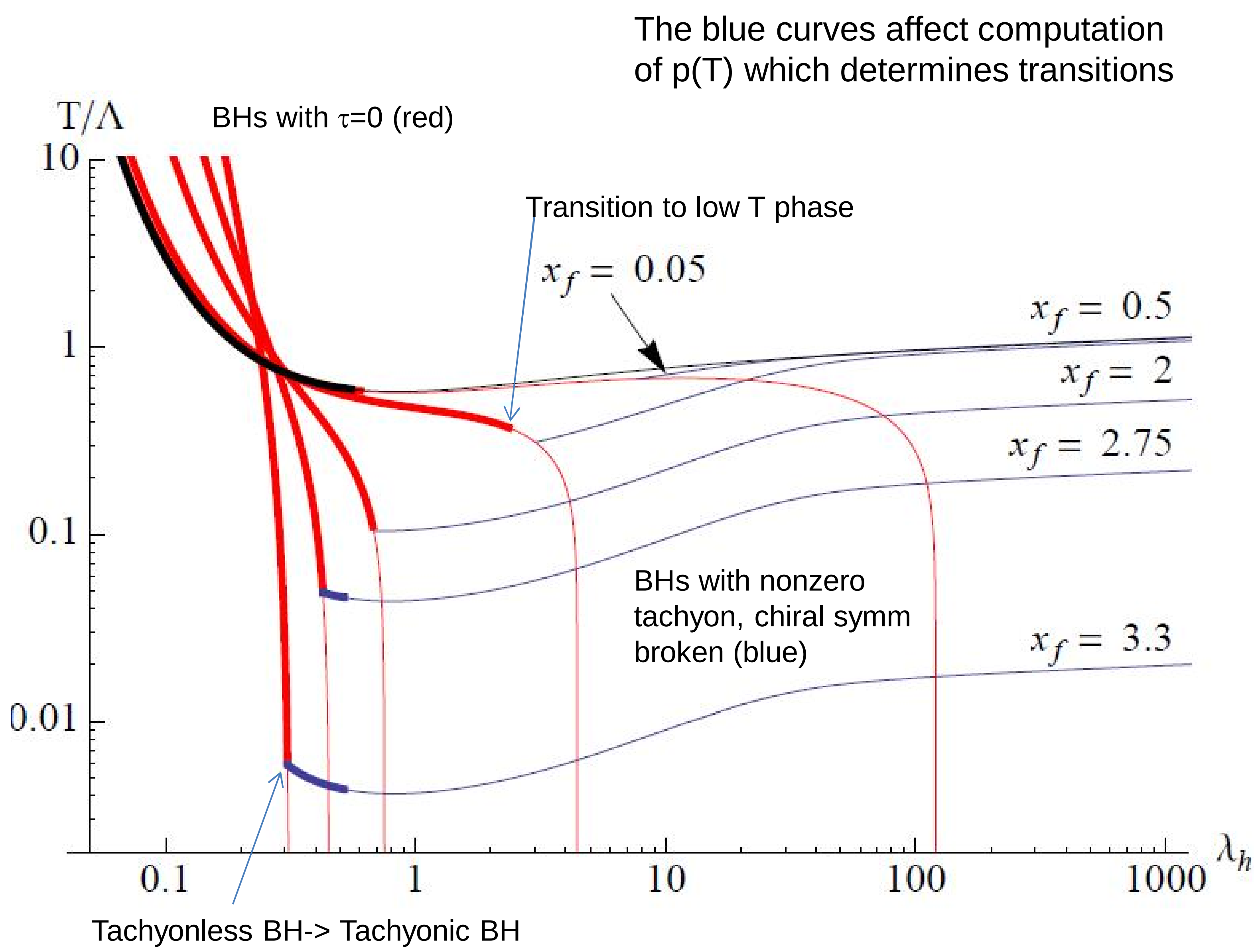
- Pick a value of coupling at horizon, $\lambda(z_h)$

