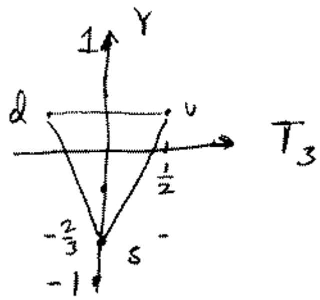
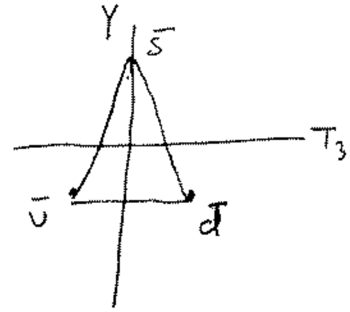
Pre QCD Strong Interactions: Spectrum, Regge, dual models

Spectrum

Flavour SU(3)!





$Q = T_3 + \frac{Y}{2}$

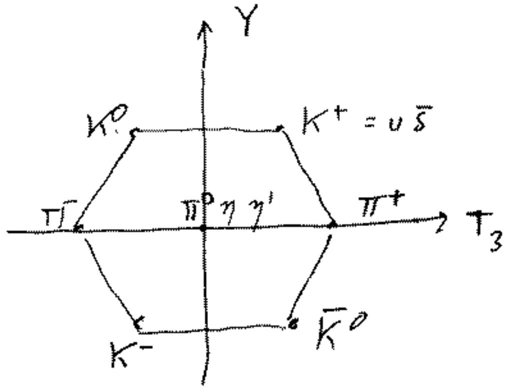
$Y = S + B$

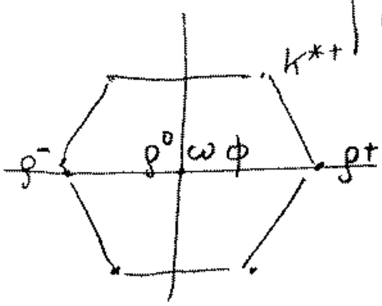
$\bar{q}'q$ states:

U: $J = L + S$, $S = 0, 1$

$P = (-)^{L+1}$

$C(\bar{q}q) = (-)^{L+S}$





3 neutrals $\bar{u}u$ $\bar{d}d$ $\bar{s}s$

isovector $\underset{u\bar{d}}{\pi^+}$ $\underset{\frac{u\bar{u}-d\bar{d}}{\sqrt{2}}}{\pi^0}$ $\underset{\bar{u}d}{\pi^-}$

Goldstone octet

isoscalars $\eta = \frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d - 2\bar{s}s)$

"η' problem" $\longrightarrow$ $\underset{958}{\eta'} \simeq (\bar{u}u + \bar{d}d + \bar{s}s)$

isovector $\rho^0(770)$

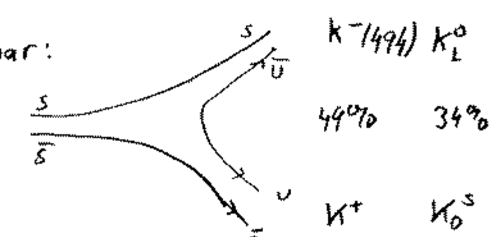
isoscalars $\begin{cases} \omega = \frac{\bar{u}u + \bar{d}d}{\sqrt{2}} & 782 \\ \varphi = \bar{s}s & 1020 \end{cases}$

$\Gamma = 150$ MeV

Dynamics

Zweig's rule: annihilation diagrams suppressed $\Leftrightarrow$ Exact for large N_c!

