The initial state of little bang and classical gluon fields

K. Kajantie

keijo.kajantie@helsinki.fi

University of Helsinki, Finland

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Work with F. Gelis and T. Lappi

Problem: How do you relate

An ultrarelativistic ($\gamma = E/M \gg 1$) heavy ($A \approx 200 \gg 1$) ion collision

and

$$L_{\text{QCD}} = \frac{1}{4}(\underbrace{\partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f_{abc} A^b_\mu A^c_\nu}_{F^a_{\mu\nu}})^2 + \\ + \sum_{udscbt} \{\bar{\psi}_f [(\underbrace{\partial_\mu + i g A^a_\mu T^a}_{D_\mu})\gamma_\mu + m_f]\psi_f\}$$

Classical EOM extremize the action:

$$D_\mu = \partial_\mu + i g A_\mu, \quad F_{\mu\nu} = (ig)^{-1}[D_\mu, D_\nu], \quad [D_\mu, F_{\mu\nu}] = J_\nu, \quad [D_\nu, J_\nu] = 0$$

Parameters: $N_c, g, m_f \Rightarrow N_c, g(\mu), m_f(\mu), \Lambda_{\text{QCD}}$ in Quantum Physics.

Scenario, parameters

$A + A$ central collision, $A \approx 200$, $\gamma =$100 (RHIC) =2250 (LHC),
$v \equiv \tanh(y_B) \Rightarrow y_B =$5.3 (RHIC) $=$ 8.4 (LHC).

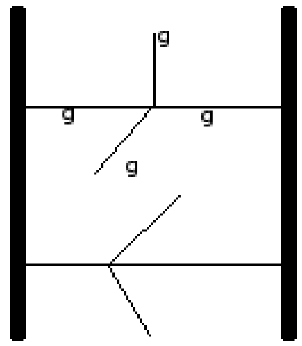
Max size $\sim 2R_A \approx$ 12 fm, max time $\sim R_A/c_s \approx$ 10 fm/c $+$ some expansion.

Perturbative picture:

A nucleus is a "cloud of partons". Small $x = p_L/E$ gluons dominate. $A + A$ is $\Sigma(gg \to gg)$. N:o of gluons saturates:

$$\partial_t N(t,x) = \underbrace{\partial_x^2 N}_{\text{diffusion}} + \underbrace{N - N^2}_{\text{logistics}}$$

Dominant momentum: saturation scale Q_s



$Q_s = 1$ GeV (RHIC) $= 2$ GeV (LHC)

Classical (+ quantum initial condition) field picture:

Dense saturated system of gluons $\Rightarrow$ large occupation numbers $\Rightarrow$ classical gluon fields.

Source of those fields? Nuclei moving with $v \approx c = 1$.