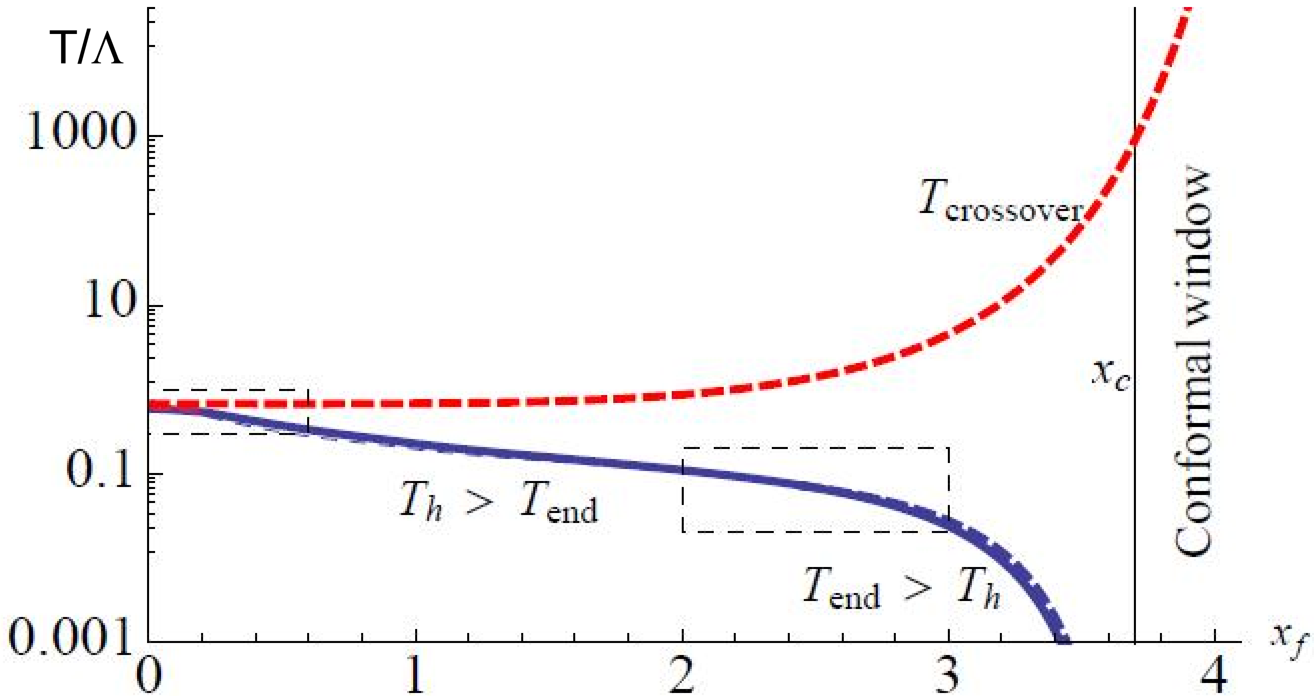
- Find $\tau(z_h)$ so that m_q=0 (or some other value)

- Compute temperature from $$4\pi T = -f'(z_h)$$

- Compute entropy density from $$sV_3 = \frac{A}{4G_5} = \frac{b^3(z_h)}{4G_5}$$

- Integrate p(T) from p'(T) = s(T). Find phase with largest p


The blue curves affect computation
of p(T) which determines transitions
T/Λ
BHs with τ=0 (red)
10
1
0.1
0.01
Transition to low T phase
$x_f = 0.05$
$x_f = 0.5$
$x_f = 2$
$x_f = 2.75$
$x_f = 3.3$
BHs with nonzero
tachyon, chiral symm
broken (blue)
0.1
1
10
100
1000
λ_h
Tachyonless BH-> Tachyonic BH

