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Jump all the way to

Type IIA sugra:

$F_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu$

$$S = \underbrace{\frac{1}{2\kappa^2}}_{\equiv \frac{1}{16\pi G_{10}}} \int d^{10}x \sqrt{-G} \left\{ R - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{12} e^{-\phi} H_3^2 - \frac{1}{4} e^{\frac{3}{2}\phi} F_2^2 - \frac{1}{48} e^{\frac{1}{2}\phi} \underbrace{G^2}_{\equiv F_4^2} \right.$$

$$\left. - \frac{1}{2304} \frac{1}{\sqrt{-G}} \varepsilon^{\mu_0 \cdots \mu_9} B_{\mu_0\mu_1} \underbrace{G_{\mu_2\cdots\mu_5}}_{\substack{\text{field strength}\\ \text{constructed from}\\ \text{the R-R 3-form } C_{\mu\nu\lambda} \equiv F_4}} G_{\mu_6\cdots\mu_9} \right\} + \text{fermions}$$

$B_2 \wedge F_4 \wedge F_4$

Fields		# of physical dofs	
NS-NS	ϕ	1	
	$B_{\mu\nu}$	28	← couples to a string
	$G_{\mu\nu}$	35	
R-R	C_μ	8	← couples to point
	$C_{\mu\nu\lambda}$	56	← " " 2d surface
NS-R	χ_a $s=\frac{1}{2}$	8	
	ψ^μ_a $s=\frac{3}{2}$	56	
R-NS	χ'_a	8	
	$\psi^{\mu\prime}_a$	56	

$128_b + 128_f$ dofs

{R: periodic BC for ψ
{NS: anti " " " ψ

R-R bispinors

$8^2 = 36 - 1 + 28 + 1 \quad d = 10 \to 8$

$2^2 = 3 - 1 + 1 + 1 \quad d = 4 \to 2$

= D branes carry RR charges

dim $= p$, p even for IIA

$\binom{8}{3} = 56$

<u>some trivial details:</u>

$$H = dB = \frac{1}{2!} \frac{\partial B_{\mu\nu}}{\partial x^\lambda} dx^\lambda \wedge dx^\mu \wedge dx^\nu$$

$$B_{\mu\nu} = \frac{1}{3!} H_{\mu\nu\lambda}\, dx^\mu \wedge dx^\nu \wedge dx^\nu$$

$$= \frac{1}{3!} \left(\partial_\mu B_{\nu\lambda} + \partial_\nu B_{\lambda\mu} + \partial_\lambda B_{\mu\nu} \right) dx^\mu \wedge dx^\nu \wedge dx^\lambda$$

Massless fields; in general:

F = k-form, $F = \frac{1}{k!} F_{\mu_1 \cdots \mu_k} dx^{\mu_1} \wedge \cdots \wedge dx^{\mu_k}$ $\begin{cases} dF = 0 \\ d{*}F = 0 \end{cases}$ Bianchi

$$dF = 0 \Rightarrow \quad = dC$$

$$C = \frac{1}{(m-1)!} C_{\mu_1 \cdots \mu_{k-1}} dx^{\mu_1} \wedge \cdots \wedge dx^{\mu_{k-1}}$$

$$F_{\mu_1 \cdots \mu_k} = \frac{1}{(k-1)!} \partial_{[\mu_1} C_{\mu_2 \cdots \mu_k]}$$

check: $H_{\mu\nu\lambda} = \frac{1}{2!} \partial_{[\mu} B_{\nu\lambda]} = \partial_\mu B_{\nu\lambda} + \partial_\nu B_{\lambda\mu} + \partial_\lambda B_{\mu\nu}$

$$\begin{matrix} +\mu\ \nu\ \lambda & +\nu\ \lambda\ \mu & +\lambda\ \mu\ \nu \\ -\ [\ \ ] & [\ \ ] & [\ \ ] \end{matrix} \leftarrow \text{cancels the } \tfrac{1}{2!}$$

A : $A_\mu dx^\mu$

B : $\frac{1}{2!} B_{\mu\nu} dx^\mu \wedge dx^\nu$

C : $\frac{1}{3!} C_{\mu\nu\lambda} dx^\mu \wedge dx^\nu \wedge dx^\lambda$

$\vdots$

p-dim <u>manifold</u>:

$$x^\mu = x^\mu(\underbrace{\ell^1, \cdots, \ell^p}_{\text{some parameters}})$$

many sets of ℓ^i may be needed

<u>orientable</u> $\left| \frac{\partial \tilde{\ell}}{\partial \ell} \right|$ positive def.

another set

Type II B sugra

$$S_{IIB} = \frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{-G} \left\{ e^{-2\phi} \left[R + 4(\nabla\phi)^2 - \frac{1}{2} H^2 \right] \right.$$

$$\left. - \frac{1}{2}\left(F_1^2 + \tilde{F}_3^2 + \tilde{F}_5^2\right) \right\} - \frac{1}{32\pi G_{10}} \int C_4 \wedge H_3 \wedge F_3 + \text{ferm's}$$

RR 1- 3- 5- forms $\quad (C_4 \wedge H_3)_{\mu_1 \cdots \mu_7} = [(C_4)_{\mu_1 \cdots \mu_4} (H_3)_{\mu_5 \mu_6 \mu_7}]$

States;

NS-NS	ϕ	1
	$B_{\mu\nu}$	28
	$R_{\mu\nu}$	35
R-R	C_0	1
$C_2 \to$	$C_{\mu\nu}$	28
$C_4 \to$	$C_{\mu\nu\lambda\kappa}$	35

+ fermion states

$H \equiv H_3 = dB$

$F_1 = dC_0$

$F_3 = dC_2 \qquad \tilde{F}_3 = F_3 - C_0 \wedge H_3$

$F_5 = dC_4 \qquad \tilde{F}_5 = F_5 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3$

self-dual!

$* \tilde{F}_5 = \tilde{F}_5$

<u>A general prototype action</u> will be:

$$S = \frac{1}{2\kappa_d^2} \int d^dx \sqrt{g} \left\{ R - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{2} \sum_{m=1} \frac{1}{m!} e^{a_m \phi} \underbrace{F_m^2}_{F_{\mu_1 \cdots \mu_m} F^{\mu_1 \cdots \mu_m}} + \cdots \right\}$$

$a_m = -\frac{m-5}{2}$

Lyng-Petersen hep-th/9909131 ch. 3.1

$\begin{cases} d = 10 \\ d = 11 \quad a_m = 0,\ \phi = 0 \quad \text{M-theory!} \end{cases}$

p-brane: source of charge for the p+1 form RR gauge field $C \Rightarrow m = p+2$ form F

3-brane: source of $C_{\mu\nu\lambda\kappa} \Rightarrow F_{\mu\nu\lambda\kappa\rho} \cong \partial_{[\mu} C_{\nu\lambda\kappa\rho]}$

$$d = p+1 + \underbrace{d-(p+1)}_{\text{dims transverse to the p-brane}}$$

$$z^\mu = (t, x^i, y^a) = (z^0, x^1, \cdots, x^p, \underbrace{y^1, \cdots y^{d-(p+1)}}_{\text{transverse}})$$

$p=3: \quad 10 = 4 + 6$