EM Weizsäcker-Williams fields: Solve Maxwell for $v \to 1$ in

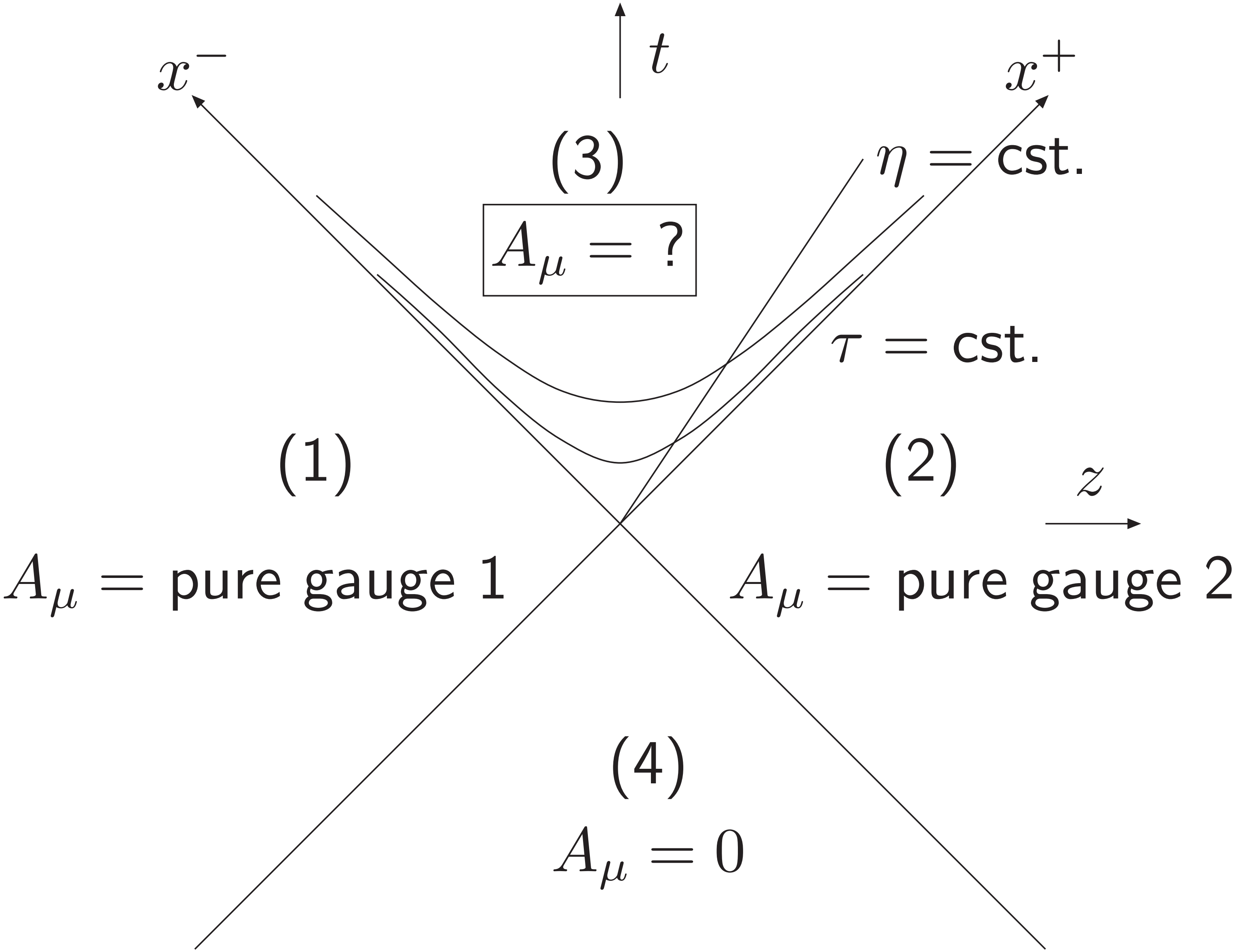
$$J^0(t, \mathbf{x}) = e\delta(x)\delta(y)\delta(z - vt), \quad J^z = vJ^0, \quad J^x = J^y = 0.$$

Nice exercise! Lots of technicalities: use $x^\pm = t \pm z$, choose gauge:

$$(A^+, A^-, \mathbf{A}_T) = (\frac{e}{2\pi}\delta(t - z)\log\frac{C}{x_T}, 0, \mathbf{0}) \to (0, 0, \underbrace{\frac{e}{2\pi}\frac{\mathbf{x}_T}{x_T^2}\theta(t - z)}_{\text{vacuum for } t>z})$$

The gauge invariant $F_{\mu\nu}$ corresponds to a $\delta(t - z)$ pulse of quasi-real photons $\equiv$ WW photons.

Photon-photon collisions: $e^+e^- \to \gamma\gamma + X \to$ .....


t
x^-
x^+
(3)
$\eta =$ cst.
$A_\mu = ?$
$\tau =$ cst.
(1)
(2)
z
$A_\mu =$ pure gauge 1
$A_\mu =$ pure gauge 2
(4)
$A_\mu = 0$

Now take the two nuclei as zero-thickness ($\delta(x^\pm)$) transverse colored disks moving in opposite directions:

$$J^\mu = \delta^{\mu +}\delta(x^-)\rho_{(1)}(\mathbf{x}_T) + \delta^{\mu -}\delta(x^+)\rho_{(2)}(\mathbf{x}_T).$$

Color is built in

$$\rho(\mathbf{x}_T) \equiv \rho^a(\mathbf{x}_T)T^a.$$

Can you physically fix the color at each point $\mathbf{x}_T$? Of course not! But one may think of drawing them from a statistical ensemble so that colors at different $\mathbf{x}_T$ are uncorrelated and, say, Gaussian-distributed at fixed $\mathbf{x}_T$:

$$\langle \rho^a_{(m)}(\mathbf{x}_T)\rho^b_{(m)}(\mathbf{y}_T)\rangle = g^2\mu^2\delta^{ab}\delta^2(\mathbf{x}_T - \mathbf{y}_T), \quad m = 1, 2.$$