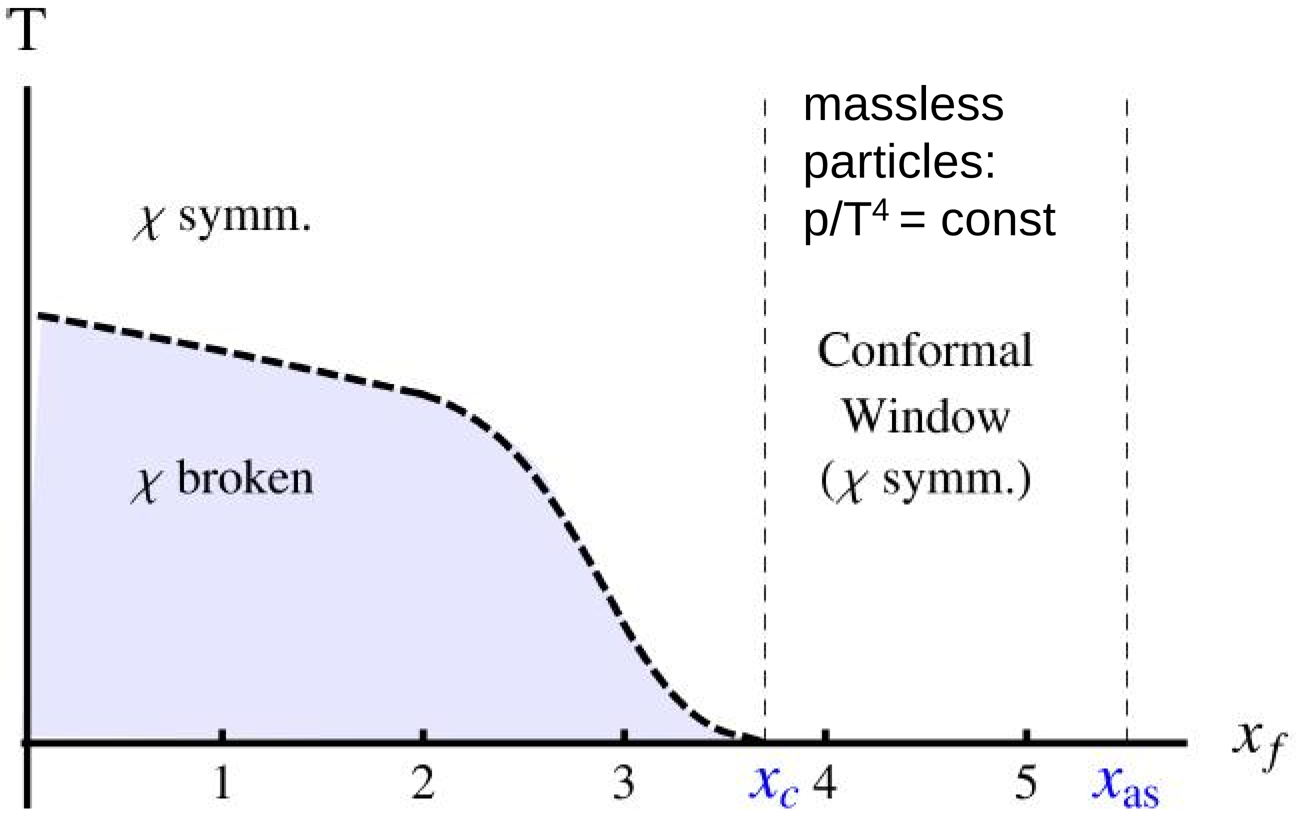


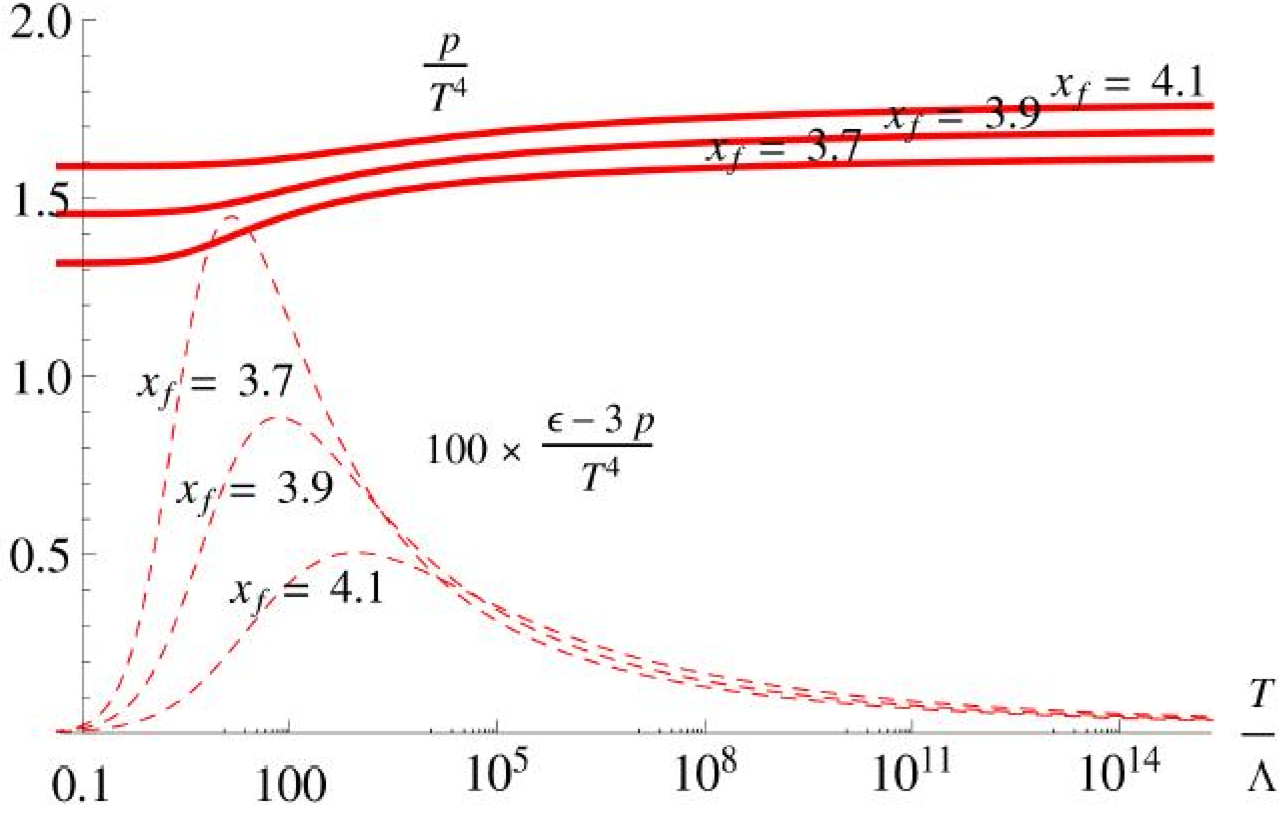
Miransky scaling hard to study numerically

Analytically: mass ~ $\exp\left(-\frac{2\hat{K}}{\sqrt{x_c - x_f}}\right)$

Thermo in conformal window?

$$N_f \gtrsim 4N_c \quad m_q = 0$$





Normalised to SB at T=infty

3. Conclusions

- The potentials $V_g(\lambda)$, $V_f(\lambda)$ are constrained but not completely: little predictive power. Offers a **framework, alternatives**

- The subtle interplay between confinement/chiral symmetry and black holes with and without tachyons is impressive (to me)

- Not a cheap simple way to solve QCD!

