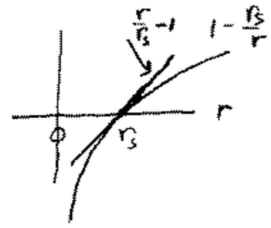
Getting T_H from the BH metric: seems like a very formal argument!

Finite T means periodicity in $\tau = it$: $f(\tau + \frac{\hbar}{T}) = f(\tau)$

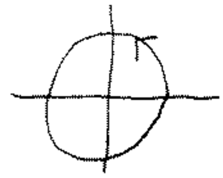


__Near__ r_s : $1 - \frac{2MG}{r} \cong \frac{1}{r_s}(r - r_s)$ $\qquad r_s = 2MG = 2\frac{M}{M_{Pl}^2}$

$$ds^2 = \frac{1}{r_s}(r-r_s)\,d\tau^2 + \frac{r_s}{r-r_s}\,dr^2 + \cdot$$

Try: $\varrho = 2\sqrt{r_s(r-r_s)}$ $\quad d\varrho = \sqrt{\frac{r_s}{r-r_s}}\,dr$ $\quad \frac{1}{4}\varrho^2 = r_s(r-r_s)$

$$ds^2 = d\varrho^2 + \frac{1}{4}\frac{\varrho^2}{r_s^2}\,d\tau^2 = d\varrho^2 + \varrho^2\, d\left(\frac{\tau}{2r_s}\right)^2$$



compare with $\quad x = \varrho \sin\varphi$, $y = \varrho\cos\varphi$ $\quad dx^2 + dy^2 = (s\varphi\, d\varrho + \varrho c\varphi\, d\varphi)^2 + (c\varphi\, d\varrho - \varrho s\varphi\, d\varphi)^2$

$$= d\varrho^2 + \varrho^2 d\varphi^2$$

Periodicity $\varphi + 2\pi$

$$\frac{\tau}{2r_s} + 2\pi \quad \text{or} \quad \tau + 4\pi r_s$$

$$\Rightarrow 4\pi r_s = \frac{1}{T_H} \qquad \boxed{T_H = \frac{1}{4\pi r_s} = \frac{1}{8\pi G M} = \frac{M_{Pl}^2}{8\pi M}} \qquad \frac{\hbar}{T_H}$$

BB radiation:

$$\frac{dM}{dt} \sim -r_s^2 T_H^4 \sim -\frac{1}{r_s^2} \sim -\frac{M_{Pl}^4}{M^2}$$

$$M^2 dM = -M_{Pl}^4\,dt \qquad \int_{M_0}^{M(t)} \frac{1}{3}M^3 = -M_{Pl}^4\, t = \frac{1}{3}M^3(t) - \frac{1}{3}M_0^3$$

$$M^3(t) = M_0^3 - 3M_{Pl}^4\, t$$

$$\Rightarrow \text{lifetime of BH} \sim \frac{M_0^3}{M_{Pl}^4} \sim \left(\frac{M_0}{M_{Pl}}\right)^3 \frac{1}{M_{Pl}}$$

__Example__ BH with lifetime $\sim \tau_{universe}$:

$$\frac{1}{H_0} \sim \tau_U \sim \frac{3}{h}\, 10^{60}\frac{1}{M_{Pl}}$$

$M = 10^{20} M_{Pl} \Rightarrow$ lifetime $= 10^{10}a \sim \tau_U$

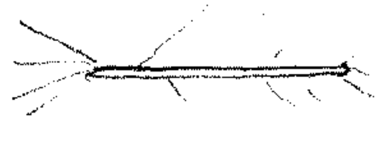
$\sim 10^{12}\,kg \qquad r_s \sim 10^{20}\frac{1}{M_{Pl}} \sim \frac{1}{0.1\,GeV} \approx 1\,fm$

__Entropy__:

$$S_{BH} = \frac{A_d}{4\hbar G_d} \Rightarrow \text{holography}$$

$\dim G_d = (\text{length})^{d-2} = \text{area}$

BH history:

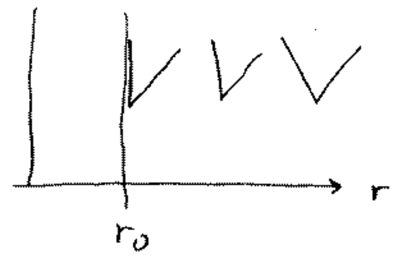


<u>More generally:</u>

$$-dt^2 = -g(r)\,dt^2 + \frac{1}{f(r)}\,dr^2 \qquad \equiv g_{00}\,dt^2 - g_{rr}\,dr^2$$

$$\underset{r\approx r_0}{\approx} -g'(r_0)(r-r_0)\,dt^2 + \frac{1}{f'(r_0)(r-r_0)}\,dr^2$$

light: $\quad \dfrac{dt}{dr} = \dfrac{1}{\sqrt{g'(r_0)\,f'(r_0)}}\;\dfrac{1}{r-r_0}$



