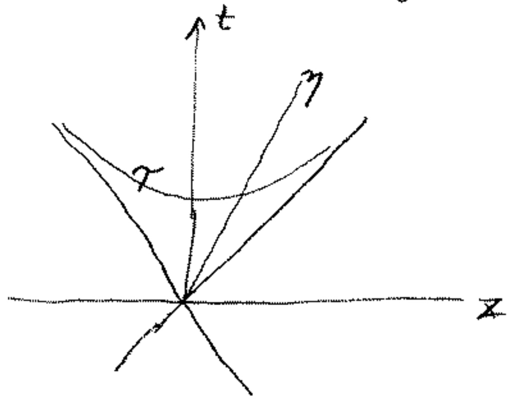
Some more motivation for string $\leftrightarrow$ QCD connection: adiabatic flow in RHIC data

23 Feb 04

Viscosity & 1+1d Bjorken flow in heavy ion collisions:



Ideal: $T^{\mu\nu} = (\varepsilon + p) u^\mu u^\nu - p g^{\mu\nu}$

$$\partial_\mu T^{\mu\nu} = 0$$

$$u^\mu = (\gamma, \gamma v) \equiv (\mathrm{ch}\,\theta, \mathrm{sh}\,\theta)$$

$$\begin{cases} t = \tau\, \mathrm{ch}\,\eta \\ z = \tau\, \mathrm{sh}\,\eta \end{cases}$$

Assume ideal similarity flow $\theta(t,z) = \eta = \frac{1}{2}\log\frac{t+z}{t-z}$

$$\Rightarrow u^\mu = (\mathrm{ch}\,\eta, \mathrm{sh}\,\eta) = \frac{1}{\tau} x^\mu$$

$$\partial_\alpha u^\mu = \frac{1}{\tau}\left(\delta_\alpha^\mu - u_\alpha u^\mu\right)$$

$$\Rightarrow \frac{d\varepsilon}{d\tau} + \frac{1}{\tau}(\varepsilon + p) = 0 \quad \Rightarrow \frac{ds}{d\tau} + \frac{1}{\tau} s = 0 \quad \Rightarrow s = s_i \frac{\tau_i}{\tau}$$

Viscosity $\quad \Delta T^{\mu\nu} = \left(\frac{4}{3}\eta + \zeta\right)\frac{1}{\tau}\left(g^{\mu\nu} - u^\mu u^\nu\right)$ for $u^\mu = \frac{1}{\tau} x^\mu$

$$\left(T_{ij} = p\,\delta_{ij} - \eta\left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\,\partial \cdot v\right) - \zeta\,\delta_{ij}\,\partial \cdot v \;, \; T_{0i} = 0 \right)$$

$$T_{00} = \varepsilon \quad \left(= 3p\,, \; T^\mu{}_\mu = 0\right) \qquad Ts = \varepsilon + p \quad s = p'(T)$$

$$\Rightarrow \varepsilon'(\tau) + \frac{1}{\tau}(\varepsilon + p) - \left(\frac{4}{3}\eta + \zeta\right)\frac{1}{\tau^2} = 0 \qquad \boxed{T\,\partial_\mu (s u^\mu) = \left(\frac{4}{3}\eta + \zeta\right)\frac{1}{\tau^2}}$$

Insert

$$p = aT^4 \quad a = \left(g_B + \tfrac{7}{8} g_F\right)\frac{\pi^2}{90} \approx \Big(\underbrace{16}_{2(N_c^2-1)} + \tfrac{21}{2} N_f\Big)\frac{\pi^2}{90} \underset{N_f = 2}{\cong} 4$$

$$s = 4aT^3 \qquad \varepsilon = 3aT^4 \qquad \eta = \eta_0 T^3 \qquad \frac{\eta}{s} = \frac{\eta_0}{4a} \quad \left(\approx \frac{1}{4\pi} \;\; \text{String th}\right)$$

$$12 a T^3 T'(\tau)\,\tau + 4aT^4 = \frac{4}{3}\eta_0 \frac{1}{\tau} T^3$$

$$\Rightarrow$$

$$\Rightarrow \quad \boxed{\tau \frac{dT}{d\tau} + \frac{1}{3} T = \frac{\eta_0}{9a} \frac{1}{\tau}}$$

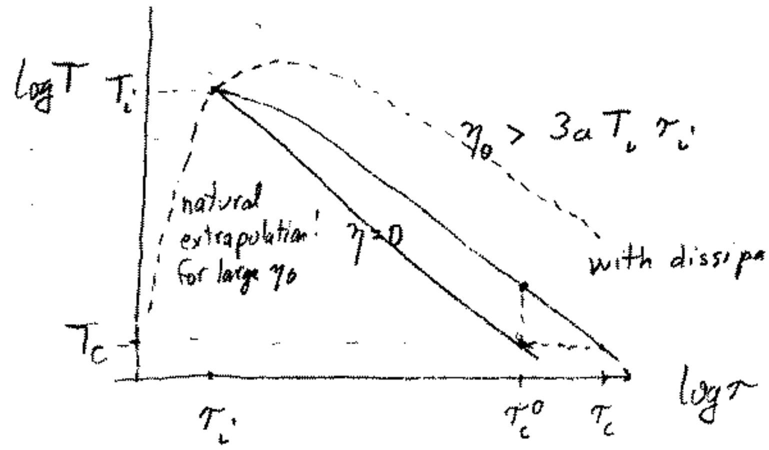
Solution: $$T = T_i \left(\frac{\tau_i}{\tau}\right)^{\frac{1}{3}} + \frac{\eta_0}{6a\tau_i}\left[\left(\frac{\tau_i}{\tau}\right)^{\frac{1}{3}} - \frac{\tau_i}{\tau}\right]$$