- Much to do: more and better numbers, other BSM theories (technicolor!), correlators, chemical potential, magnetic fields, theta vacua,....

Overflow

’t Hooft, Witten, Veneziano limits

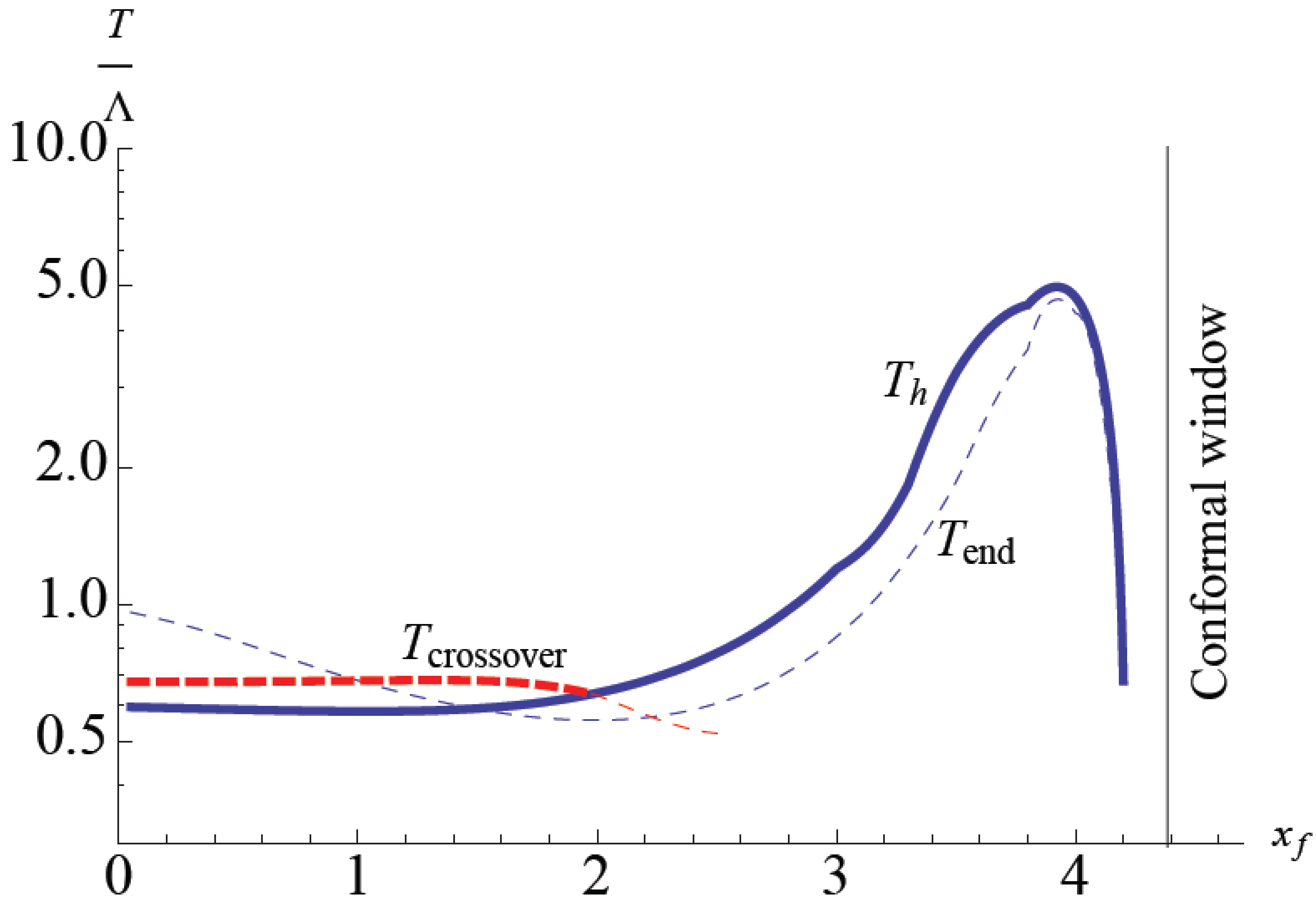
’t Hooft limit: λ fixed, N_c large

Chiral anomaly:

$$\partial_\mu(\bar{\psi}\gamma^\mu\gamma_5\psi) \sim g^2 N_f\, \tilde{F}_{\mu\nu}F^{\mu\nu} = \frac{\lambda}{N_c} N_f\, \tilde{F}_{\mu\nu}F^{\mu\nu}$$