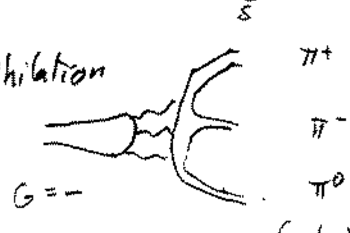
Planar:



$\Gamma = 4.4$ MeV $\sim \frac{1}{45\,\text{fm}}$

Small because of $Q = 1020 - 2 \cdot 494 = 32$ MeV

Annihilation



$\Rightarrow \Gamma(3\pi) \sim 0.66\,\text{MeV} \sim \frac{1}{300\,\text{fm}}$ small due to QCD dynamics

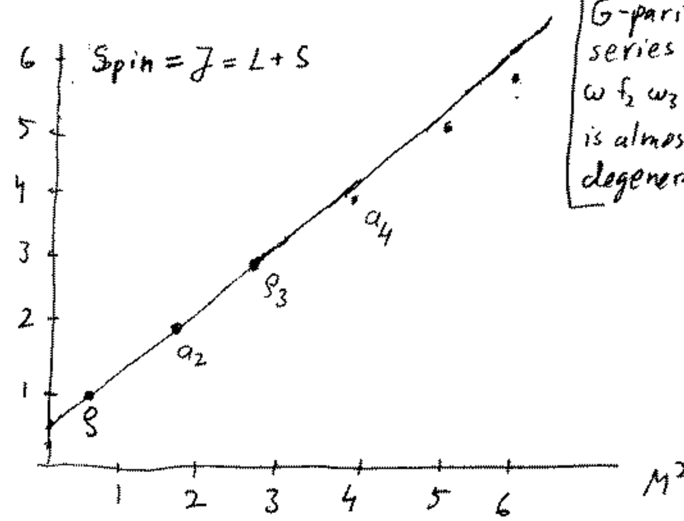
$\Gamma(2\pi)$ tiny: G-parity

Chew - Frautschi plot $I=1,\ S=0,\ B=0$ from PDG

${}^{2S+1}L_J$		m^2	J^{PC}	I^G	$\alpha' = (J-\frac{1}{2})/M^2$
${}^3S_1$	ρ	0.59	1^{--}	1^+	0.85
${}^3P_2$	a_2	1.74	2^{++}	1^-	0.86
${}^3D_3$	ρ_3	2.86	3^{--}	1^+	0.87
${}^3F_4$	a_4	4.16	4^{++}	1^-	0.84
	ρ_5	2.35^2 5.52	5^{--}	1^+	0.81
	a_6	2.45^2 6.00	6^{++}	1^-	0.92

PDG: "omitted from summary table" "needs confirmation"

search for $\pi^- p \rightarrow \underbrace{\underbrace{\pi^+\pi^-\pi^0}_{\omega}\ \pi^0}_{\rho_5}\ n$



the opposite G-parity series $\omega\ f_2\ \omega_3\ f_4$ .. is almost degenerate

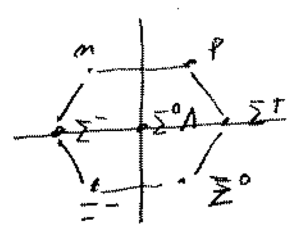
Remarkably (?) good fit to

$$\boxed{\alpha(t) = \frac{1}{2} + 0.85 M^2}$$

In 1970 Loma Koli Proceedings
p. 132 an experimental plot was given up to $J = 13$!
Real optimism!

Baryons

8



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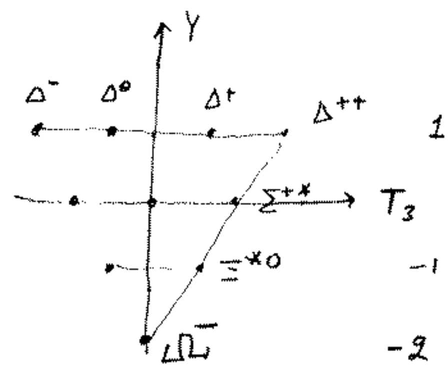
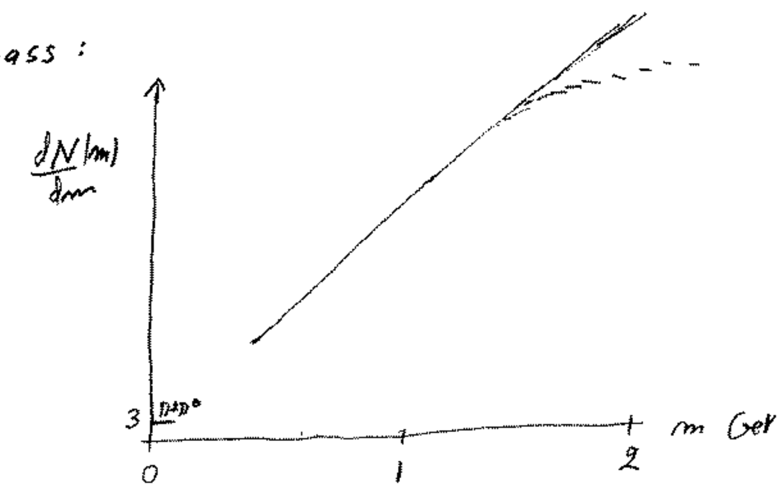


