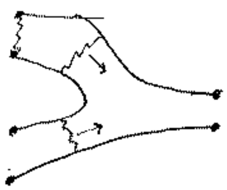
U(N) gauge theory

Given open strings with fixed boundary conditions we have an open string theory with interactions of the type

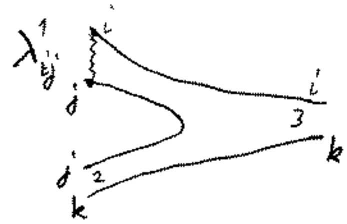
fixed end points



For this vertex for the tachyon ($\alpha' M^2 = -4$) of the bosonic string, see, eg. Polchinski I (6.4.1) etc

To get gauge theory (SYM) take a 10D superstring with external spin 1 massless state ($\alpha' M^2 = \sum_1^\infty \alpha_{-m} \cdot \alpha_m + \sum t \psi_{-t} \cdot \psi_t - a$, $t = \frac{1}{2}$ or 0,
$\psi^\alpha_{-\frac{1}{2}} |0 k\rangle \Rightarrow A^\alpha \quad \psi^i_{-\frac{1}{2}} |0, k\rangle \Rightarrow \phi^i$, $\alpha = 0, 1, \cdots, p$; $i = p+1, \cdots 9$)

and add "color" to the end points



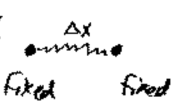
$\Rightarrow \mathrm{Tr}\, \lambda^1 \lambda^2 \lambda_3$

these Chan-Paton factors were originally introduced to impose flavor symmetry on the Veneziano amplitude

$\sim \dfrac{\Gamma(-\alpha_s)\Gamma(-\alpha_t)}{\Gamma(-\alpha_s - \alpha_t)}$ $\quad \pi\pi \to \pi\pi$, $K\pi \to K\pi$, $\eta K \to \eta K$

End points are fixed on branes

If branes are separated at some distance the string has mass $\frac{\Delta x}{\alpha'}$



"Higgs

(though we started from a $M=0$ state in the usual superstring with free ends)

To have massless quanta put the branes on top of each other, like multiple charges in a point:

EM: point = 0-brane couples to 1form A related to $F = dA$

$\Rightarrow$ p-brane couples to $(p+1)$-form $A \Rightarrow (p+2)$-form $F = dA$

$$\text{dual} = [D-(p+2)]\text{-form } \tilde{F}$$
$$= d-1 \quad \text{if} \quad D = 1 + p + d = (1+3) + 6$$

Elementary electric p-brane

$$Q = \frac{1}{\Omega_{d-1}} \int_{S^{d-1}} \tilde{F}_{d-1}$$ Noether charge

$p=0$ $d=3$ $F=F_2$ $\Rightarrow$ Gauss
$D=4$ $\tilde{F}=\tilde{F}_2$

$$= \frac{1}{\Omega_{d-1}} \frac{1}{(d-1)!} \int d^{d-1}x\, \varepsilon^{\mu_1 \cdots \mu_{d-1}} \tilde{F}_{\mu_1 \cdots \mu_{d-1}}$$

Magnetic $(D-p-4)$-brane

$A \sim D-p-3$

$F \sim D-p-2$

$$P = \frac{1}{\Omega_{d-1}} \int_{S^{d-1}} F_{d-1}$$ Topological charge

$$\frac{1}{\Omega_{p+2}} \int_{S^{p+2}} F_{p+2}$$

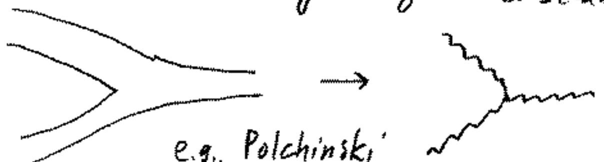
For the extremal brane on p. 15: $d=6$

$$F_{0ijkr} = \varepsilon_{ijk} \frac{1}{\left[1 + \frac{Q}{2r^4}\right]^2} \cdot \frac{Q}{r^5}$$

$$Q = N g_s \frac{(2\pi\sqrt{\alpha'})^{d-2}}{\Omega_{d-1}} \underset{d=6}{=} N g_s \frac{1}{\pi^3}(2\pi\sqrt{\alpha'})^4 = N g_s \cdot 4\pi\alpha'^2 \cdot 4$$

multiple charge on a brane

Gauge theory arises in the $\alpha' \to 0$ limit, e.g.
(also adjoint gluinos & scalars)



e.g., Polchinski 6.15.15

, i.e. $$\frac{1}{g^2} \int d^Dx \left[-\frac{1}{4} \mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu} + O\left(\frac{\alpha'}{g^2} \mathrm{Tr} FFF\right) \right]$$