Witten: at fixed λ, N_f large N_c switches of the anomaly

Veneziano: keep N_f/N_c fixed at large N_c



$\frac{T}{\Lambda}$
10.0
5.0
2.0
1.0
0.5
0
1
2
3
4
x_f
T_h
T_{end}
$T_{\text{crossover}}$
Conformal window

Gauge/gravity duality

$$\langle \exp[i \int \overset{-4}{d^4x}\, \overset{\Delta_-}{\phi_0(x)}\, \overset{4-\Delta_-}{\mathcal{O}(x)}] \rangle$$

$$\exp[i \int d^4x \int dz \sqrt{-g}\mathcal{L}_{\text{grav}}[g_{\mu\nu}, ..., \phi(x,z)]]$$

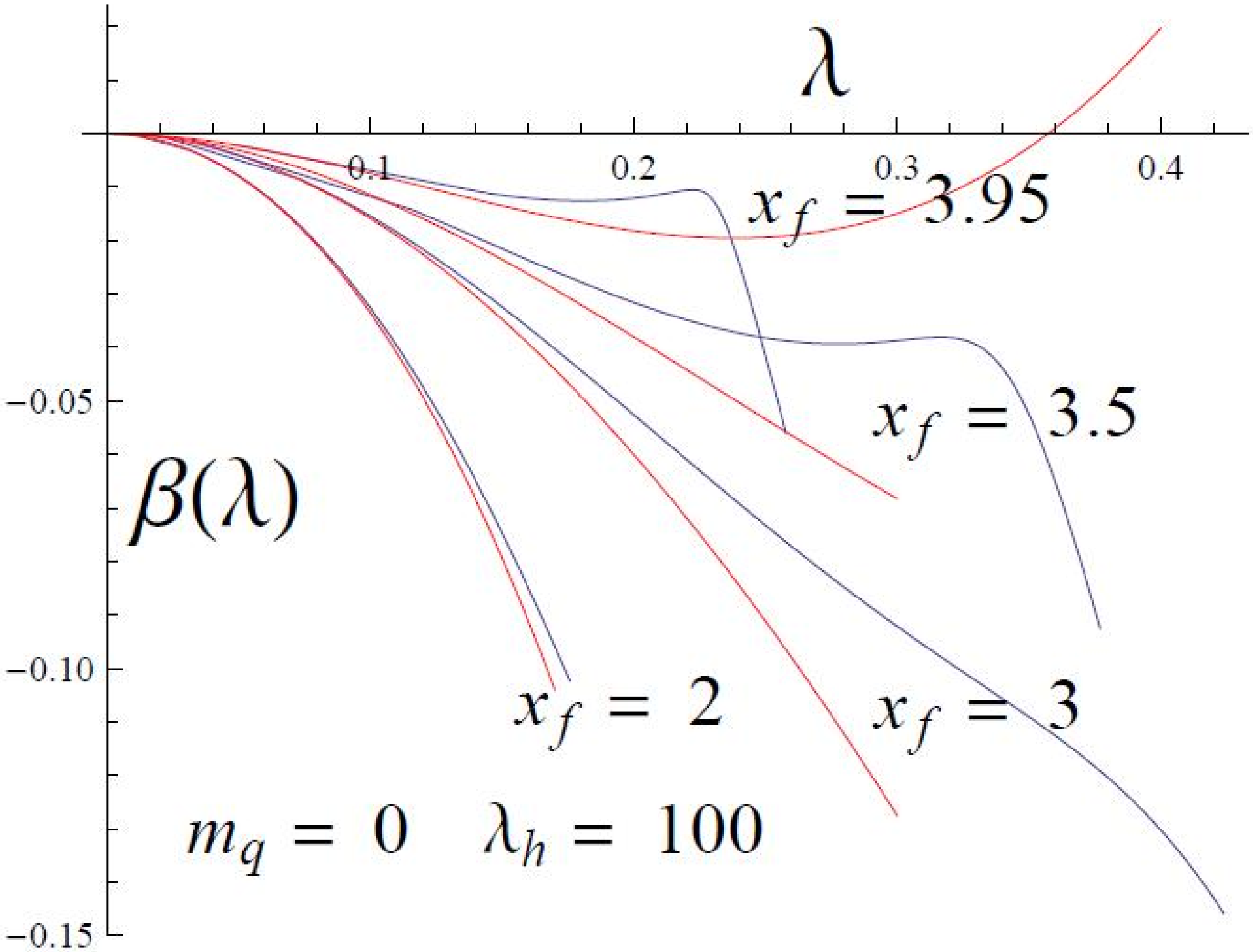
$$\phi(x,z) = \phi_0(x) z^{\Delta_-} + \langle\mathcal{O}\rangle z^{4-\Delta_-} + ...$$

Dofs of gravity ~ area, not volume!