Essential parameter:

$$g^2\mu \sim 1...2\,\mathrm{GeV} \quad \leftrightarrow \quad Q_s.$$

This charging is a Brownian process (McLerran, Helsinki, 1982),

$$\langle x^2\rangle = Dt \Rightarrow \langle \mu^2\rangle = \mathrm{const}\, A^{1/3}$$

.

What about solving

$$[D_\mu(A), F_{\mu\nu}(A)] = J^\nu$$

for $A_\mu \equiv A_\mu^a T^a$?

Is

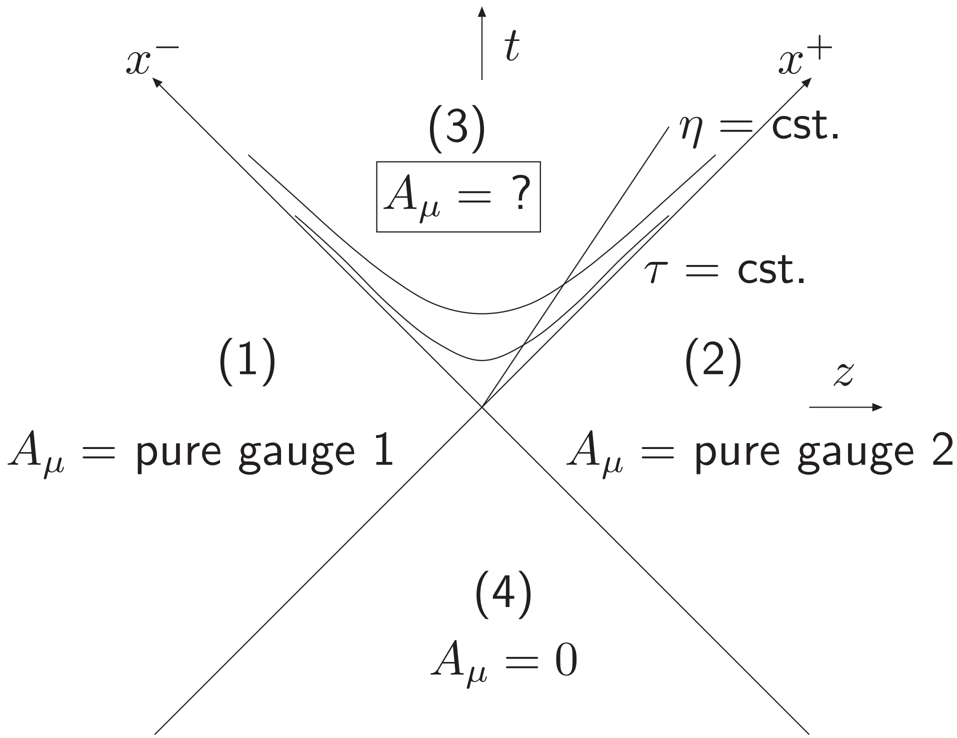
$$[D_\mu, J^\mu] = 0$$

satisfied? Yes, for a proper gauge choice!

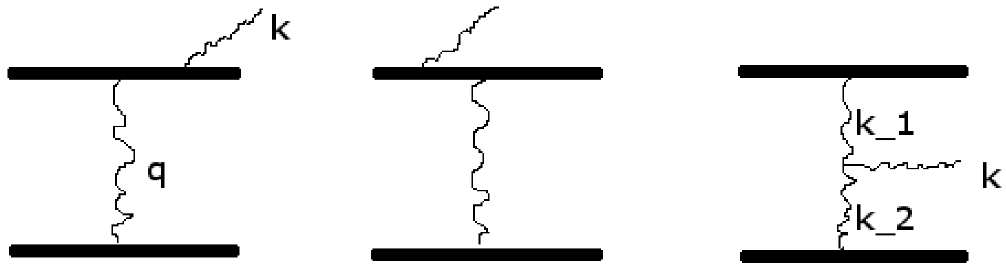
Since J_μ lives on the light cone, one will actually be solving $[D_\mu, F_{\mu\nu}] = 0$ with boundary conditions on the light cone:

$A_i(\mathbf{x}_T) = U(\mathbf{x}_T)\partial_i U^{-1}(\mathbf{x}_T)$,

$U(\mathbf{x}_T) \in$ SU(3) is vacuum,
$F_{\mu\nu} = 0$



Solving $[D_\mu, F^{\mu\nu}] = J^\nu$ iteratively in charge densities ρ_1, ρ_2 of the two currents:



$$A^\mu_{12,a}(k) = \frac{-ig}{k^2} \int \frac{d^4 k_1}{(2\pi)^4} \, c^\mu(k, k_1, k_2) \quad \underbrace{A^+_{1,b}(k_1) A^-_{2,c}(k_2)}_{\text{WW fields from both currents}}$$

$$c^\mu(k, k_1, k_2) = ... \quad \text{famous Lipatov vertex}$$

Classical fields also give perturbative weak-field tree-level gluon emission!

We want strong fields!

For $\tau > 0$ want

$$A_\mu(\tau, \eta, \mathbf{x}_T) = (\underbrace{A_\tau = 0}_{\text{gauge choice}}, \underbrace{A_\eta(\tau, \mathbf{x}_T)}_{\sim\text{longit.}}, \mathbf{A}_T(\tau, \mathbf{x}_T))$$

which are then converted to energy in and number of gluons.

Remarkable initial condition: matching to two vacua below the light cone obtain:

$$\begin{aligned} A^i(\tau = 0, \mathbf{x}_T) &= A^i_{\text{vac1}}(\mathbf{x}_T) + A^i_{\text{vac2}}(\mathbf{x}_T), \\ A^\eta(\tau = 0, \mathbf{x}_T) &= \frac{1}{2} ig[A^i_{\text{vac1}}(\mathbf{x}_T), A^i_{\text{vac2}}(\mathbf{x}_T)]. \end{aligned}$$

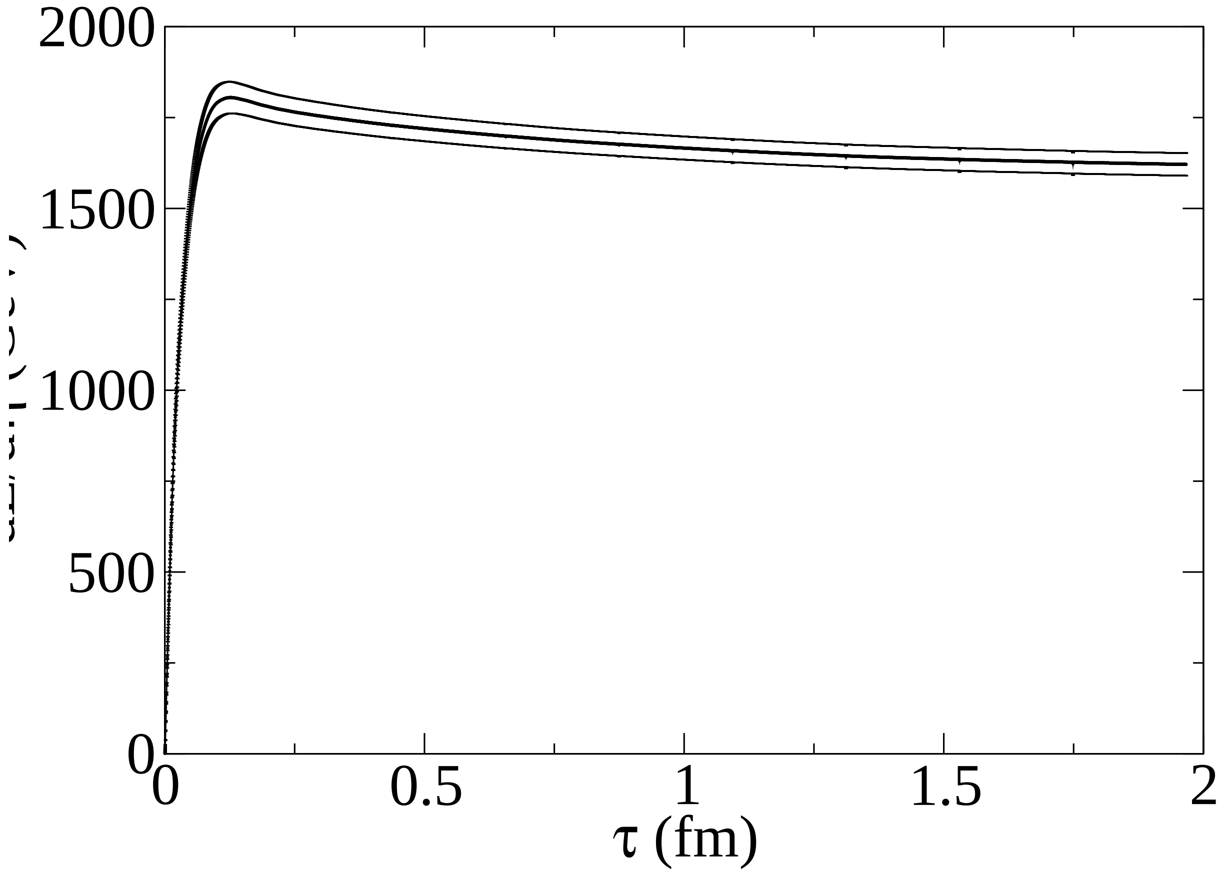
For Non-Abelian theory sum (nor commutator) of two vacua is NOT a vacuum!!