Δ^{++}

$\Omega = sss$

Plot all of today's 3182 u,d,s -states as a function of mass:



{dual models
{string theories
also have exp. density of states!

$$\Rightarrow \frac{dN}{dm}(m) \cong \frac{1}{m^3} e^{bm} \qquad b \approx \frac{1}{m_\pi} \sim \frac{1}{\Lambda_{QCD}}$$

Hagedorn spectrum (1965)

$$Z = \int dE \frac{dN(E)}{dE} e^{-\frac{E}{T}} \sim \int dE \; e^{\frac{E}{m_\pi} - \frac{E}{T}}$$

Singularity at $T \approx \Lambda_{QCD}$

Was T_{max} in 1965-72; now it is T_c!

Exercise: (a) Show that the density of states of N $m=0$ particles of $m=0$ in a box V is $\quad \rho(E,N) = \frac{1}{N!(3N-1)!} \frac{1}{E}\left(\frac{VE^3}{\pi^2}\right)^N$

(b) Show that the grand canonical density of states is

$$\rho(E,z) = \sum_{N=1}^{\infty} z^n \rho(E,N) \approx c\, E^{-5/8} e^{bE^{3/4}} \qquad b = \frac{4}{3}\left(\frac{3zV}{\pi^2}\right)^{\frac{1}{4}}$$

Hard to get e^{bE}!! Need diverging thermodynamics!

(c) You know that the partition functions (ideal massless class. gas) are

$$Z(T,N) = \frac{1}{N!}\left(\frac{VT^3}{\pi^2}\right)^N, \quad Z(T,\mu) = e^{\frac{zV}{\pi^2}T^3}$$

Derive $\rho(E,N)$, $\rho(E,z)$ by inverting the Laplace $Z = \int_0^\infty dE\, \rho(E)\, e^{-E/T}$

2 → 2 scattering

Nuovo Cimento 1959, 60

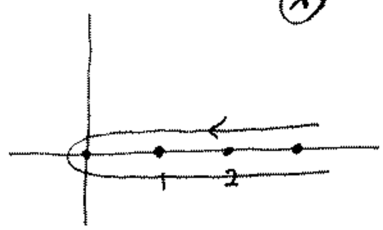
Regge poles:

(1) Potential scattering

$$f_\ell(E) = \frac{e^{2i\delta_\ell}-1}{2ip} \qquad E = \frac{p^2}{2m}$$

$$\frac{d\sigma}{d\Omega} = |f(\theta, E)|^2 \qquad f(z, E) = \sum_{\ell=0}^{\infty} (2\ell+1)\, f_\ell(E)\, P_\ell(z)$$

λ



$$= \frac{1}{2i} \int_C \frac{d\lambda}{\sin\pi\lambda} (2\lambda+1)\, f(\lambda, E)\, \underbrace{(-)^\lambda P_\lambda(z)}_{z\to\infty}$$

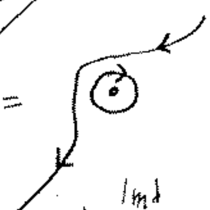
$$\frac{1}{\sqrt{\pi}} \frac{\Gamma(\lambda+\frac{1}{2})}{\Gamma(\lambda+1)} (2z)^\lambda$$

$$\sin\pi\lambda = \sin(\pi\ell + \pi x) = \cos\pi\ell \cdot \pi x = (-)^\ell\, \pi x$$

$$\lambda = \ell + x$$

$$\oint \frac{d\lambda}{\sin\pi\lambda} f(\lambda) = \oint \frac{d\lambda}{\pi(-)^\ell(\lambda-\ell)} f(\lambda) = 2i(-)^\ell f(\ell)$$

