3-brane metric in 10d: 25Feb04 brane excitations at finite $T \checkmark$

Our goal is the metric ("non-extremal black three-brane")

horizon at $r = r_0$

$$ds^2 = \frac{1}{\sqrt{1+\frac{R^4}{r^4}}}\left[-\left(1-\frac{r_0^4}{r^4}\right)dt^2 + dx^2+dy^2+dz^2\right] + \sqrt{1+\frac{R^4}{r^4}}\left[\frac{1}{1-\frac{r_0^4}{r^4}}dr^2 + r^2 d\Omega_5^2\right] \quad \circledast$$

brane in ground state → "Extremal": $r_0 = 0$ "near-extremal": $r_0 \ll R$ "throat approximation": $r \ll R$

$r_0 \ll r \ll R$: $ds^2 = \frac{r^2}{R^2}(-dt^2 + d\vec{x}^2) + \underbrace{\frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2}_{\text{6 compactified dims}}$

<u>p-brane ansatz was:</u>

$$ds^2 = g_{\mu\nu}dz^\mu dz^\nu = (\pm) B\,dt^2 + C^2\underbrace{\left[(dx^1)^2 + \cdots + (dx^p)^2\right]}_{\text{parallel space}} + F^2 dr^2 + G^2 r^2 \overbrace{d\Omega_{d-1}^2}^{\text{metric of unit sphere in transverse space}}$$

$$\underset{=10}{D} = 1 + p + 1 + d-1 \qquad \boxed{\Omega_{2n-1} = \frac{2\pi^n}{(n-1)!} \qquad \Omega_{2n} = \frac{2(2\pi)^n}{(2n-1)!!}}$$

$$z^\mu = (\, t \quad \underset{i=1,\dots,p}{x^i} \quad \underset{a=1,\dots,d}{y^a}\,)$$

$$D = 1 + p + d$$

$$r^2 = (y^1)^2 + \cdots + (y^d)^2$$

and the functions $B(r)$ $C(r)$ $F(r)$ $G(r)$ ($\to 1$ for $r \to \infty$)

are determined by solving $g_{\mu\nu}$ by extremalizing

$$S = (\mp)\frac{1}{2\kappa_D^2}\int d^Dx\,\sqrt{g}\left\{R - \frac{1}{2}\partial^\mu\phi\,\partial_\mu\phi - \frac{1}{2}\sum_m \frac{1}{m!}e^{a_m\phi}F_m^2 + \cdots\right\} \qquad a_m = \frac{5-m}{2}$$

+ Mink $(-+++\cdots)$

− Eucl. $(+++\cdots)$

$$\Rightarrow \begin{cases} R^\mu{}_\nu = \frac{1}{2}\partial^\mu\phi\,\partial_\nu\phi + \overbrace{e^{a\phi}F^2}^{\approx} \\ \nabla^2\phi = \frac{a_m}{2m!}F_m^2 \\ \partial_\mu\left[\sqrt{g}\,e^{a\phi}F^{\mu\nu_2\cdots\nu_m}\right] = 0 \end{cases}$$

two-parameter (r_0, $\overset{R}{\uparrow}{Q}$) $\Rightarrow$ family of sol's

$\Rightarrow$ $\boxed{\text{for } m=5 \text{ one can choose } \phi = 0 \text{ !! } (a_5 = 0)}$ "dilaton decouples", metric can be of form $AdS_q \times S^{D-q}$

$\Rightarrow AdS_5 \times S^5$

$$n = 5 \;: \quad \begin{cases} R^{\mu}{}_{\nu} = \dfrac{1}{2\cdot 5!}\left[5 F^{\mu\nu_2\nu_3\nu_4\nu_5} F_{\nu\nu_2\nu_3\nu_4\nu_5} - \dfrac{4}{D-2}\,\delta^{\mu}{}_{\nu}\, F_5^2\right] \\ \partial_\mu\left(\sqrt{g}\, F^{\mu\nu_2\cdots\nu_5}\right) = 0 \end{cases}$$

For the sol'n ⊛ one further has $\boxed{* F_5 = F_5}$

$F_m \Rrightarrow$

$$(*F)^{\mu_{m+1}\cdots\mu_D} = \frac{1}{m!}\underbrace{\varepsilon^{\mu_1\cdots\mu_D}} F_{\mu_1\cdots\mu_m}$$

$$= \frac{1}{\sqrt{|g|}}\,\tilde{\varepsilon}^{\mu_1\cdots\mu_D}$$

$\phi = 0$ is also possible for $D = 11$:

$AdS_4 \times S^7 \qquad -dt^2 + \underbrace{dx^2 + dy^2}_{2\text{-brane}} + dr^2 + d\Omega_7^2$

$AdS_7 \times S^4 \qquad -dt^2 + \underbrace{dx_1^2 + \cdots + dx_5^2}_{5\text{-brane}} + dr^2 + d\Omega_4^2$

$\int A_\mu dx^\mu \qquad \frac{1}{2!}\int B_{\mu\nu}\, dx^\mu \wedge dx^\nu \qquad \frac{1}{3!}(\cdots) \qquad \frac{1}{4!}\int C_{\mu\nu\lambda\sigma}\, dx^\mu \wedge \cdots \wedge dx^\sigma$

$F_2 = dA$ couples to point 0d $\qquad F_3 = dB$ couples to string 1+1d $\qquad F_4$ $\gamma\sigma$ $\qquad F_5 = dC$ couples to D3, 1+3d