(while $\partial_i\chi_1 + \partial_i\chi_2 = \partial_i(\chi_1 + \chi_2)$).

Set up the numerical computation on a, say, 512×512 transverse lattice (Krasnitz, Venugopalan, Lappi).

Parameters: $g^2\mu, R_A$.

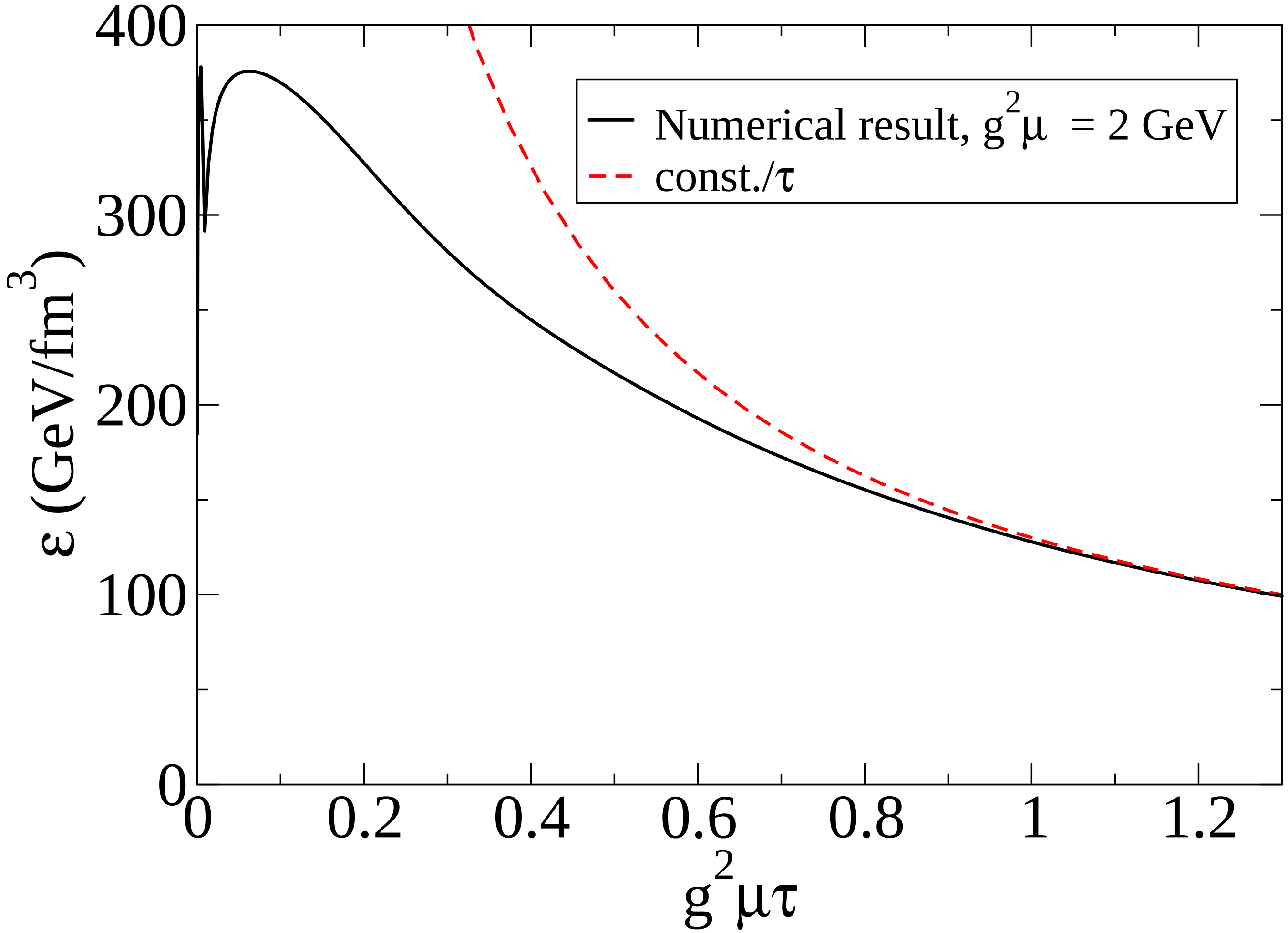
Main output: energy density plotted as $dE/d\eta = \mathrm{V}\epsilon = \pi R_A^2 \tau\, \epsilon$:



$$g^2\mu = 2 \text{ GeV} \Rightarrow 1/g^2\mu = 0.1 \text{ fm}$$

Sudden rise at $\tau = 1/Q_s$, then $\epsilon\tau =$ const, no thermalisation, other physics.

But you can as well plot $\epsilon(\tau)$:



$\Rightarrow$ gluon production is instantaneous, all the action is on light cone.

Creation of little bang; followed by thermalisation, expansion, hadronisation,...

Find analytically:

$$\epsilon(\tau = 0) = \langle \int \frac{d^2\mathbf{x}_T}{\pi R_A^2} \frac{H(\mathbf{x}_T)}{\tau}|_{\tau=0} \rangle$$

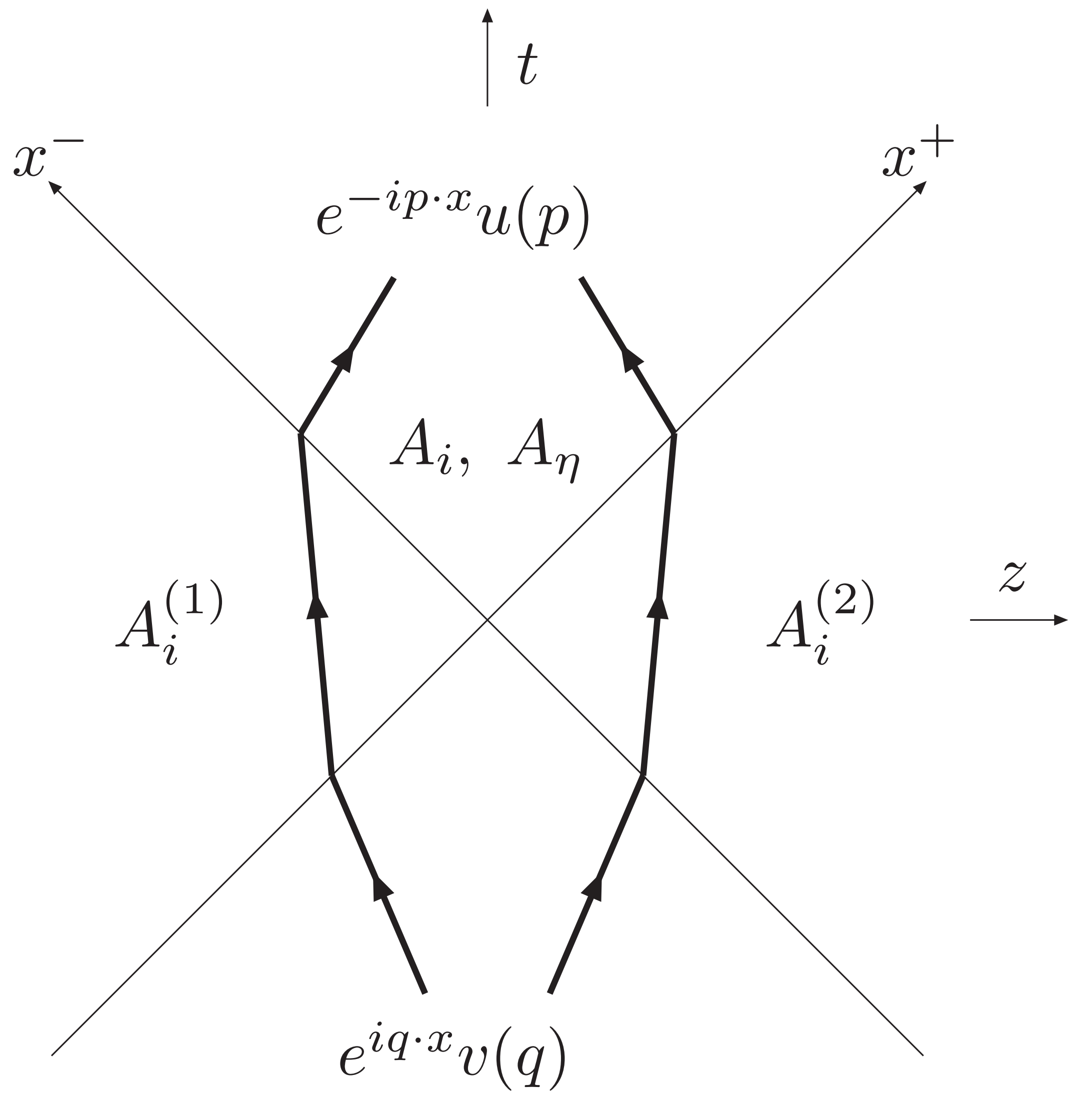
$$\frac{H(\mathbf{x}_T)}{\tau}|_{\tau=0} = g^2(\delta_{ij}\delta_{kl}+\epsilon_{ij}\epsilon_{kl})\mathrm{Tr}[A_i^{(1)}(\mathbf{x}_T), A_j^{(2)}(\mathbf{x}_T)]\,[A_k^{(1)}(\mathbf{x}_T), A_l^{(2)}(\mathbf{x}_T)]$$

Remember, independently for the two nuclei

$$A_i = \frac{i}{g} U\partial_i U^\dagger, \quad U = e^{i\Lambda}, \quad -\partial_T^2\Lambda = g\rho \quad \rho = \text{stochastic source}$$