(1) go Euclidian, $\tau = it$

(2) transform to

$$d\rho^2 + \rho^2\,d\varphi^2 \qquad \varphi \to \varphi + 2\pi \text{ - periodic!}$$

$$ds^2 = \frac{1}{f_0'(r-r_0)}\,dr^2 + g_0'(r-r_0)\,d\tau^2$$

$$\rho = 2\sqrt{\frac{1}{f_0'}(r-r_0)} \qquad \rho^2 = \frac{4}{f_0'}(r-r_0) \qquad 2\rho\,d\rho = \frac{4}{f_0'}\,dr$$

$$dr = \frac{f_0'}{4}\cdot 4\sqrt{\frac{1}{f_0'}(r-r_0)}\;d\rho \qquad dr^2 = f_0'(r-r_0)\,d\rho^2$$

$$\Rightarrow ds^2 = d\rho^2 + g_0'\cdot\frac{1}{4}\rho^2 f_0'\,d\tau^2 = d\rho^2 + \rho^2\cdot\underbrace{\frac{g_0' f_0'}{4}\,d\tau^2}_{d\left(\frac{\tau}{2/\sqrt{g_0' f_0'}}\right)^2}$$

periodic under $\tau \to \tau + \dfrac{4\pi}{\sqrt{g_0' f_0'}}$

$$\boxed{T_H = \frac{1}{4\pi}\sqrt{g'(r_0)\,f'(r_0)}}$$

If the period is not 2π, there is a conical singularity; Singularity is removed only for this period

Usual BH: $\quad g_0' = \frac{1}{r_s} = f_0' \qquad T_H = \dfrac{1}{4\pi r_0}$

Two other cases:

(1) de Sitter in "static coordinates" see later!

$$ds^2 = -(1 - H^2 r^2)\,dt^2 + \frac{1}{1 - H^2 r^2}\,dr^2 - \cdots$$

$$r_0 = \frac{1}{H} \qquad g'(r_0) = -2H = f'(r_0)$$

$$\Rightarrow \boxed{T_H = \frac{H}{2\pi}} \qquad \left(R(t) \sim e^{Ht} \right)$$

(2) BH in AdS_{m+1}

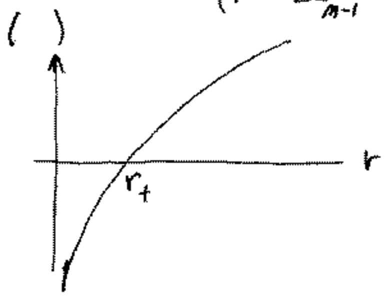
$$ds^2 = -\left(1 - \frac{w_m G_{m+1} M}{r^{m-2}} + \frac{r^2}{b^2}\right) dt^2 + \frac{1}{(\text{same})}\,dr^2 + r^2 d\Omega^2_{m-1}$$

$\uparrow$ anti dS $\qquad b \simeq \frac{1}{H}$

$m \Leftrightarrow d-1 \qquad G_d = \frac{1}{M^{d-2}}$

$$w_m = \frac{16\pi}{(m-1)\Omega_{m-1}} \qquad w_3 = \frac{16\pi}{2 \cdot 4\pi} = 2 \qquad \dim G_{m+1} = \frac{1}{M^{m-1}}$$

$$[G_{m+1} M] = \frac{1}{M^{m-2}} \equiv r_s^{\,m-2}$$



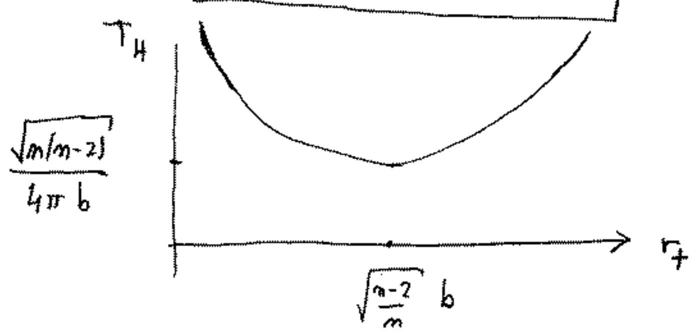
$$g'(r_+) = \frac{2 r_+}{b^2} + \frac{(m-2) w_m G_{m+1} M}{r_+^{m-1}} = \frac{2 r_+}{b^2} + \frac{m-2}{r_+}\left(1 + \frac{r_+^2}{b^2}\right)$$

$$= \frac{m-2}{r_+} + \frac{m r_+}{b^2}$$

$$\frac{w_m G_{m+1} M}{r_+^{m-1}} \cdot r_+ = 1 + \frac{r_+^2}{b^2}$$

$$\Rightarrow \boxed{T_H = \frac{m r_+^2 + (m-2) b^2}{4\pi r_+ b^2}}$$

This metric will be related to the 3brane metric on p. 15



$$\boxed{S = \frac{\Omega_{m-1} r_+^{m-1}}{4 G_{m+1}} = \frac{\text{"area of horizon"}}{4 G_{m+1}}}$$

$\boxed{AdS_{m+1}}$ $\qquad d \Leftrightarrow m+1$

$4 = 3 + 1$