D_p branes

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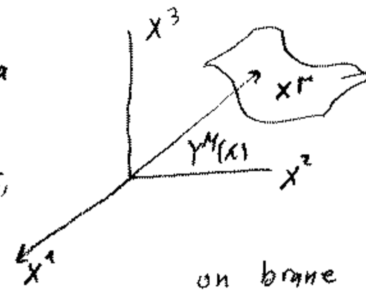
We have discussed "p branes", $z^\mu = (t, x^1, \dots, x^p, \underbrace{y^1, \dots, y^{d-(p+1)}}_{\text{transverse}})$

often $x^M = (x^1, x^2, \quad , x^{4+m})$

"Physics of m extra dimensions"
Csaki, TASI lectures, hep-ph/0404096



Mink. space as a $1+3$ d brane at position $Y^M(x^0, x^1 x^2 x^3)$

$ds^2 = g_{MN}\, dx^M dx^N$

on brane $ds^2 = \underbrace{g_{MN}^{(Y)} \dfrac{\partial Y^M}{\partial x^\mu} \dfrac{\partial Y^N}{\partial x^\nu}}\, dx^\mu dx^\nu$

$T_p \equiv \dfrac{\text{energy}}{V_p} \sim \dfrac{2\pi}{(2\pi\sqrt{\alpha'})^{p+1}} \dfrac{1}{g_s}$

Massive in weak coupling, $M \sim \frac{1}{g^2}$, like monopoles

SM fields $\varphi(x)\ A_\mu(x)\ \psi(x)$ $\rightarrow g_{\mu\nu}$

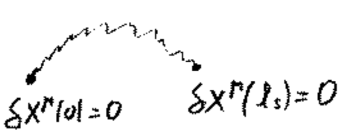
feel only $g_{\mu\nu}$

Dirichlet

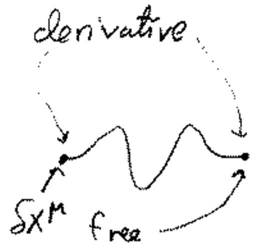
We need special p-branes of string theory, Dp-branes, where "open strings can end". Have to start from string theory basics.

We also need large N_c

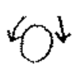
The new, at first somewhat trivial looking element is that even for open strings one can think of fixing the end points (where? is the question and the answer is: in Dp branes!)



while one first would let the end points move freely and only fix the derivative.



For closed strings the low en. eff theory was Sugra



$\int d^{10}x\ \mathcal{L}(\underbrace{G_{\mu\nu}, B_{\mu\nu}, \phi}_{\text{massless states}})$ (p.6 - -)

For open strings we will get $\int d^{10}x \mathcal{L}(A_\mu, \dots)$ $\underset{N=1\ \text{Susy}}{}$

$\Rightarrow N=4\ U(N)$ SYM

String action, oscillator expansions: $\qquad + \int d^2\sigma \sqrt{-h}\,(R^{(2)} - 2\lambda^{(2)})$

$T = \frac{1}{\pi l_s^2} = \frac{1}{2\pi\alpha'}$

$$S = -\frac{T}{2}\int_{\tau_i}^{\tau_f} d\tau \int_0^{\pi} d\sigma\, \sqrt{-h}\, h^{ab}_{(\tau\sigma)}\, \overset{\eta_{\mu\nu}}{G_{\mu\nu}}\, \partial_a X^\mu \partial_b X^\nu = S(h_{ab}, \partial_a X^\mu)$$

(1) world sheet reparam. invariance $\sigma^a \to \sigma'^a(\sigma)$ $\qquad \begin{cases} h = \det h_{ab} \\ h^{ac} h_{cb} = \delta^a{}_b \end{cases}$

(2) D-dim Poincaré ($G_{\mu\nu} = \eta_{\mu\nu}$)