=



λ

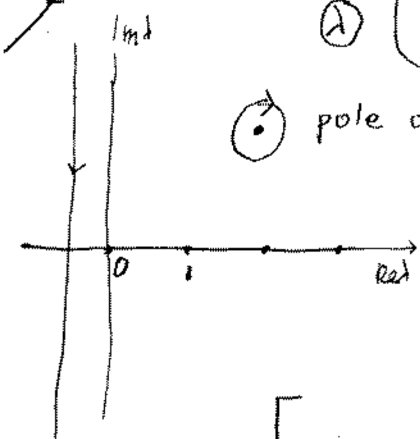
$$f(\ell) \equiv \frac{(-)^\ell}{2i} \oint \frac{d\lambda}{\sin\pi\lambda} f(\lambda)$$

pole of $f(\lambda, E)$

= Regge pole at $\lambda = \alpha(E)$

= pole of partial wave amplitude in the complex ℓ plane

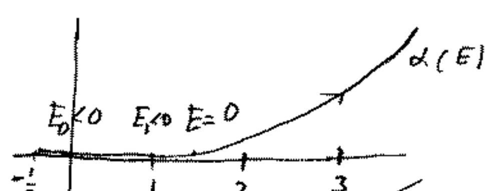
=



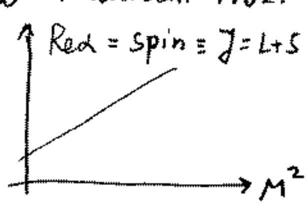
$$\left[f(z, E) = \sum_{n=1}^{N} \frac{\gamma_n(E)}{\sin\pi\alpha_n(E)} P_{\alpha_n(E)}(-z) + \text{cut} \right.$$

$$\left[\begin{array}{lll} \mathrm{Im}\,\alpha(E) \geqslant 0 & \text{for } E>0 & \\ \qquad\quad = 0 & \qquad E<0 & \text{bound state} \end{array} \right.$$

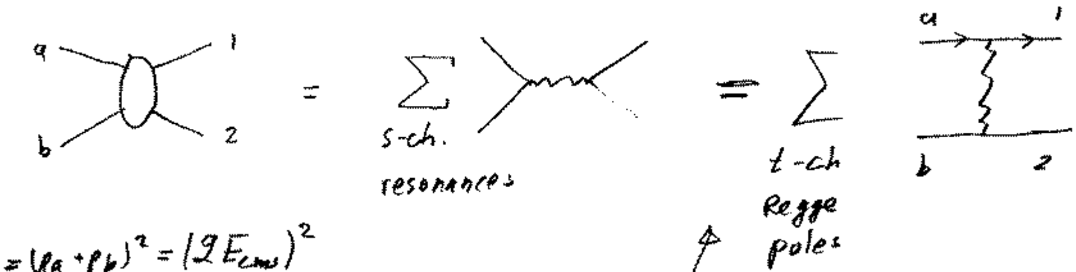
Regge trajectory:
(example)



Chew - Frautschi 1962:

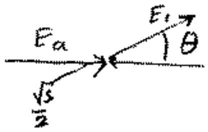


(2) 2 → 2 in particle physics



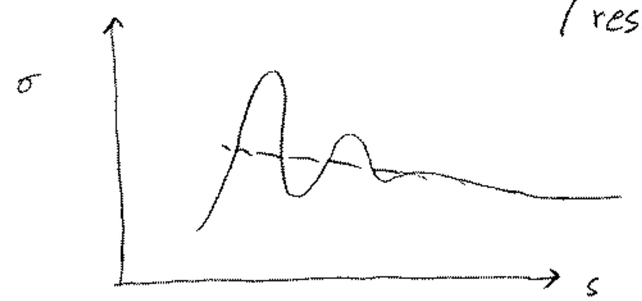
$s = (p_a + p_b)^2 = (2E_{cms})^2$

$t = (p_a - p_1)^2 = -|\bar{p}_a - \bar{p}_1|^2 = -2p_s^2(1-\cos\theta) = -\left(\frac{s}{2} - 2m^2\right)(1-z)$



