Sketch an example: Thermo of $N=4$ SuSy YM in $d=4$

Supersymmetrize:

$$S = \int d^dx \left\{ -\frac{1}{4} F^{\overset{1,2,\cdots,N^2-1}{a}}_{\underset{A,B=0,1,\cdots,d-1}{AB}} F^{ABa} - \frac{1}{2} \bar\lambda^a i \underset{\text{d-dim Dirac matrices}}{\Gamma_A} \underset{\text{adjoint fermion}}{(D^A\lambda)}^{\overset{\text{color}}{a}} \right\}$$

DiVecchia
hep-th/9803096
sect 9-

$$[D_A, \lambda] = \partial_A\lambda - ig[A_A, \lambda]$$

$$(D_A\lambda)^a = \partial_A \lambda^a - g f_{abc} A_A^b \lambda^c$$

Work out susy transformations:

$$\delta A_A^a = \frac{i}{2}\left(\bar\lambda^a \Gamma_A \xi - \bar\xi \Gamma_A \lambda^a\right)$$

$$\delta\lambda_a = \sigma_{AB} F_a^{AB} \xi \qquad \delta\bar\lambda = -\bar\xi \sigma_{AB} F_a^{AB}$$

classic problem: does not simply close to form an algebra!! ⇒ group

(later!), find constraints for d and N = the number (from closure of SuSy generators;

$$\{ \underset{\substack{\uparrow \\ \text{this is Lorentz group} \\ \text{representation index } a=1,2 \\ \Lambda \sim M \in SL(2,C)}}{Q_a^i}, \underset{\nwarrow \, \Lambda \sim M^*}{\bar Q_{\dot a}^j} \} = 2(\sigma_\mu)_{a\dot a} P^\mu \delta^{ij} \qquad i,j = 1,\cdots,N$$

physical boson dofs $d-2$ = fermion dofs $\frac{1}{4} 2^{\frac{d}{2}}$

$N=1$ for $d=10$ if λ is a Weyl-Majorana spinor

⇒ $N=4$ in $d=4$ is obtained by dimensional reduction:

$$x^A = (x^0, x^1, x^2, x^3, \underbrace{x^4, \ldots, x^9}_{\substack{\text{assume fields do not depend on} \\ \text{these coordinates (compactified)}}})$$

$$\Rightarrow A_a^{A=4,\cdots,9}(x^0, \vec{x}) \text{ become } \overset{6}{\text{adjoint scalars}}$$

" with interactions from, say

$$F_{ij}^2 \sim (A_i A_j)^2$$

$\Rightarrow \mathcal{L}$ for $\left\{\begin{array}{l} d=4 \\ N=4 \end{array}\right.$ susy YM with

Di Vecchia, (9.29)
(10.9) ← superfield formalism

	dofs $/(N^2-1)$		
one vector field	$1\cdot 2 = 2$	bosonic	
6 scalars	$6\cdot 1 = 6$		
4 fermions	$4\cdot 2 = 8$		

So this is QCD ($N_f=0$, pure Y-M) decorated with scalars and fermions. Maybe it can help in solving Y-M?

But the theory is conformally invariant; coupling does not run (p. 4)

$$\mu\frac{\partial g(\mu)}{\partial\mu} = \beta(g) = 0$$

add'l dofs cancel each other!

So g is just some parameter!

Thermodynamics $\quad \frac{F}{T} = -p\frac{V}{T}$

Ideal gas limit $\quad p = a\frac{\pi^2}{90}T^4 = -\frac{F}{V} \qquad a = g_B + \frac{7}{8}g_F$

$$= d_A\cdot\frac{\pi^2}{6}T^4 \qquad\qquad = \left(8+\frac{7}{8}\cdot 8\right)d_A = 15\cdot \underset{N^2-1}{\underbrace{d_A}}$$

Corrections can be worked out $\boxed{\text{(Nieto-Tytgat, hep-th/9906147)}}$ + others

$$p = d_A\frac{\pi^2}{6}T^4\left(1 - \frac{3}{2\pi^2}\underbrace{C_A g^2}_{\equiv \frac{1}{2}\text{ 't Hooft}} + \frac{3+\sqrt{2}}{\pi^3}\left(C_A g^2\right)^{3/2} + O(g^4\ln g)\right)$$

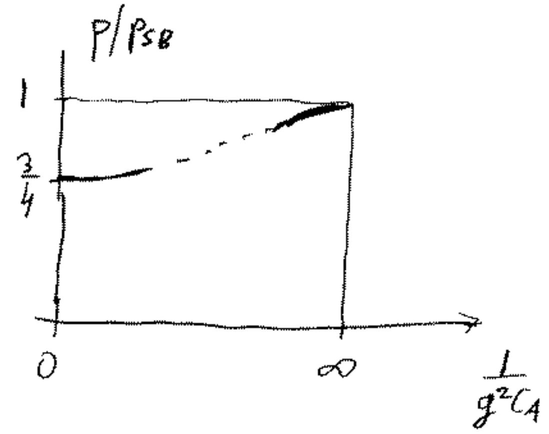
not done yet!