(3) Weyl: $h_{ab} \to e^{\phi} h_{ab}$

Classical EOM: $\qquad \frac{\partial \mathcal{L}}{\partial \varphi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \varphi} = 0 \qquad \mathcal{L}(\varphi, \partial_\mu \varphi)$

① $\partial_a \frac{\partial \mathcal{L}}{\partial \partial_a X^\mu} = 0 \Rightarrow \boxed{\partial_a\left[\sqrt{-h}\, h^{ab} \partial_b X^\mu\right] = 0} \qquad \mu = 1, \dots, d \qquad \nabla_a^2 X^\mu = 0$

② $\frac{\partial \mathcal{L}}{\partial h_{ab}} = 0$ or $\frac{\partial \mathcal{L}}{\partial h^{ab}} = 0 \qquad \delta(\sqrt{-h}\, h^{ab}) = \underbrace{\delta(\sqrt{-h})}_{-\frac{1}{2}\sqrt{-h}\, h_{cd}\, \delta h^{cd}} \cdot h^{ab} + \sqrt{-h}\, \delta h^{ab}$

p. 20: $T_{cd} = -\frac{2}{\sqrt{-h}} \frac{\delta S}{\delta h^{cd}}$

$= -\frac{2}{\sqrt{-h}} \cdot \left(\frac{T}{-2}\right) \cdot$

$$= \left[-\frac{1}{2}\sqrt{-h}\, h_{cd} + \sqrt{-h}\, \delta_{ac}\delta_{bd}\right] \delta h^{cd}$$

$$\Rightarrow \left(-\frac{1}{2} h_{cd} h^{ab} + \delta^a_c \delta^b_d\right)\sqrt{-h}\; \eta_{\mu\nu}\, \partial_a X^\mu \partial_b X^\nu = 0$$

$$\boxed{= \frac{1}{T} T_{cd} = \boxed{\underbrace{\partial_c X^\mu \partial_d X_\mu}_{\equiv\, \tilde{h}_{cd}} - \frac{1}{2} h_{cd}\, h^{ab} \partial_a X^\mu \partial_b X^\nu = 0}}$$

$$\equiv \tilde{h}_{cd} = \frac{1}{2} h_{cd}\, h^{ab} \tilde{h}_{ab} \qquad \text{solved by } \tilde{h}_{cd} = h_{cd} = \underbrace{\partial_c X^\mu \partial_d X_\mu}_{\text{metric induced on the string by } \eta_{\mu\nu}}$$

$$1 = \frac{1}{2} h^{ab} h_{ab} = \frac{1}{2}\delta^b_b$$

$$\Rightarrow S_{cl} = -\frac{T}{2}\int d^2\sigma\, \sqrt{-h}\, h^{ab} h_{ab} = -T \int d^2\sigma \sqrt{-\det \partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}}$$

$$= -T \int dA$$

$$T^c_c = h^{cd} T_{cd} = h^{cd} \partial_c X \partial_d X - \underbrace{\frac{1}{2} h^{cd} h_{cd}}_{=1}\, h^{ab} \partial_a X \partial_b X = 0$$

Gauge fixing

$$h_{ab} \underset{\substack{\uparrow \\ \sigma \to \sigma'}}{=} e^{\phi(\tau\sigma)} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \underset{\substack{\uparrow \\ \text{conformal}}}{=} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \eta_{ab} \qquad \sqrt{-h} = 1$$

Now the EOM are $\quad S = -\frac{T}{2}\int d^2\sigma \left(\partial_\tau X^\mu \partial_\tau X_\mu - \partial_\sigma X^\mu \partial_\sigma X_\mu\right) = -\frac{T}{2}\int d^2\sigma\, \partial_a X^\mu_\mu \partial^a X^\mu$

① $\quad \partial^2 X^\mu = (\partial_\tau^2 - \partial_\sigma^2) X^\mu = 0 \Rightarrow X^\mu = e^{i\lambda\tau} \cdot e^{\pm i\lambda\sigma} = e^{i\lambda(\tau \pm \sigma)}$