The initial energy density of little bang is given by the ensemble average of Tr product of two commutators of vacuum fields

Now that you have A_μ, does it produce $q\bar{q}$ pairs? Strong or time dependent fields produce particles.



The matrix element is

$$M_\tau(p,q) \equiv \int \frac{\tau \mathrm{d}z \mathrm{d}^2\mathbf{x}_T}{\sqrt{\tau^2+z^2}} \phi^\dagger_{\mathbf{p}}(\tau,\mathbf{x})\gamma^0\gamma^\tau\psi_{\mathbf{q}}(\tau,\mathbf{x}) \ .$$

Now you have to set up a truly 1+3 d computation for integrating $\psi_{\mathbf{q}}(\tau,\mathbf{x})$ using Dirac.

Lattice spacing: $(N_T a)^2 = \pi(6.7\,\mathrm{fm})^2 \Rightarrow a = 12\,\mathrm{fm}/N_T \approx 0.05$ fm.

Number count: $\psi^c_{\mathbf{q}}(\tau,\mathbf{x})$ has $180^2\times 400$ numbers for $\mathbf{x}$, 3 for $c=3$ colors, 2·4 for ψ, a total of 1.2 GB single precision. This set is integrated forward in steps of $d\tau = 0.02a$ in 500 steps to get to $\tau = 0.25$ fm.