AdS_5 has boundary at z=0 and scale L

N_c, $g^2 N_c$ large

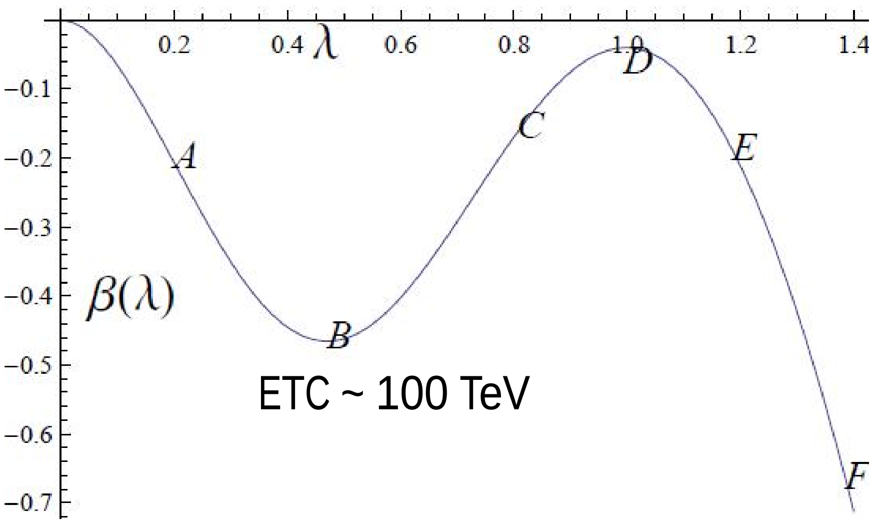
Appearance of ”walking” with increasing N_f/N_c:



Below conformal window: quasiconformal, walking technicolor

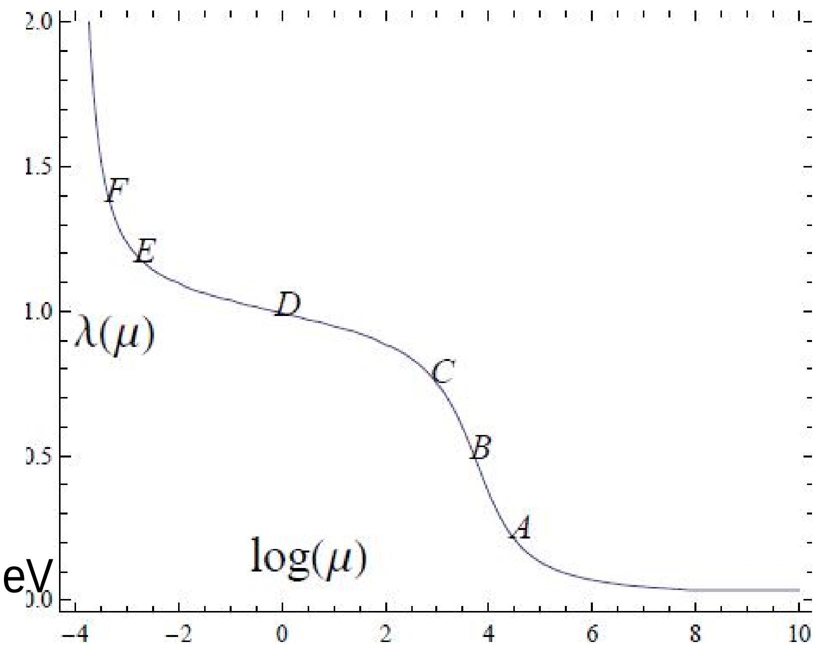
Alanen-Kajantie-Tuominen 1003.5499, Alanen-Alho-Kajantie-Tuominen 1107.3362

$$\beta(\lambda) = -c\lambda^2 \frac{(1-\lambda)^2 + e(N_f)}{1 + a\lambda^3} \qquad c = 8, \ \ a = 1, \ \ e = 0.01$$