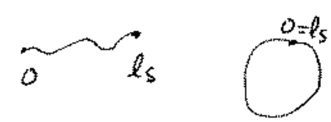
$\quad$ + boundary cond. $\qquad \partial_+ \partial_- X^\mu = 0$

② $\quad T_{cd} = \partial_c X \partial_d X - \frac{1}{2}\eta_{cd}\, \partial^a X \partial_a X$

$$T^c_c = 0 \qquad \begin{cases} T_{00} = \partial_0 X \partial_0 X - \frac{1}{2}\left(\partial_0 X \partial_0 X - \partial_\sigma X \partial_\sigma X\right) = \frac{1}{2}\left(\partial_\tau X \partial_\tau X + \partial_\sigma X \partial_\sigma X\right) = 0 \\ T_{11} = \partial_\sigma X \partial_\sigma X + \frac{1}{2}\left(\qquad \right) = \quad - \quad '' - \qquad = 0 \\ T_{01} = T_{10} = \partial_\tau X \cdot \partial_\sigma X = 0 \end{cases}$$

$$\begin{cases} T_{++} = T_{--} = 0 \\ T_{+-} = T_{-+} = 0 \end{cases}$$

String boundary conditions:



$$S = -\frac{T}{2}\int d^2\sigma\, \partial_a X_\mu \partial^a X^\mu$$

$X^\mu \to X^\mu + \delta X^\mu$

$$\delta \int d^2\sigma \mathcal{L}(\partial_a X) = \int d^2x \frac{\partial \mathcal{L}}{\partial \partial_a X} \cdot \partial_a \delta X = \underbrace{\int d^2\sigma\, \partial_a \left(\frac{\partial \mathcal{L}}{\partial \partial_a X} \delta X\right)}_{\text{surface term}} - \underbrace{\int d^2\sigma \left(\partial_a \frac{\partial \mathcal{L}}{\partial \partial_a X}\right)}_{\text{EOM}} \delta X$$

$$\int d^2\sigma\, \partial_a \left[\underbrace{\frac{\partial \mathcal{L}}{\partial \partial_a X_\mu}} \delta X_\mu\right] - \int d^2\sigma \underbrace{\left(\partial_a \partial^a X^\mu\right)}_{=0 \ \text{EOM}} \delta X_\mu$$

$$\int d^2\sigma\, \partial_a \left[\partial^a X^\mu \delta X_\mu\right]$$

$$= \underbrace{\int d\tau \left. \partial^\sigma X^\mu \delta X_\mu \right|_0^{l_s}}_{\text{crucial term}} + \int d\sigma \left. \partial^\tau X^\mu \underbrace{\delta X_\mu}_{=0 \text{ at } \pm\infty} \right|_{-\infty}^{\infty}$$

$$0 = \Big|_{\sigma=0}^{\sigma=l_s} \partial_\sigma X^\mu(\tau,\sigma)\, \delta X_\mu(\tau,\sigma) = \partial_\sigma X^\mu(l_s)\,\delta X_\mu(l_s) - \partial_\sigma X^\mu(0)\,\delta X_\mu(0)$$

$\boxed{\text{closed}}$

simple to satisfy!

$$X^\mu(\sigma + \overset{\pi\pi}{l_s}) = X^\mu(\sigma) \qquad X^\mu = e^{i2m(\tau\pm\sigma)} \qquad 0 \le \sigma \le \pi$$

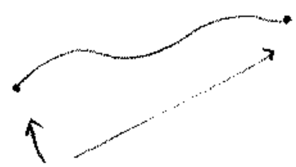
often 2π for closed

$$\partial_\sigma X^\mu(\sigma + \overset{\pi\pi}{l_s}) = \partial_\sigma X^\mu(\sigma)$$

$$\delta X_\mu(0) = \delta X_\mu(l_s)$$

L and R movers separate!