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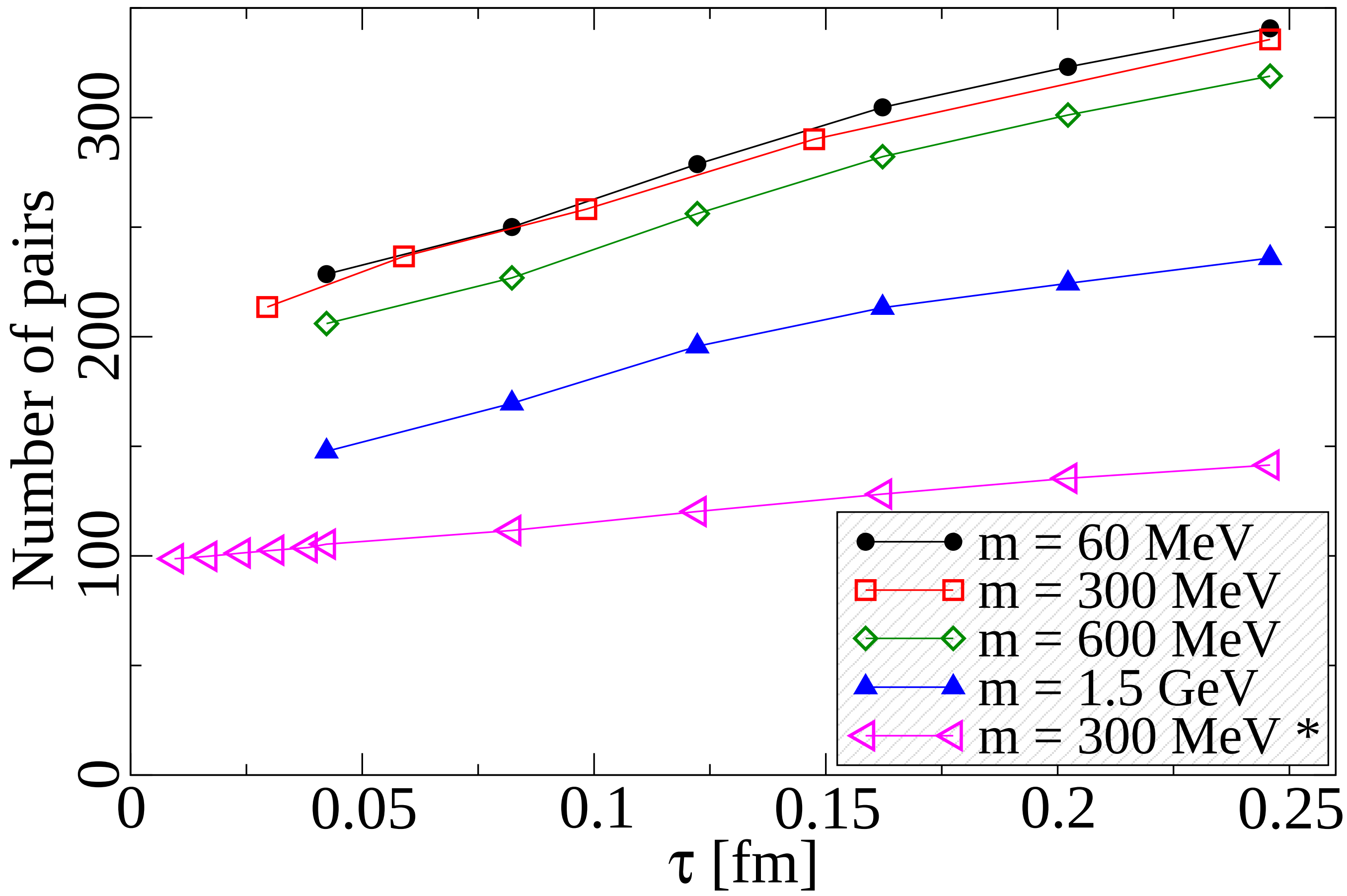
$q\bar{q}$ pairs are also produced instantaneously:



Conclusions

- Gluons dominate the wave function of a fast-moving hadron. Thus one thought that the initial state of little bang would be dominantly gluonic, out of chemical equilibrium (the Color-Glass-Condensate picture assumes it is entirely gluonic).
- Parametrically, pairs are suppressed by g^2 from the $q + \bar{q} \rightarrow g$ vertex (”suppressed”? $g \approx 2$)
- In chemical equilibrium, counting dofs, for 16 gluons there are $\approx N_F \times$ 10 $q + \bar{q}$’s
- Our numerical result suggests that the intense gluonic fields produce this amount of $q + \bar{q}$’s instantaneously.
- Experimental implications: thermal dilepton production is not suppressed by lack of $q + \bar{q}$’s.