QCD & String Theory

conformal gauge $h_{ab} = e^{\varphi(\tau,\sigma)}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$

QCD	String Theory
Very well known	??

QCD

Local $SU(3)$

Parameters:

$N_c = 3$

$N_f = 1 \quad m_u(\mu) \sim 4$ MeV

$2 \quad m_d(\mu) \sim 7$ "

$3 \quad m_s(\mu) \sim 100$ "

$4 \quad m_c(\mu) \sim 1.5$ GeV

$5 \quad m_b(\mu) \sim 5$ "

$6 \quad m_t(\mu) \sim 175$ "

$\Lambda_{QCD} \qquad \mu \frac{\partial}{\partial\mu}\left[m(\mu)\,\bar\psi\psi(\mu)\right] = 0$

Integration const. of

$$\mu \frac{dg(\mu)}{d\mu} = \beta(g) = -\beta_0 g^3 - \cdots$$

The problem of QCD is manifest even if $m_q = 0$. Then the problem is "generation of mass gap"

$m_{Glueball} > 0$

$= \# \cdot \Lambda_{QCD}$

"breaking of conformal invariance"

Lorentz inv & unitarity require that Weyl symmetry be preserved on quantum level

String Theory

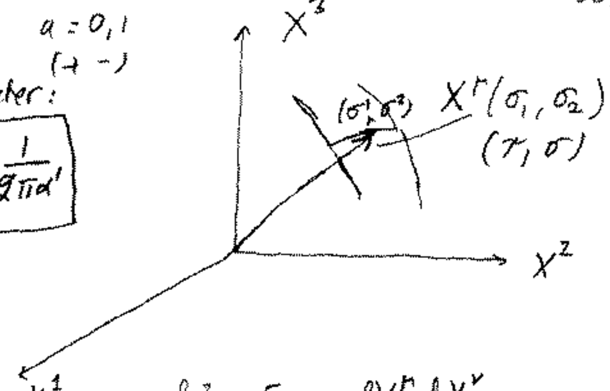
{ world sheet reparam. inv.
{ conformal inv.

$$S = \frac{T}{2}\int d^2\sigma \sqrt{h}\, h^{ab}\, G^{\mu\nu}(X)\, \partial_a X_\mu\, \partial_b X_\nu + \text{SUSY}$$

$a = 0,1$
$(+\ -)$

only parameter:

$$\boxed{T = \frac{1}{\pi l_s^2} = \frac{1}{2\pi\alpha'}}$$



$$ds^2 = G_{\mu\nu}(X)\, dX^\mu dX^\nu$$

Invariances:

1. "worldsheet reparametrisations"

   $ds^2 = h_{ab}\, d\sigma^a d\sigma^b \qquad \sigma^a \to \sigma'^a$

   $\int d^2\sigma \sqrt{h}$ (scalar) is inv. <u>2 param.</u>

2. Weyl

   $h_{ab} \to \Lambda(\sigma)\, h_{ab}$ "conformal"

   $h = \det h_{ab} \to \Lambda^d h$ <u>1 param</u>

   h^{ab} is the inverse of h_{ab}

   $\begin{pmatrix}a & b\\ c & d\end{pmatrix}^{-1} = \frac{1}{ad-bc}\begin{pmatrix}d & -b\\ -c & a\end{pmatrix} \to \frac{1}{\Lambda}(\ )^{-1}$

   or $h^{ai} h_{ib} = \delta^a_b$

   $h^{ab} \to \frac{1}{\Lambda} h^{ab}$ (any d)

   $\sqrt{h}\, h^{ab} \to \underbrace{\Lambda^{\frac{d}{2}-1}}_{=1 \text{ for } d=2} \sqrt{h}\, h^{ab}$

Further: the whole mass spectrum

all the amplitudes $A(\pi^- p = \pi^0 n)$

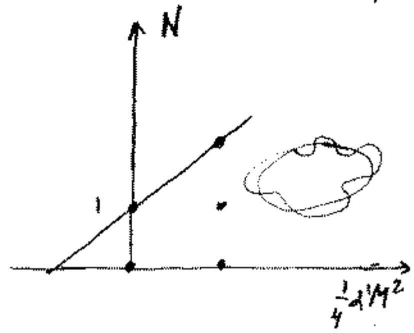
More on this later!!

$\frac{1}{2}(N_L + N_R) \nearrow d = 26 \Rightarrow$ Weyl inv. on quantum level

$N - \frac{d-2}{24} = N - 1$ Regge.

Excitation spectra of type $\frac{1}{4}\alpha' M^2 = N + \frac{d-2}{2}\underbrace{\sum_1^\infty n}_{\zeta(-1) = -\frac{1}{12}}$ $\qquad J = \alpha(t) = \alpha_0 + \alpha' t +$

(math. gives $d = 26$!!)