Open



endpoints free

$\delta X_\mu(0)\ \delta X_\mu(l_s)$ anything

$$\boxed{\partial_\sigma X^\mu(0) = \partial_\sigma X^\mu(l_s) = 0} \quad \text{Neumann}$$

no momentum flows in from ends

However one may also require, even for open strings,

$$\boxed{\delta X_\mu(l_s) = \delta X_\mu(0) = 0} \quad \text{Dirichlet (fixed boundary)}$$

open string mode expansion $e^{i\lambda(\tau\pm\sigma)} \to e^{im\tau}(e^{im\sigma} + e^{-im\sigma})$ L & R movers related by Neumann

N-N BC

$$X^\mu(\tau,\sigma) = \underbrace{x^\mu + l_s^2 p^\mu \tau}_{\text{CMS motion}} + i l_s \sum_{m\neq 0} \underbrace{\frac{1}{m}\alpha^\mu_m \overset{\cos(\frac{\pi m\sigma}{l})}{\cos(m\sigma)}\, e^{-im\tau}}_{\text{oscillation}}$$

$$S = \int d^2\sigma\, \frac{-T}{2}(\partial_\tau X^\mu \partial_\tau X_\mu - \partial_\sigma X \partial_\sigma X) \qquad P^\mu = \frac{\partial \mathcal{L}}{\partial \partial_\tau X^\mu} = -T\partial_\tau X^\mu$$

$$H = \int d\sigma\,(P_\mu \partial_\tau X^\mu - \mathcal{L}) = \int d\sigma \left[-T\partial_\tau X_\mu \partial_\tau X^\mu + \frac{T}{2}(\partial_\tau X \partial_\tau X - \partial_\sigma X \partial_\sigma X)\right]$$

$$= -\frac{T}{2}\int d\sigma \underbrace{(\partial_\tau X \partial_\tau X + \partial_\sigma X \partial_\sigma X)}_{\sim T_{00} = T_{11}\ \underbrace{=0}_{\text{classically}}}$$

$$\Rightarrow\ P^\mu = -T\int_0^\pi d\sigma\ l_s^2 p^\mu = -T l_s^2 \pi\, p^\mu = -p^\mu \qquad \text{total momentum of string}$$

$$T = \frac{1}{l_s^2 \pi}$$

Oscillators :

$$\text{Closed}: \quad X^\mu = x^\mu + l_s^2 p^\mu \tau + \frac{i}{2} l_s \sum_{n \neq 0} \frac{1}{n}\Big[\alpha^\mu_n \, e^{-2in(\tau-\sigma)} + \tilde{\alpha}^\mu_n \, e^{-i2n(\tau+\sigma)}\Big]$$

($\log z\bar{z}$, z^{-n}, $\bar{z}^{-n}$)

$$\alpha^{\mu\,+}_m = \alpha^\mu_{-m}$$

$$H = -\sum_{m\neq 0}\left(\alpha^\mu_{-m}\,\alpha_{\mu m} + \tilde{\alpha}^\mu_{-m}\,\tilde{\alpha}_{\mu m}\right) - \frac{1}{2} l_s^2 p^\mu p_\mu$$

Fourier comp L_m

Impose $T_{++} = T_{--} = 0$ (like Gauss $DE = \rho$ in $A_0 = 0$ gauge)

$[L_m, L_n] = (m-n)L_{m+n} + \frac{D}{12} m(m^2-1)\,\delta_{m+n,0}$ must be $N_L = N_R$

$$\Rightarrow p_\mu p^\mu = M^2 = \frac{2}{\alpha'}\left\{\sum_{m=1}^{\infty}\sum_{i=1}^{D-2}\left(\alpha^i_{-m}\,\alpha^i_m + \tilde{\alpha}^i_{-m}\,\tilde{\alpha}^i_m\right) + 2\cdot\underbrace{\frac{2-D}{24}}\right\} = \frac{2}{\alpha'}\left(N_L + N_R - 2\right)$$

only the $D-2$ "transverse" states should be counted

$\mu = 0$ & $D-1$ cancel

$$= \frac{D-2}{2}\underbrace{\sum_1^\infty m}_{\zeta(-1) = -\frac{1}{12}}$$

"light cone gauge"

can keep $h_{ab} = \eta_{ab}$ by $\begin{cases}\text{a conformal reparametrisation } \eta_{ab} \to \Lambda\eta_{ab} \\ \text{Weyl transf. } \Lambda\eta_{ab} \to \eta_{ab}\end{cases}$