$$= \left(1 + \frac{\eta_0}{6a\tau_i T_i}\right) T_i \left(\frac{\tau_i}{\tau}\right)^{\frac{1}{3}} - \frac{\eta_0}{6a\tau} \quad \Bigg| \Rightarrow \frac{\tau}{\tau_i} = \frac{T_i^3}{T^3}\left(1 + \frac{\eta_0}{6a\tau_i T_i}\right)^3$$

$$f = \frac{a}{x^{1/3}} - \frac{b}{x} \qquad f' = -\frac{a}{3}\frac{1}{x^{4/3}} + b\frac{1}{x^2} \qquad f'(1) = -\frac{a}{3} + b$$

$$T'(\tau_i) \cong \frac{\eta_0}{6a\tau_i} - \frac{1}{3}\cdot\left(T_i + \frac{\eta_0}{6a\tau_i}\right) = \frac{\eta_0}{9a\tau_i} - \frac{1}{3} T_i$$

$$\tau_i T'(\tau_i) = -\frac{1}{3}\left(T_i - \frac{\eta_0}{3a\tau_i}\right)$$



with dissipation T is larger at the same τ or same T is reached at later τ

To have "small" dissipation (linear response):

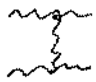
String th gives $\frac{\eta}{s} \sim \frac{\eta_0}{4a} \sim 0.1 \ll T_i \tau_i \approx 0.5$ $\Rightarrow$ ideal fluid

$$\eta_0 \ll 3a\, T_i\, \tau_i = 3 \cdot \underbrace{1.75}_{\text{pure glue}} \cdot \underbrace{0.53}_{\text{saturation model}} \simeq 2.8$$

$$\frac{1}{\Delta y} S \sim \tau\, s(\tau) \sim \tau T^3$$ assuming a is constant (actually and hopefully $a = a_g \to a_g + a_q$, quarks thermalise!)

S at some fixed T_c has grown by $$\boxed{\frac{\tau_c}{\tau_c^0} = \left(1 + \frac{\eta_0}{6a\tau_i T_i}\right)^3 = \frac{S(\eta)}{S(\eta = 0)}}$$

Estimates of η



Kinetic theory:

$$\eta \simeq \tau_c \overset{\sim N_c^2 T^4}{p} \qquad \tau_c = \frac{1}{n v \sigma} \sim \frac{1}{T^3 \frac{1}{T^2}(g^2 N_c)^2 \ln \frac{1}{g^2 N_c}} \qquad g^2 N_c \ll 1$$

$$\Rightarrow \eta \sim \frac{1}{T (g^2 N_c)^2 \ln \frac{1}{g^2 N_c}} N_c^2 T^4 \sim \frac{1}{g^4 \ln \frac{1}{g^2 N_c}} T^3$$

Carefully: Arnold-Moore-Yaffe

$$\eta \sim 86 \frac{T^3}{g^4 \ln \frac{1}{g}} \qquad \begin{matrix} N_c = 3 \\ N_f = 2 \\ g \ll 1 \end{matrix}$$

$$\sim 1.1 \frac{T^3}{\alpha_s^2 (\ln \frac{1}{\alpha_s} - 2.5)}$$

However: in the region $2T_c < T < 10 T_c$ $\quad 1.7 > g > 1.3$

$$\ln \frac{1}{g} < 0 \;!$$

meaningless estimate in the range of exp!

These (meaningless) estimates give

- "large" η $\left(\frac{\eta}{s} \sim \frac{1}{g^4} \gtrsim 1 \right)$
- no thermalisation (small σ's)

Data (v_2 measurements) find thermalisation!

hep-th/0104066

String - QCD (Son et al) $\quad \eta = \frac{\pi}{8} N_c^2 T^3 \quad (g^2 N_c \gg 1)$

$N=4$ SUSY YM

For $p(T)$ we had (p 3):

$$p = \underbrace{\tfrac{3}{4}}_{\text{correction for } g^2 N_c \gg 1} \cdot \frac{\pi^2}{6} N_c^2 T^4$$

$$\Rightarrow s = \frac{\pi^2}{2} N_c^2 T^3$$

$$\boxed{\frac{\eta}{s} = \frac{\pi}{8} \cdot \frac{1}{\frac{\pi^2}{2}} = \frac{1}{4\pi}} \Rightarrow \boxed{\frac{\eta_0}{4a} \sim 0.1 \ll T_i \quad T_i \sim 0.5 \quad \Rightarrow \text{thermalisation!}}$$

small!