But how do you describe coupling to brane? → p. 67

Reminder: string coupling g_s

$$e^{-S} = e^{-\frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{-h}\left\{ \cdots\cdots + \alpha' R\phi\right\}} \sim \left[e^{-\langle\phi\rangle}\right]^{2-2\tilde{g}} \sim (g_s^2)^{\tilde{g}-1}$$

$$\chi = \frac{1}{4\pi}\int d^2\sigma\sqrt{-h}\, R = \text{Euler number} = 2 - 2\tilde{g}$$ ← genus

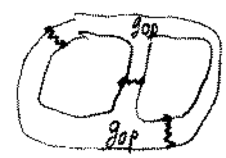
$\exists$ a potential for dilaton $\Rightarrow \exists\langle\phi\rangle$. Then $g_s = e^{\langle\phi\rangle}$

Why is this "string coupling"?

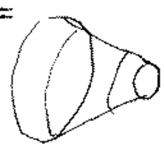
$$\left[\begin{array}{l}\phi \to \phi - c\\ \Rightarrow g_s \to e^c g_s\end{array}\right.$$

open string loop

2loop open string



(pull out internal circle
=



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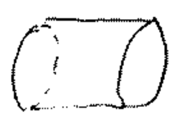
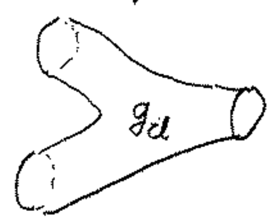


closed string propag

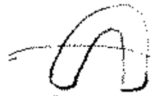
⇓ pull out



closed string vertex

$$\Rightarrow g_{open}^2 = g_{cl}$$

Adding one handle costs g_s^2 (since $\tilde{g} \to \hat{g}+1$) or g_{cl}^2 :



$$\Rightarrow \boxed{g_{open}^2 \cong g_{cl} \cong g_s \cong e^{\langle\phi\rangle}}$$

$$\left\{\begin{array}{lll} g_s^2 & \text{is} & \text{handle exp. parameter}\\ \frac{1}{N_c^2} & & \text{— " —}\end{array}\right.$$

constants appear!

$$\left\{\begin{array}{l} g_{open}^2 = \dfrac{2\pi^{D/2}}{2^{(10-D)/4}}\, g_{cl}\\ g_{cl} = (2\pi)^2 g_s\end{array}\right.$$

<u>Born-Infeld action</u> Proc. R. Soc. A144 (1934) 425

ℓ^2 some param of dim $\frac{1}{\mathrm{GeV}^2}$

$$L = \frac{1}{\ell^4}\left\{\sqrt{-\det g_{\mu\nu}} - \sqrt{-\det(g_{\mu\nu} + \ell^2 F_{\mu\nu})}\right\} \qquad \pi\ell^2 = \frac{1}{T} = 2\pi\alpha'$$

$$= \frac{1}{\ell^4}\left\{1 - \sqrt{1 - \ell^4\underbrace{(E^2-B^2)}_{-\frac{1}{2}F_{\mu\nu}F^{\mu\nu}} - \ell^8\underbrace{(\bar{E}\cdot\bar{B})^2}_{\frac{1}{4}F^*_{\mu\nu}F^{\mu\nu}}}\right\}$$

$g_{\mu\nu} = \eta_{\mu\nu}$

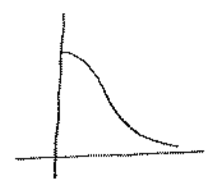
$d = 4$

$$\begin{cases} E^i = F_{0i} \\ B^i = -\frac{1}{2}\varepsilon_{ijk}F_{jk} \end{cases} \qquad \begin{vmatrix} 1 & E & 0 & 0 \\ -E & -1 & 0 & 0 \\ 0 & 0 & -1 & -B \\ 0 & 0 & B & -1 \end{vmatrix} = -1 + E^2 - B^2 + E^2B^2$$

$$= \frac{1}{2}(E^2-B^2) + \ell^4\left[\frac{1}{2}(\bar{E}\cdot\bar{B})^2 + \frac{1}{8}(E^2-B^2)^2\right] + \dots$$

$\bar{E} \leftrightarrow \bar{B}$ <u>duality</u> invariant

$$\rho(\bar{x}) = q\delta^3(\bar{x}) \Rightarrow E(r) = \frac{q}{\sqrt{r^4 + \ell^4 q^2}}$$



$$\Rightarrow \text{finite} \int d^3x\,(\bar{D}\cdot\bar{E} - L) \qquad \bar{D} = \frac{\partial L}{\partial \bar{E}}$$