- Gr. state $N = 0$ $\quad \alpha' m^2 = -4$

  non-oscillating string tachyon

- 1st excited, massless, $N = 1$

  $d^2 = \underbrace{\frac{d(d+1)}{2} - 1}_{G_{\mu\nu}} + \underbrace{\frac{d(d-1)}{2}}_{B_{\mu\nu}} + \underbrace{1}_{\phi}$

  spin 2 $\quad$ spin 1 $\quad$ spin 0

  # of dofs is $\frac{(d-1)(d-2)}{2} - 1$

- $N = 2$, $\frac{1}{4}\alpha' M^2 = 1$

Superstring

$$S = -\frac{T}{2}\int d^2\sigma \sqrt{h}\left[h^{ab}\partial_a X^\mu \partial_b X_\mu + i\,\bar{\psi}^\mu\, i\gamma^a \partial_a \psi_\mu\right]$$

$$\text{susy}\quad \begin{cases} \delta X^\mu = \bar{\xi}\psi^\mu \\ \delta\psi^\mu = -i\gamma^a\partial_a X^\mu \xi \end{cases}$$

$\boxed{\text{Weyl inv.} \Rightarrow d = 10}$

Strings in String backgrounds

"massless fields $G_{\mu\nu}$ .. have nontrivial VEV's"

$$S = -\frac{T}{2}\int d^2 \sqrt{h}\left\{h^{ab} G_{\mu\nu}(X)\partial_a X^\mu \partial_b X^\nu + \varepsilon^{ab} B_{\mu\nu}(X)\partial_a X^\mu \partial_b X^\nu + \frac{\alpha}{2} R_h(X)\,\phi(X)\right\}$$

$e\int d\tau \frac{dx^\mu}{d\tau}A_\mu = e\int dx^\mu A_\mu \rightarrow \frac{T}{2}\int B_{\mu\nu}(X)\underbrace{\partial_a X^\mu \partial_b X^\nu d\sigma^a \wedge d\sigma^b}_{dX^\mu \wedge dX^\nu}$

point $\rightarrow$ string

dilaton

Chain of arguments:

- S is NOT conformally invariant for general X, in fact

$$\frac{1}{\sqrt{h}} T^a{}_a = \frac{1}{2\pi} h^{ab}\beta^G_{\mu\nu}\,\partial_a X^\mu \partial_b X^\nu + \dots$$

- The β-functions can be computed in the limit $\alpha' \to 0$

$$\frac{1}{\alpha'}\beta^G_{\mu\nu} = R_{\mu\nu} - \frac{1}{4}\underbrace{H_{\mu\alpha\beta}}\, H_\nu{}^{\alpha\beta} + \nabla_\mu \nabla_\nu \phi \quad + O(\alpha')$$

(next page) $\qquad \partial_\mu B_{\alpha\beta} + \partial_\alpha B_{\beta\mu} + \partial_\beta B_{\mu\alpha} \Rightarrow H = dB$

- Want $\begin{cases}\text{Weyl} \\ \text{conf. invariance}\end{cases} \Rightarrow \beta^G_{\mu\nu} = 0$, ...

[generalizes $d = 26$ etc. to string backgrounds !!]

A_μ vs $B_{\mu\nu}$

1. Coupling of a point particle to A_μ: $J^\mu A_\mu$ $\quad = \rho u^\mu$

$$\int d^4x\left[-\frac{1}{4}F_{\mu\nu}^2 - J^\mu A_\mu\right] - m\int ds \qquad J^\mu = (\rho\gamma, \rho\gamma\vec{v})$$

$$J^\mu(x) = e\int dx^\mu\, \delta^4(x - x(\tau)) \qquad e = \int d^3x\, J^0(x)$$

$$= \int d^4x\left(-\frac{1}{4}F_{\mu\nu}^2\right) - e\int dx^\mu A_\mu(x(\tau)) - m\int ds$$

$$\int d\tau\, \frac{dx^\mu}{d\tau} A_\mu(x(\tau)) \qquad m\int d\tau\sqrt{\eta_{\mu\nu}\dot{x}^\mu\dot{x}^\nu}$$

2. Coupling of a string to $B_{\mu\nu}$:

$$\left(m\int ds \Rightarrow -T\int dA \quad \text{Nambu-Goto} \right.$$

$$\left. \Rightarrow \frac{T}{2}\int d^2\sigma\sqrt{h}\, h^{ab}\,\partial_a X^\mu \partial_b X_\mu \right)$$

$$-\frac{T}{2}\int d^2\sigma\; B_{\mu\nu}\,\varepsilon^{ab}\partial_a X^\mu \partial_b X^\nu \qquad B_{\mu\nu} = -B_{\nu\mu}$$

$$\equiv -\frac{T}{2}\int B_{\mu\nu}\, dX^\mu \wedge dX^\nu \qquad d\sigma^a \wedge d\sigma^b \equiv \varepsilon^{ab} d^2\sigma$$

p-form is a totally antisymmetric $(0,p)$ tensor

$(1,0)$ is vector, $(0,1)$ is dual vector, mapping from vectors to R

Components of vectors $\qquad \omega = \omega_\mu\,\hat{\theta}^{(\mu)} \qquad \hat{\theta}^{(\nu)}\hat{\varepsilon}_{(\mu)} = \delta^\nu_\mu$