ETC ~ 100 TeV

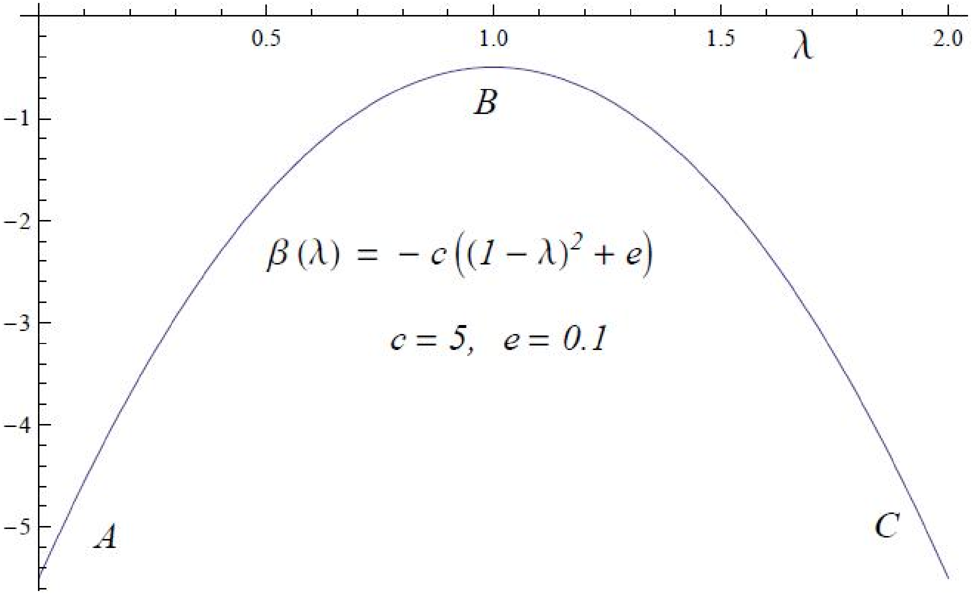
TC ~ 246 GeV



Coupling runs -> condensate walks

Coupling walks -> condensate runs (want this)

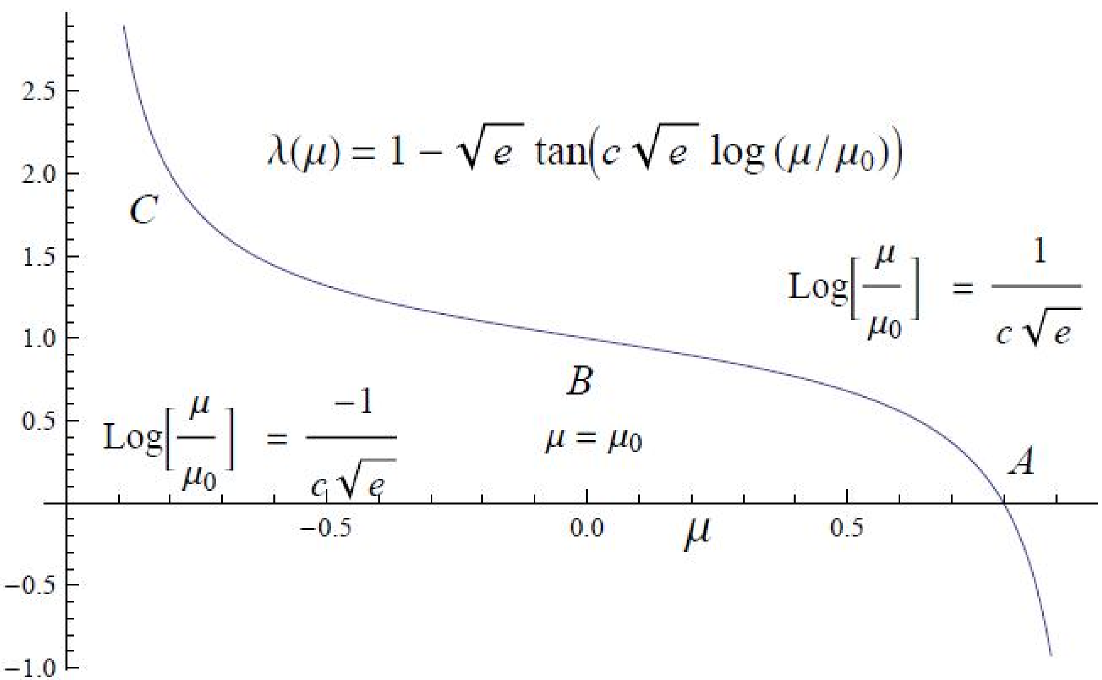
Approaching conformality:
KT-Miransky scaling:



$$\frac{d\lambda}{d\log\mu} = -c[(1-\lambda)^2 + e]$$

$$\lambda(\mu) = 1 - \sqrt{e}\tan(c\sqrt{e}\log\frac{\mu}{\mu_0})$$

$$\frac{\mu_{\rm IR}}{\mu_{\rm UV}} = C\, e^{-\frac{\pi}{\sqrt{e}}}$$

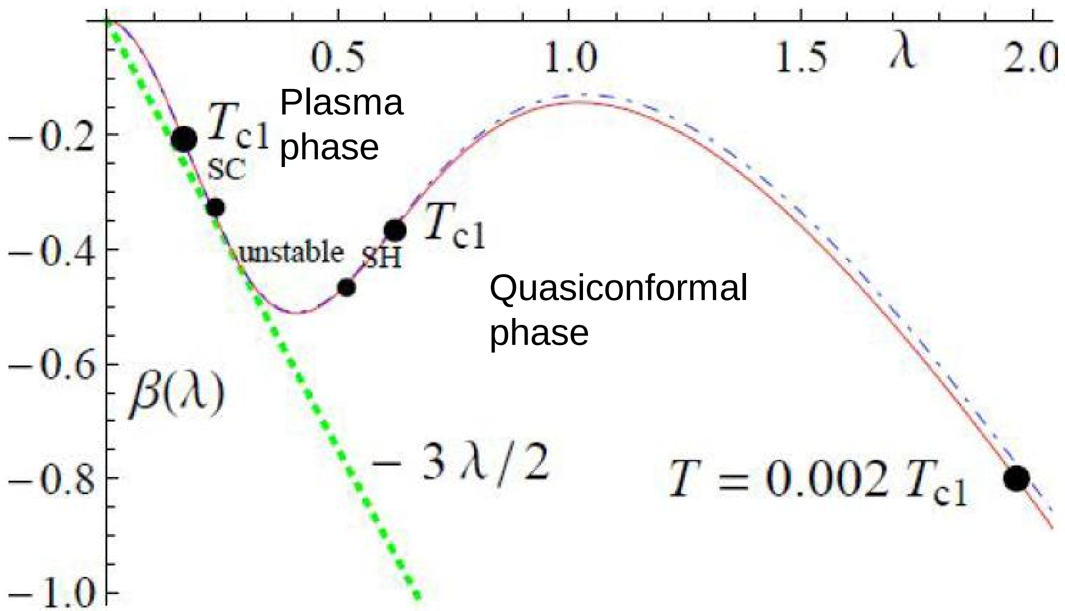


Model 1: build N_f dependence in the beta function

Alanen-Kajantie-Tuominen 1003.5499 Alanen-Alho-Kajantie-Tuominen 1107.3362

$$\beta(\lambda) = -c\lambda^2 \frac{(1-\lambda)^2 + e}{1 + \frac{2}{3} c\, \lambda^3}$$

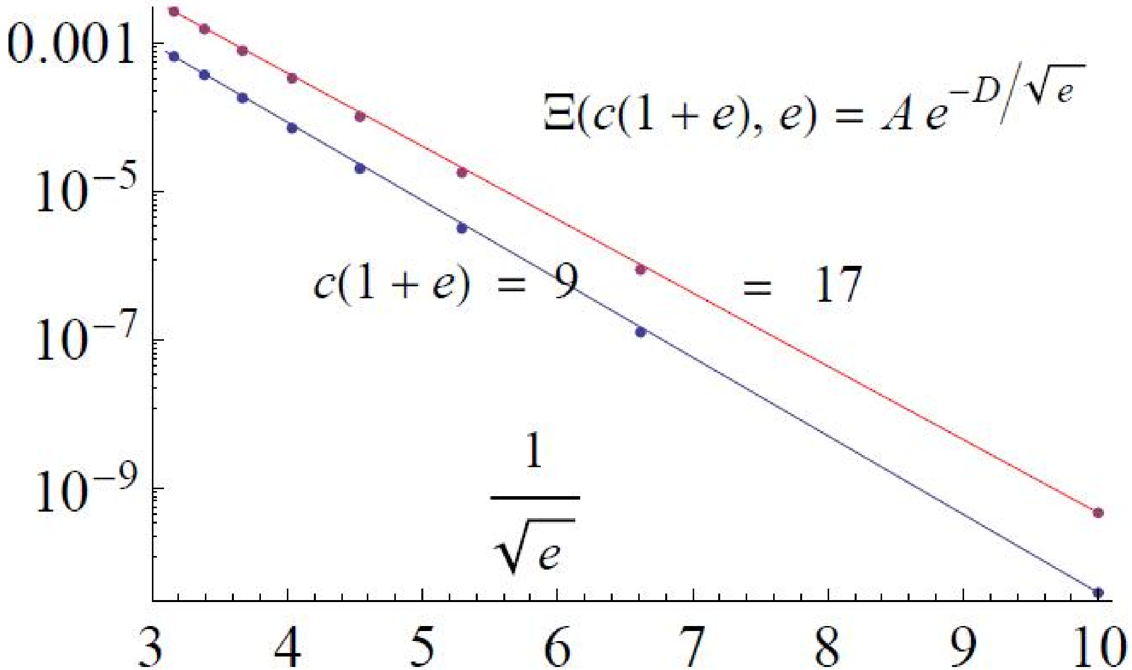
Fix two scales, $\Lambda_{\mathrm{ETC}} \sim 10^3 \Lambda_{\mathrm{TC}}$ 2 transitions, 3 phases



Confining, chiral
symm broken phase

When e approaches 0 all masses should also approach zero, conformality

4d prediction: $\exp\left[-\left(\frac{2}{3}+\frac{1}{c}\right)\frac{\pi}{\sqrt{e}}\right]$



5d computation

Path to this prediction: ansatz for 5d bulk metric, solve numerically Einstein's equations, solve numerically scalar field equations in this background, compute eigenvalues of Schrödinger equation. Striking that the result is as predicted!