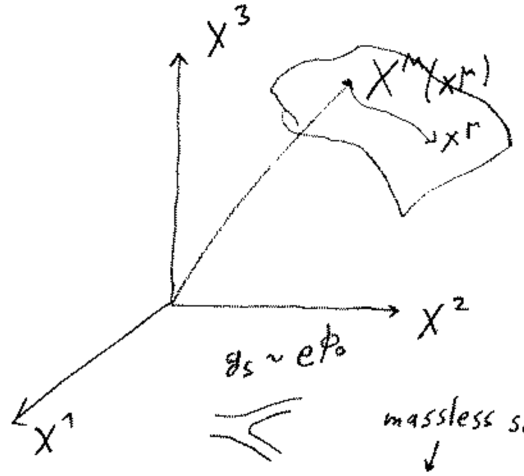
$$L_{Eucl} = -1 + \sqrt{1 + \ell^4\underbrace{(E^2+B^2)}_{\frac{1}{2}F^2} + \ell^8\underbrace{(\bar{E}\cdot\bar{B})^2}_{(\frac{1}{4}FF^*)^2}} = \sqrt{\left(1 + \frac{\ell^4}{4}FF^*\right)^2 + \frac{\ell^4}{4}(F-F^*)^2} - 1$$

$E^2 \to -E^2$

$L \to -L$

$$\geq \frac{\ell^4}{4}FF^* = \text{ if } F = F^* \quad !$$

D-brane actions analogously with $g_{\mu\nu} + \pi\ell_s^2 F_{\mu\nu}$;



brane at $X^M(x^\mu)$
$\uparrow$ internal coord's

$$\begin{cases} g_{\mu\nu} = g_{MN}(X(x))\,\partial_\mu X^M(x)\,\partial_\nu X^N(x) \\ b_{\mu\nu} = B_{MN}(X(x)) \quad - \;\; '' - \end{cases}$$

$\uparrow$ massless spin 2 & 1 fields of the closed string theory IIB in the $\alpha' \to 0$ limit

$g_s \sim e^{\Phi_0}$

massless scalar $\downarrow$

$$S_p = T_p \int d^{p+1}x \; e^{-\Phi} \sqrt{-\det\left(g_{\mu\nu} + b_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}\right)}$$

$\uparrow$ tension of p-brane

$\downarrow$ massless vector field from open strings (which were found by T-duality)

$\downarrow$ scaling with p

$$= \frac{\text{energy}}{V_p} = (\text{energy})^{p+1} = \frac{1}{(2\pi\sqrt{\alpha'})^{p-1}} \cdot T_1$$

$$= \frac{1}{(2\pi\sqrt{\alpha'})^{p-1}}\,\frac{1}{2\pi\alpha'}\cdot\frac{1}{g_s} \sim \frac{2\pi}{g_s(2\pi l_s)^{p+1}}$$

This came from explicit p-brane soln's given earlier

Expand for small α' ($\alpha' F \ll 1$):

$$\sqrt{-\det(\dots + 2\pi\alpha' F_{\mu\nu})} \;\hat{=}\; \frac{1}{2}(\pi l_s^2)^2 F_{\mu\nu}F^{\mu\nu}$$

$$S = \frac{1}{g_s}\frac{1}{(2\pi)^p}\frac{1}{l_s^{p+1}}\frac{1}{2}\pi^2 l_s^4 F^2 = \frac{4\pi}{g_s}\cdot\frac{1}{(2\pi)^{p-1}}\frac{1}{l_s^{p-3}} = \frac{4\pi}{g_s(2\pi l_s)^{p-3}}\underbrace{\frac{1}{(2\pi)^2}}_{\text{some convention}}$$

Should find $\boxed{\dfrac{g_{YM}^2}{4\pi} = g_s(2\pi l_s)^{p-3}} \Rightarrow \dim g_{YM}^2 = (\text{GeV})^{4-(p+1)}$

Parameters, validity

String theory $S = -\frac{T}{2}\int d^2\sigma \sqrt{-h}\, h^{ab}\, G_{\mu\nu}(x)\,\partial^\mu X_a\, \partial^\nu X_b + \cdots$

$\Downarrow$ small α', only massless excitations

Sugra $S = \frac{1}{(2\pi)^7 \alpha'^4}\int d^{10}x \sqrt{-g}\left\{ e^{-2\phi}\left[R + 4(\nabla\phi)^2\right] - F \text{ term}\right\}$