$V = V^\mu\,\hat{\varepsilon}_{(\mu)}$ ← basis vectors $\qquad (\theta, \varepsilon)\ \ \langle\theta|\varepsilon\rangle$

# of lin. indep. p-forms in d-dim vector space is $\binom{d}{p}$: $\sum = (1+1)^d$

$d=4$	$\binom{4}{0}=1$	$\binom{4}{1}=4$	$\binom{4}{2}=6$	$\binom{4}{3}=4$	$\binom{4}{4}=1$
	F	$F_1 = F_\mu dx^\mu$	$F_2 = \frac{1}{2}F_{\mu\nu}\, dX^\mu\wedge dX^\nu$	$F_3 = \frac{1}{3!}F_{\mu\nu\lambda}\, dX^\mu\wedge dX^\nu\wedge dX^\lambda$	$\frac{1}{4!}F_{\mu\nu\lambda\rho}\, dX^\mu \ldots$
			01 02 03 12 13 23	012 123 230 301	0123

$$dF = \frac{1}{m!}\frac{\partial F_{\mu_1\cdots\mu_m}}{\partial x^\nu}\, dX^\nu \wedge dX^{\mu_1} \cdots \wedge dX^{\mu_m}$$

dual $\quad (*F)_{\mu_1\cdots\mu_{m-p}} = \frac{1}{p!}\,\varepsilon^{\nu_1\cdots\nu_p}{}_{\mu_1\cdots\mu_{m-p}}\, F_{\nu_1\cdots\nu_p}$

• The eqs $\beta^G_{\mu\nu} = 0$, $\beta^B_{\mu\nu} = 0$, $\beta^\phi_{\mu\nu} = 0$ can be obtained from a field theory action

strings appear as point particles

$$(\alpha')^{d-2} S = \int d^dx \, \underbrace{\sqrt{G}\, e^{-\phi}}_{\Downarrow} \left[R + (\nabla\phi)^2 - \frac{1}{12} H^2 + \frac{D-26}{3} \right] + O(\alpha')$$

$$\sqrt{G}\; R_E \quad \text{by } G^E_{\mu\nu} = e^{-\frac{2\phi}{d-2}} G_{\mu\nu}$$

$$\frac{1}{\alpha'}\beta^B_{\mu\nu} = \nabla^\alpha \left[e^{-\phi} H_{\mu\nu\alpha} \right] = 0 \qquad \text{"Maxwell"}$$

$$3\frac{1}{\alpha'}\beta^\phi_{\mu\nu} = \frac{d-26}{3\alpha'} + (\nabla\phi)^2 - 2\Box\phi - R + \frac{1}{12} H^2 = 0 \qquad \text{"scalar"}$$

$$\frac{1}{\alpha'}\beta^G_{\mu\nu} = R_{\mu\nu} - \frac{1}{4} H_{\mu\alpha\beta} H_\nu{}^{\alpha\beta} + \nabla_\mu \nabla_\nu \phi = 0 \qquad \text{"gravity"}$$

On Weyl scaling:

$$\tilde{G}_{\mu\nu} = e^{\sigma\phi} G_{\mu\nu}$$

$$\boxed{\begin{aligned} &\tilde{G}_{\mu\nu} = e^{2\Omega} G_{\mu\nu} \\ &\Rightarrow \tilde{R} = e^{-2\Omega}\left[R - 2(d-1)\nabla^2\Omega - (d-2)(d-1)\,\partial_\mu\Omega\,\partial^\mu\Omega \right] \end{aligned}}$$

$$\Rightarrow \sqrt{G}\, e^{-\phi} R = \sqrt{\tilde{G}}\, e^{-[\sigma(d-2)+2]\frac{\phi}{2}} \left\{ \tilde{R} + \sigma(d-1)\frac{1}{\sqrt{\tilde{g}}}\partial_\mu\left(\sqrt{\tilde{g}}\,\partial^\mu\phi\right) - \frac{1}{4}\sigma^2 (d-1)(d-2)\, \partial_\mu\phi\,\partial^\mu\phi \right\}$$

$$\sigma = -\frac{2}{d-2}$$

Gives the "Einstein frame": $\sqrt{G}\, e^{-\phi} = \sqrt{\bar{G}}\, e^{\frac{2}{d-2}\phi}$

$$(\alpha')^{d-2} S = \int d^dx \sqrt{G} \left[R - \frac{1}{d-2}(\nabla\phi)^2 - e^{-\frac{4\phi}{d-2}} \frac{1}{12} H^2 + e^{\frac{2}{d-2}\phi} \frac{d-26}{3} \right]$$

Cf <u>Brans – Dicke</u>

$$\int d^4x \sqrt{g} \left[f(\phi) R + \frac{1}{2} \nabla^\mu\phi \nabla_\mu\phi - V(\phi) \right]$$