$E = \frac{\sqrt{s}}{2} \quad p = \sqrt{\frac{s}{4} - m^2}$

not sum; either or / duality! (resonance - Regge)



t channel:

$z_t = \cos\theta_t$:
$$\begin{cases} t = (p_a + p_1)^2 = (2E_t)^2 \\ s = (p_a - p_b)^2 = -2p_t^2(1 - z_t) \qquad p_t^2 = \frac{t}{4} - m^2 \end{cases}$$

$$P_\alpha(-z_t) \sim (-z_t)^\alpha \sim \left(\frac{s}{2m^2 - \frac{1}{2}t}\right)^{\alpha(t)} \qquad z_t = 1 + \frac{s}{2p_t^2} = 1 - \frac{2s}{4m^2 - t}$$

Regge 2→2 amplitude

$$\boxed{A(s,t) = \underbrace{f(t)}_{\text{real, some fn of } t} \cdot \left(\frac{s}{s_0}\right)^{\alpha(t)} \underbrace{\xi(t)}_{\text{phase factor}}}$$

sum over various $\alpha(t)$!

phase factor from $1 \pm (-)^l = 1 \pm e^{-i\pi\alpha}$
$$\hat{=} \begin{cases} e^{-i\frac{\pi}{2}\alpha} & C = + \\ i e^{-i\frac{\pi}{2}\alpha} & C = - \end{cases}$$

$$\frac{d\sigma}{dt} = \frac{1}{16\pi\,\lambda(s, m_a^2, m_b^2)} |A(s,t)|^2$$

$\lambda \to (s - m_a^2 - m_b^2)^2 - 4m_a^2 m_b^2$

$$\Rightarrow |f(t)|^2 \left(\frac{s}{s_0}\right)^{2\alpha(t)-2} = |f(t)|^2 \left(\frac{s}{s_0}\right)^{2\alpha(0)-2} \cdot e^{\underbrace{2\alpha'(0)\ln\frac{s}{s_0}}_{\text{shrinking forward peak}} \cdot t}$$

$$\boxed{\sigma_{tot}(ab \to X) = \frac{1}{\sqrt{\lambda(s, m_a^2, m_b^2)}} \operatorname{Im} A(s, t=0)}$$

Skip here detailed discussion of

- quantum numbers of Regge poles
- spin ⇒ many amplitudes, each a complex number

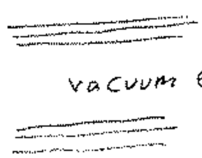
  phase shift analyses

{Pomeron trajectory
{Vacuum

$$\alpha_P(t) = 1 + 0.08 + 0.25\,t \qquad \text{(Landshoff et co)}$$

Dominates large s:

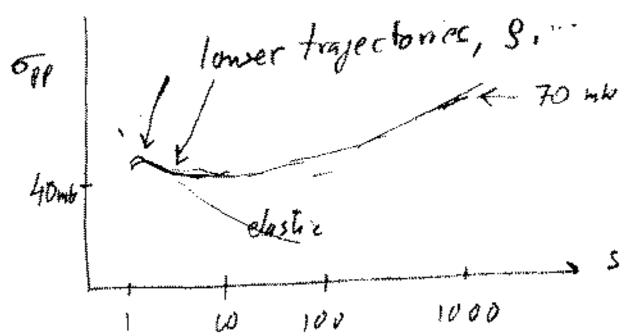
$$\sigma_{tot}(ab) = f_{ab}\cdot\left(\frac{s}{s_0}\right)^{0.08}$$