$\Downarrow$ $g_{E\mu\nu} = \sqrt{g_s e^{-\phi}}\, g_{\mu\nu}$ $\quad R_E = \frac{1}{\sqrt{g_s}} e^{\phi/2} R$ (p. 8)

$$\sqrt{-g_E} = (g_s e^{-\phi})^{10}\sqrt{-g}$$

$$\sqrt{-g}\, e^{-2\phi} R = \sqrt{-g_E}\,\frac{1}{g_s^2} R_E + \cdots$$

$$= \frac{1}{(2\pi)^7}\frac{1}{\alpha'^4 g_s^2}\int d^{10}x \sqrt{-g_E}\left[R_E - \frac{1}{2}(\nabla\phi)^2 \cdots\right.$$

Computed from graviton-graviton scattering in string theory and linearised ($g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$) sugra

$\int dx\left[(\partial h)^2 + \kappa h (\partial h)^2 + \cdots\right]$

$$2\kappa_{10}^2 = 16\pi G_{10} = (2\pi)^7 \alpha'^4 g_s^2 = \frac{(2\pi\, l_s)^8}{2\pi} g_s^2 \quad (\text{now } l_s^2 \equiv \alpha')$$

$\Rightarrow$ 10 dim Planck length $l_{Pl}^{10} = g_s^{\frac{1}{4}} l_s$

$\Downarrow$ classical solutions (EOM on p. 15) for $g_{\mu\nu}$ F $\phi = \text{const}$

$$ds^2 = \frac{1}{\sqrt{1 + \frac{R^4}{r^4}}}\left[-\left(1 - \frac{r_0^4}{r^4}\right)dt^2 + d\vec{x}^2\right] + \sqrt{1 + \frac{R^4}{r^4}}\left[\frac{1}{1 - \frac{r_0^4}{r^4}}\, dr^2 + r^2 d\Omega^5\right]$$

$r_0 = 0$: $\boxed{R^4 = 4\pi g_s N \alpha'^2} = \frac{1}{4} Q \leftarrow$ source of F at $r=0$ (p. 64) (brane at $r=0$)

$$\left(\frac{g_{YM}^2}{4\pi} = g_s\right)$$

- For this class sol'n to be valid one must have $R \gg l_{Pl}^{10}$

$$4\pi g_s N \alpha'^2 \gg g_s \alpha'^2 \Rightarrow N \gg 1$$

- Small α' means $\frac{\alpha'^2}{R^4} \cong \frac{1}{g_s N} \ll 1 \Rightarrow g_s N \gg 1$

$$(g_{YM}^2 N \gg 1)$$

String $\leftrightarrow$ SYM quantitatively but sketchily:

$$Z_{\text{String}}[\phi_0] = \int \mathcal{D}\phi(x,r)\, e^{-\underbrace{S[\phi(x,r)]}_{\text{bulk field}}} \overset{\begin{cases} N \gg 1 \\ g_s N \gg 1 \end{cases}}{\cong} e^{-S_{SUGRA}[\phi_0^{cl}(x)]}$$

$\phi(x,0) = \phi_0(x)$

Generating functional of SYM correlators is the SUGRA action!!

$$\beta F = S_{\text{sugra}}\left[\begin{smallmatrix}\text{on } AdS_5 \\ \times S_5\end{smallmatrix}\right]$$

$$= Z_{SYM}[\phi_0] = \left\langle e^{\int d^4x\, \mathcal{O}(x)\phi_0(x)} \right\rangle \equiv e^{-\Gamma[\phi_0]} \overset{\text{finite } T}{=} e^{-\beta F}$$

$$= \sum \frac{1}{m!} \int \prod_1^m d^4x_i \underbrace{\langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_m) \rangle}_{} \phi_0(x_1) \cdots \phi_m(x_m)$$

exp. value of $\mathcal{O}(x)$ in SYM, like $\mathcal{O}(x) = \mathrm{Tr}F^2$

$$\langle \mathrm{Tr}F^2(x)\, \mathrm{Tr}F^2(y) \rangle \sim \langle \mathcal{O} \rangle^2 + C e^{-M|x-y|}$$

to get the glueball mass

Ex 1 $\quad F_{SYM}$:

What is the relevant S_{SUGRA}?

$$ds^2 = \frac{r^2}{R^2}\Big[\underbrace{-\big(1-\tfrac{r_0^4}{r^4}\big)dt^2 + d\vec{x}^2}_{\tau = it \;\Rightarrow\; S^1 \times R^3}\Big] + \frac{R^2}{r^2}\frac{1}{1-\frac{r_0^4}{r^4}}dr^2 + R^2 d\Omega_5^2$$

$$\mathrm{Vol}(S^5) = R^5 \Omega_5 = \pi^3 R^5$$

$1 - \frac{r_0^4}{r^4} \sim 4(r-r_0)$ $r \approx r_0$ $\Rightarrow \beta = \frac{1}{T} = \frac{\pi R^2}{r_0}$ (p. 27)