coupling, QCD in
the large N_c limit, $C_A = N_c$

$\boxed{\text{Manohar, hep-ph/9802419}}$

But, remarkably, proceeding via string theories, one can show (conjecture?) that in the strong coupling limit

$$p = d_A \frac{\pi^2}{6} T^4 \left[\frac{3}{4} + \overset{\mathcal{O}(\alpha')\text{ correction}}{\frac{45\zeta(3)}{64\sqrt{2}} \frac{1}{(g^2 C_A)^{3/2}}} + \cdots \right]$$



Gubser et al
hep-th/9602135
/9805156

$$g^2 C_4 = \frac{R^4}{g(\alpha')^2} \leftarrow \text{compact. radius}$$

large $g^2 C_4$ is small α'/R^2, field theory limit of string theory

This is what we shall try to understand.

Lots of eqs. on the way there!

$$\underline{\mu\frac{\partial g}{\partial\mu} = \beta(g) = -\beta_0 g^3 - \beta_1 g^5 - \beta_2 g^7 - \cdot}$$

QCD : $\int d^4x \, -\frac{1}{4}(F^a_{\mu\nu})^2 \Rightarrow \quad \beta_0 = \frac{1}{16\pi^2}\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right)$

1 loop, in general

$$\beta_0 = \frac{1}{16\pi^2}\left[\frac{11}{3}C_2(G) - \frac{4}{3}\underbrace{N_f^{Dirac}}_{\cdot\frac{1}{2}N^{Major}}\underset{\sim}{C_2}(F) - \frac{1}{6}N_S^{real}\,C_2(S)\right]$$

$f_{akl}f_{bkl} = C_2(G)\delta_{ab}$ $\quad$ $Tr T_F^a T_F^b = C_2 \cdot \delta_{ab}$ $\quad$ $Tr T_S^a T_S^b = C_2\delta_{ab}$

SUSY QCD's (many of them!)

N = 1 $\quad -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + \theta\frac{g^2}{32\pi^2}F^a_{\mu\nu}\tilde{F}^{\mu\nu a} + \frac{1}{2}\bar{\lambda}^a i\gamma^\mu (D_\mu\lambda)^a$ (just like the any d but for the $F\tilde{F}$ term!

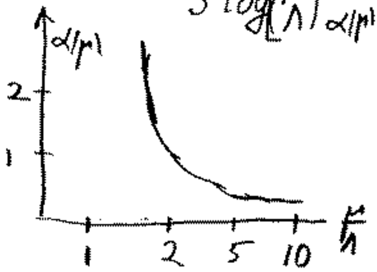
A^a_μ $\quad$ gluino { 4-comp real Maj. spinor or 2-comp compl. Weyl spinor

Shifman et co $\quad \langle\lambda\lambda\rangle = \langle\lambda^{a\alpha}\lambda^a_\alpha\rangle \neq 0!$

$$\Psi_{Dir} = \begin{pmatrix}\varphi_\alpha \\ \bar{\psi}^{\dot{\alpha}}\end{pmatrix} \begin{matrix} D^{(\frac{1}{2},0)} \\ D^{(0,\frac{1}{2})}\end{matrix} \qquad \Psi_{Maj} = \begin{pmatrix}\varphi_\alpha \\ \bar{\varphi}^{\dot{\alpha}}\end{pmatrix}$$

$$\beta_0 = \frac{1}{16\pi^2}\left(\frac{11}{3} - \frac{2}{3}\right)N_c = \frac{3N_c}{16\pi^2}$$

In fact, exactly $\quad \beta(g) = -\beta_0 g^3 \cdot \frac{1}{1 - \frac{N_c}{2\pi}\alpha} \Rightarrow \alpha(\mu) = \frac{2\pi}{3\log\left[\left(\frac{\mu}{\Lambda}\right)^3\frac{1}{\alpha(\mu)}\right]}$



N = 4	We had	1 Vector	$\frac{11}{3}$
	(all adjoint	4 Fermions	$-\frac{4}{3}\cdot\frac{1}{2}\cdot 4$
	$C_2^{all} = N_c$	6 Scalars	$-\frac{1}{6}\cdot 6$

$$\frac{11}{3} - \frac{8}{3} - 1 = 0 \quad !!$$

In 1977 it was noticed that even $\beta_1 = 0$ (Jones)

In 1983 Brink et al $\beta = 0$ to all orders "conjectured"

superfield formalism

"indications that."

"is believed to vanish to all orders in PT"