$\Rightarrow X^+(\tau,\sigma) = \dfrac{X^0 + X^{D-1}}{\sqrt{2}} = x^+ + l_s^2 p^+ \tau$ no oscillators, just $\sim a + b\tau$

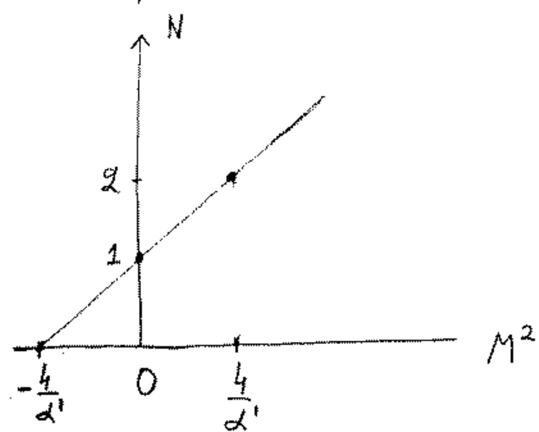
then X^- can be solved in terms of X^i, p^+, some x^-

$M^2 = 2p^+p^- - p^i p^i$ = as above

Low-lying states :

$N_L = N_R = 0 \quad M^2 = -\frac{4}{\alpha'}$ non-oscillating quantum string is a tachyon

$N_L = N_R = N \quad \frac{1}{4}\alpha' M^2 = (N-1) \quad N = \frac{1}{4}\alpha' M^2 + 1$



$N_R = N_S = 1$ $\quad \underbrace{\alpha^\mu_{-1}\tilde{\alpha}^\nu_{-1}}\,|0, k^\mu\rangle \quad M^2 = 0$

note two indices because L, R independent

can get $G_{\mu\nu}$ $B_{\mu\nu}$ ϕ

This was discussed earlier

$$M^2 = \frac{1}{\alpha'}\left\{\sum_{m=1}^{\infty}\sum_i \alpha^i_{-m}\alpha^i_{+m} + \frac{2-D}{24}\right\} = \frac{1}{\alpha'}(N-1)$$

<u>Open string</u> with D-D BC $\delta X^\mu = 0$ at ends

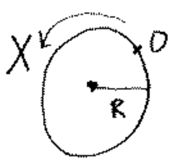
$$X^\mu(\tau,\sigma) = c^\mu\left(1-\frac{\sigma}{\pi}\right) + d^\mu\frac{\sigma}{\pi} - l_s\sum_{m\neq 0}\frac{1}{m}\alpha^\mu_m e^{-im\tau}\sin m\sigma$$

$$X^\mu(\tau, 0) = c^\mu \qquad X^\mu(\tau,\pi) = d^\mu$$

[Poincaré inv. is lost!!]

<u>Getting to Dbranes via compactification & T-duality :</u>

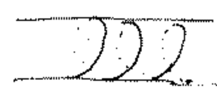
<u>Compactify</u> one coordinate for a <u>closed string</u>



$0 < X < 2\pi R \qquad p = \frac{2\pi}{L}m = \frac{2\pi}{2\pi R}m = \frac{m}{R}$

$X(\tau, \sigma)$, $0<\sigma<\pi$

$$X(\tau,\sigma+\pi) = X(\tau,\sigma) + n\,2\pi R$$



$$\Rightarrow \quad M^2 = \frac{2}{\alpha'}(N_L + N_R - 2) + \left(\frac{m}{R}\right)^2 + \left(\frac{n}{\alpha'}R\right)^2$$