AdS_{m+1}, $m=4$

Then $$S_{SUGRA} = -\frac{1}{\underbrace{16\pi G_{10}}_{=(2\pi)^7 g_s^2 \alpha'^4}} \int d^{10}x \sqrt{-g_{10}}\,[R_E + \cdots] = -\frac{\pi^3 R^5}{16\pi G_{10}} \int d^5x \sqrt{-g_5}\,(R_5 - 2\Lambda)$$

$$\underbrace{\left(R_5 + \frac{4\cdot 3}{R^2}\right)}_{= -\frac{2\cdot 4}{R^2}}$$

$$\sqrt{-g_5} = \frac{r^3}{R^3}$$

$$= +\frac{\pi^3 R^5}{16\pi G_{10}} \frac{2\cdot 4}{R^2} \int d^5x\, \frac{r^3}{R^3}$$

$$R_5 = -\frac{4\cdot 5}{R^2}$$

$$= -\frac{\pi^2}{8} N^2 V_3 T^3$$ after some clever subtractions

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eq (27) —

<u>Ex. 2</u> Glueball masses in QCD_3 (Aharony et al, hep-th/990555 p. 202)

in $ds^2 = \frac{r^2}{R^2}\left\{-\left(1-\frac{r_0^4}{r^4}\right)dt^2 + d\vec{x}^2\right\} + \cdots$ $\quad R^4 = 4\pi g_s N \alpha'^2$

compactify t: $\bigcirc$ R_0 like T! (For QCD_4 you need $AdS_7 \times S^4$ in $D=11$)

[fermions antiperiodic : disapper ! $\boxed{\text{susy broken!}}$
[scalars also get mass (not protected by susy) "QCD-like" theory

$$\Rightarrow ds^2 = R^2\left\{ u^2\left[\left(1-\frac{u_0^4}{u^4}\right)d\tau^2 + \underbrace{dx'^2 + dx^2{}^2 + dx^3{}^2}_{QCD_3^{eucl}}\right] + \frac{du^2}{\left(1-\frac{u_0^4}{u^4}\right)u^2} + d\Omega_5^2\right\}$$

Euclidean BH →
τ = angular coord

$$u_0 = \frac{1}{2R_0}$$

QCD_3^{eucl}
⇓ Mink
$-dt^2 + dx^2{}^2 + dx^3{}^2$

$$g_3^2 N = g_4^2 N \cdot \frac{1}{2\pi R_0} \quad \left(\simeq g_4^2 N \cdot T\right)$$

lattice : $g_3^2 a \to 0$ continuum ← ($R_0 \to 0$: SUGRA not valid, must use full string theory !)
$\qquad\to \infty$ strong coupling

To compute $\langle F^2 \cdot F^2\rangle$ need φ_0. Since $F^2 = O^+$, φ_0 is the dilaton ϕ.

$\Rightarrow$ solve the EOM of ϕ in the AdS_5 background:

$$\partial_\mu \sqrt{-g}\, g^{\mu\nu}\partial_\nu \phi = 0$$
$$\phi = f(u)\cdot e^{i k \cdot x}$$

$$\Rightarrow \partial_u\left[u(u^4-u_0^4)\,\partial_u f(u)\right] + M^2 u\, f(u) = 0 \qquad M^2 = -k^2 \;\begin{pmatrix} M^2 = k_0^2 - \vec{k}^2 \\ = -(k_4^2+\vec{k}^2)\end{pmatrix}$$

$x = u^2$: $\quad f'' + \left(\frac{1}{x} + \frac{1}{x-1} + \frac{1}{x+1}\right)f' + \frac{M^2}{4x(x^2-1)} f = 0$ $\quad$ 4 regular singularities at $x = 0\ \pm 1,\ \infty$

$f(u\to\infty) = \frac{1}{u^4} \qquad f'(u_0) = 0 \qquad \int du \sqrt{g}\,|f|^2 < 0$

eigenvalues
$$\Rightarrow M^2 = \frac{1.44\, m(m+1)}{R_0^2}$$

seems disastrous, one obtains a strong coupling result $M \sim \frac{1}{a}$ while we know that $M \sim g_3^2 N_c$

However, this SUGRA appro works only if $g_{YM}^2 N >> 1$ so
$M_{string} = \frac{1}{\sqrt{\alpha'}} = \frac{(g_{YM}^2 N)^{1/4}}{R}$ one is in a different limit.