⇒ model by 2 gluon exch

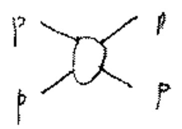
3 gluons ⇒ C=-1

odderon



$$\frac{d\sigma}{dt} \sim \frac{s_0^2 F(t)}{s^2}\left(\frac{s}{s_0}\right)^{2.16+0.5t} \sim \underbrace{f(t)}_{\substack{\text{some}\\ \text{form factor}}}\, e^{-0.5\log\frac{s}{s_0}|t|}\left(\frac{s}{s_0}\right)^{0.16}$$

Large angle scattering



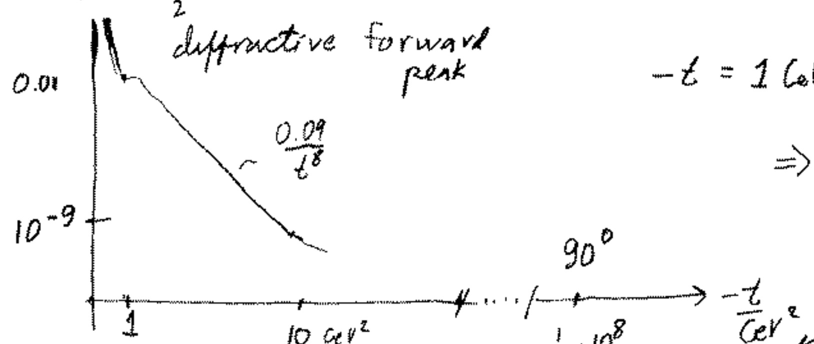
$$t = -2q^2(1-\cos\theta) \qquad q = \sqrt{\frac{s}{4} - m^2} \sim \frac{\sqrt{s}}{2} \sim E_{cm}$$

$$\theta = 90^\circ \qquad t = -2\cdot\left(\frac{s}{4} - m^2\right) = -\frac{s}{2} + 2m^2 = u$$

$$s + t + u = 4m^2 \qquad \frac{-t}{s} = +\frac{1}{2} - \frac{2m^2}{s}$$

$$-t(90^\circ) = \frac{1}{2}\cdot 10^8\, GeV^2$$

LHC: $\frac{\sqrt{s}}{2} = 5000\, GeV$



totally unmeasurable!

$$-t = 1\, GeV^2 = \frac{s}{2}\cdot\frac{\theta^2}{2} = \left(\frac{\sqrt{s}}{2}\theta\right)^2$$

$$\Rightarrow \theta \sim \frac{2\, GeV}{\sqrt{s}} \sim \frac{1}{5000}$$

$\boxed{\text{You cannot kick a proton with 1000 GeV to } 90^\circ \text{ without breaking it!}}$

$1 = 0.389\, mb\, GeV$

Forward direction $\mathrm{Re}A \ll \mathrm{Im}A$

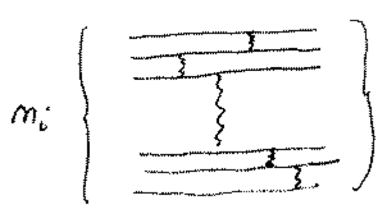
$$\frac{d\sigma}{dt} = \frac{1}{16\pi s^2}|A|^2 \sim \frac{1}{16\pi}\sigma_T^2 \approx \frac{1}{16\pi}\, 100\cdot 100\, mb\, \frac{1}{0.389\, GeV} \approx 500\, \frac{mb}{GeV^2}$$

$$\sigma_{tot} = \frac{1}{s}\,\mathrm{Im}A \qquad \leq \frac{1}{m_\pi^2}\log^2 s$$

at large t : $\frac{d\sigma}{dt} \sim \frac{0.09}{t^8}$

Scaling rule

assume only dimensional variables are s, t



$$A(m_i \to m_f) = \frac{1}{\text{energy}^{\,m_i + m_f - 4}}$$

[$A(2\to 2)$ = dimless, adding one part to final state adds $\frac{d^3p}{2E} \sim (\text{energy})^2$ which has to be compensated by $1/\text{energy}$ in A]

$$\Rightarrow \frac{d\sigma}{dt} = \frac{1}{s^2}|A|^2 = \frac{1}{s^2}\,\frac{1}{s^{m_i+m_f-4}}$$