KK: $g^5_{MN}(x^\mu, x^4) = \{g_{\mu\nu}(x^\mu),$ $A_\mu(x^\mu) = g^5_{\mu 4}(x^\mu),$ $g_{44}(x^\mu)\}$ $\quad x^4 \to x^4 + \lambda(x^\mu)$, $A_\mu \to A_\mu + \partial_\mu\lambda$

one winding has the energy $\quad \underset{\text{energy/length}}{T}\cdot 2\pi R \equiv \frac{R}{\alpha'} \quad \left(2\pi T = \frac{1}{\alpha'}\right)$

T-duality : $M(m, n, R) = M(n, m, \underbrace{\tfrac{\alpha'}{R}})$

radius of T-dual theory

$\Rightarrow R \geq \sqrt{\alpha'}$

Now work out how T-duality : $\frac{R}{\sqrt{\alpha'}} \to \frac{\sqrt{\alpha'}}{R}$

affects closed & open string Fourier series

For compact closed coordinate

$$X = x + \overset{2\alpha'}{\widetilde{l_s^2}}\, p \underset{\frac{1}{2}(\sigma_+ + \sigma_-)}{\underset{\downarrow}{\tau}} + 2mR \underset{\frac{1}{2}(\sigma_+ - \sigma_-)}{\underset{\downarrow}{\sigma}} + S(\sigma_+) + S(\sigma_-) \equiv \left[X_R(\sigma_-) + X_L(\sigma_+)\right]$$

$$\begin{cases} X_L = \frac{1}{2}x + (\alpha' p + mR)\sigma_+ + S(\sigma_+) \\ X_R = \frac{1}{2}x + \underbrace{(\alpha' p - mR)\sigma_-}_{} + \underbrace{S(\sigma_-)}_{} \end{cases} \qquad l_s = \sqrt{2\alpha'}$$

$$+\alpha'\left(\frac{m}{R} - \frac{mR}{\alpha'}\right)\sigma_- \qquad + i l_s \sum_{m \neq 0}{}' \frac{\alpha_m}{m} e^{-im(\tau - \sigma)}$$

$$= \sqrt{\alpha'}\left(\frac{m}{R/\sqrt{\alpha'}} - m\frac{R}{\sqrt{\alpha'}}\right)$$

T-duality: $R \to \frac{\alpha'}{R}$, $m \leftrightarrow m$

compact coord only:

$$\begin{cases} \partial_+ X = \alpha'\left(\frac{m}{R} + \frac{mR}{\alpha'}\right) + l_s \sum_{m \neq 0} \tilde{\alpha}_m e^{-im(\tau+\sigma)} \to +\partial_+ X \\ \partial_- X = \alpha'\left(\frac{m}{R} - \frac{mR}{\alpha'}\right) + l_s \sum \alpha_m e^{-im(\tau - \sigma)} \to -\partial_- X \end{cases}$$

$\tilde{\alpha}_m \to +\tilde{\alpha}_m$, $\alpha_m \hookrightarrow -\alpha_m$

sign ch for R movers

$$\boxed{\begin{aligned} X_L &\to X_L(\tau+\sigma) \\ X_R &\to -X_R(\tau-\sigma) \end{aligned}}$$

<u>Open</u> string has no winding modes since ends are free.