$(\text{energy}^2 = s)$

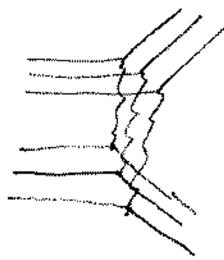
$$\boxed{\frac{d\sigma}{dt} = \frac{1}{s^{m_i+m_f-2}}\cdot f\left(\frac{t}{s}\right)}$$

fixed angle

$$\Rightarrow \frac{d\sigma^{\pi p}}{dt} = \frac{1}{s^8} f\left(\frac{t}{s}\right) \qquad \frac{d\sigma^{pp}}{dt} = \frac{1}{s^{10}} f\left(\frac{t}{s}\right) \qquad \frac{d\sigma^{\gamma p}}{dt} = \frac{1}{s^6} f\left(\frac{t}{s}\right)$$

$n_i = n_f = 5$ $\qquad$ $n_i = n_f = 6$ $\qquad$ $n_i = n_f = 4$

Landshoff claims, though that the leading diag is

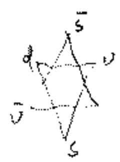


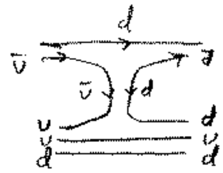
$$\Rightarrow \frac{d\sigma}{dt} = \frac{1}{s^2}\left(\frac{\alpha_s^2}{t^2}\right)^3 \sim \frac{1}{s^8}\left(\frac{s}{t}\right)^6$$

$$\sim \frac{1}{t^8}\cdot\left(\frac{t}{s}\right)^2$$

$$\approx \frac{0.09}{t^8} \quad \text{experiment}$$

<u>$\pi^- p \to \pi^0 n$:</u>





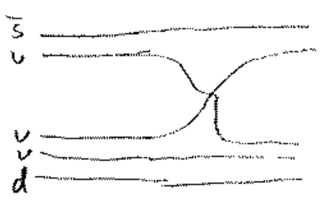
$$A(s,t) \sim \beta(t)\left(\frac{s}{s_0}\right)^{\frac{1}{2}+\alpha' t}$$

$$\frac{d\sigma}{dt} \simeq \frac{1}{16\pi s^2}\,\beta^2\left(\frac{s}{s_0}\right)^{1+2\alpha' t}$$

$$\approx \frac{1}{16\pi s_0^2}\,\beta^2\,\frac{s_0}{s}\; e^{2\alpha' t \ln\frac{s}{s_0}}$$

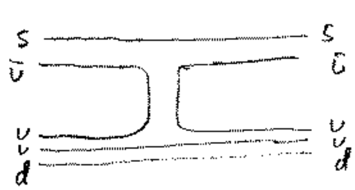
$$\Rightarrow \sigma = \frac{1}{16\pi s s_0}\int_0^{-s\to\infty} dt\, \underbrace{\beta^2(t)}_{\#\cdot e^{-2bt}} \approx \frac{1}{16\pi s}\,\frac{1}{s_0}\cdot\frac{\#}{2b} \sim \frac{1}{s}$$

<u>$K^+ p \to K^+ p$</u>



can only draw a non-planar diag: suppressed

<u>$K^- p \to K^- p$</u>



unsuppressed!

Veneziano amplitude

Cern Preprint TH 924, 19 Jul 1968

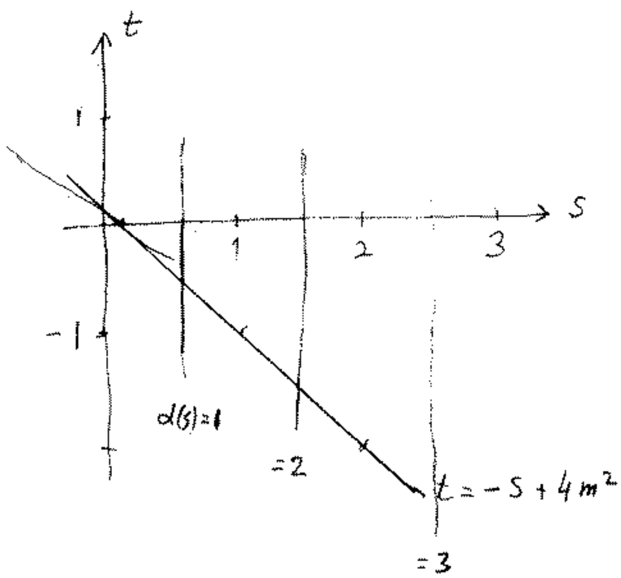
$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