But take the plane wave expansion

$$X^{\mu = \text{compact}}_{\text{open}} = x^\mu + l_s^2 p^\mu \underset{\frac{1}{2}(\sigma_+ + \sigma_-)}{\tau} + \frac{i l_s}{2} \sum_m \frac{1}{m} \alpha^\mu_m \left[e^{-im(\tau+\sigma)} + e^{-im(\tau - \sigma)}\right] \qquad p^\mu = \frac{m}{R}$$

↑ one set of α^μ_m

$$= \frac{1}{2}x^\mu + \frac{1}{2} l_s^2 p^\mu (\tau + \sigma) + \frac{i l_s}{2} \sum_m \frac{1}{m} \alpha^\mu_m e^{-im(\tau+\sigma)} = X_L(\tau+\sigma)$$

$$+ \frac{1}{2}x^\mu + \qquad -\sigma \qquad '' \qquad -\sigma \qquad + X_R(\tau + \sigma)$$

and do T-duality map:

$$X = X_L + X_R \longrightarrow X_L - X_R$$

$$= l_s^2 p \sigma + l_s \sum \frac{1}{m}\alpha_m \frac{i}{2}\left(e^{-im(\tau+\sigma)} - e^{-im\tau}\cdot e^{+im\sigma}\right)$$

$$\boxed{\begin{gathered} X = l_s^2 p \sigma + l_s \sum_{m\neq 0} \frac{1}{m}\alpha_m \sin(m\sigma)\, e^{-im\tau} \\ X(\tau,0) = 0 \quad X(\tau,\pi) = \pi l_s^2 p = \pi l_s^2 \frac{m}{R} = 2\pi\alpha' \frac{m}{R} \end{gathered}}$$

for compact coordinate

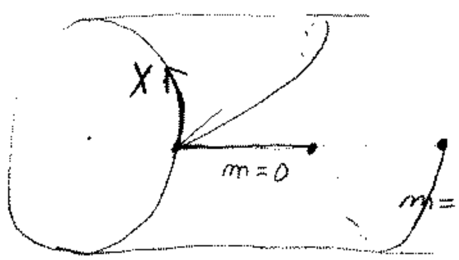
So one obtained an open string with fixed end points!

Compactify $D-1-p$ spacelike dim's, do T-duality on all of them, strings will be stuck in a $D-1-(D-1-p) = p$-dimensional hyperplane, <u>Dp brane</u>

$$X = l_s^2 p \sigma = 2\alpha' \frac{m}{R}\sigma \equiv 2m\tilde{R}\sigma \qquad \tilde{R} = \frac{\alpha'}{R}$$

originally a momentum $\frac{m}{R}$

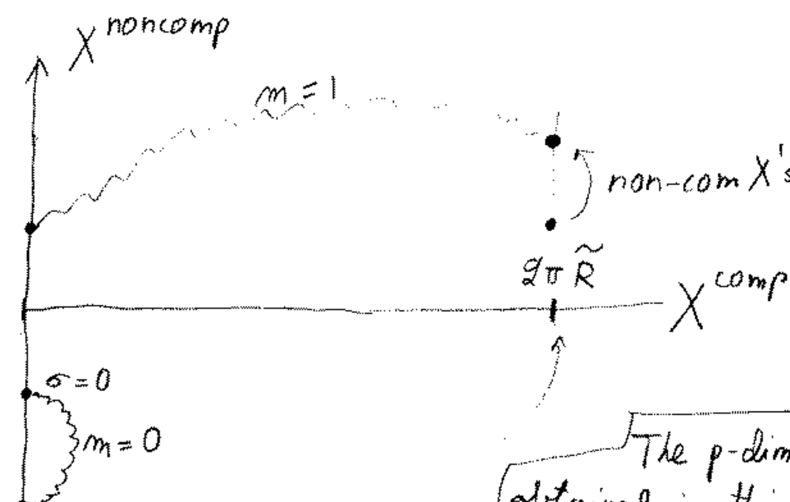
ends up being winding after T-duality map



non-compact X's

$2m\tilde{R}\cdot\sigma + l_s \alpha_1 \sin\sigma\, e^{-i\tau}$

<u>Or</u>:



non-com X's have changed, but $X^{comp} = 0 = 2\pi\tilde{R}$

For the superstring T-duality maps, say, IIA $\leftrightarrow$ II B

The p-dimensional subspaces Dp obtained in this way are the same as the p branes obtained as soln's of low end closed string eff. theories

Proof = ?