Consider the ansatz:

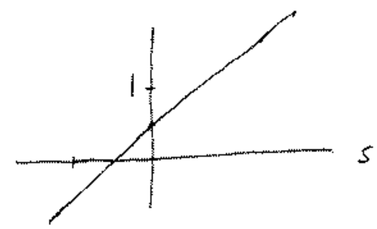
$$A\left(\begin{smallmatrix}\pi^+ & & \pi^+\\ & \bigcirc & \\ \pi^- & & \pi^-\end{smallmatrix}\right) = A(s,t) = \frac{\Gamma(1-\alpha(s))\,\Gamma(1-\alpha(t))}{\Gamma(1-\alpha(s)-\alpha(t))}$$

$$\alpha(s) = \alpha_0 + \alpha' s \approx \tfrac{1}{2} + s$$

$$s = (2E)^2 = 4(\bar{p}^2 + m^2) \qquad t = -2\bar{p}^2(1-\cos\theta) = -2\left(\tfrac{s}{4} - m^2\right)(1-\cos\theta)$$



$$0 \geq -t \geq s - 4m^2$$



(1) s-channel poles:

$t \leq 0 \quad \alpha(t) \leq +\frac{1}{2} \qquad 1-\alpha(t) \geq \frac{1}{2}$ no poles

$s \geq 4m^2 \quad \Gamma(1-\alpha(s))$ has poles at $1-\alpha_s = -m \quad \alpha(s) = m+1 = 1, 2, \ldots$

$\rho, A_2, \ldots$

$$1-\alpha_s = -m: \quad \frac{\Gamma(1-\alpha_t)}{\Gamma(-m-\alpha_t)} \cdot \frac{(-)^m}{\Gamma(m+1)} \cdot \frac{1}{1-\alpha_s+m}$$

$$= \underbrace{\frac{-\alpha_t}{1-\alpha_s}} + \frac{-\alpha(t)[1+\alpha(t)]}{2-\alpha_s} + \ldots$$

$$-\frac{\alpha_0 + \alpha' t}{1-\alpha_0-\alpha' s} = \frac{-\alpha_0/\alpha' - t}{m_\rho^2 - s} = \frac{a + b\cos\theta}{m_\rho^2 - s}$$

nice p-wave pole!

on real axis!!

etc.

add $\alpha(s) = \alpha_0 + \alpha' s + i\,\mathrm{Im}\,\alpha(s)$!

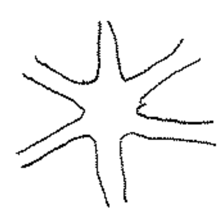
(2) Regge: $\begin{cases} \text{large } s \leftarrow \alpha(s) \gg 1 \\ t \text{ finite} \end{cases}$ $\qquad \dfrac{\Gamma(z+a)}{\Gamma(z+b)} \sim z^{a-b}\left[1 + \dfrac{(a-b)(a+b-1)}{2z} + \cdots\right]$

$$A(s,t) \to \Gamma(1-\alpha(t))\left[1-\alpha(s)\right]^{\alpha(t)}$$

$$\to \Gamma(1-\alpha(t))\, e^{-i\pi\alpha(t)} \underbrace{(\alpha' s)^{\alpha(t)}}_{\text{just the Regge type behaviour!}}$$

So one has an explicit analytic realisation of crossing symmetry, resonance - Regge duality, built in

However; this is not all the physics, just a model!

Generalised to 

Later obtained from CFT as an open string tree level tachyon scattering amplitude !