7. Stringy effects: compactifying extra dims, T-duality,

Logic: (1) Open strings with fixed end points seem mysterious

(2) Observe compactified closed strings have a special stringy symmetry: T-duality

(3) Do T-duality on open strings, find fixed end points

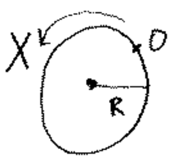
<u>Open string</u> with D-D BC $\delta X^\mu = 0$ at ends

$$X^\mu(\tau,\sigma) = c^\mu\left(1-\frac{\sigma}{\pi}\right) + d^\mu\frac{\sigma}{\pi} - l_s \sum_{n\neq 0} \frac{1}{n}\alpha^\mu_n e^{-in\tau}\sin n\sigma$$

$$X^\mu(\tau,0) = c^\mu \qquad X^\mu(\tau,\pi) = d^\mu \qquad \left[\begin{array}{l}\text{Poincaré inv. is}\\ \text{lost!!}\end{array}\right]$$

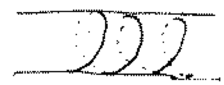
<u>Getting to Dbranes via compactification & T-duality:</u>

<u>Compactify</u> one coordinate 25 for a <u>closed string</u>



$$0 < X < 2\pi R \qquad p = \frac{2\pi}{L} m = \frac{2\pi}{2\pi R} m = \frac{m}{R}$$

$$\underset{0<\sigma<\pi}{X(\tau,\sigma)} \qquad X(\tau,\sigma+\pi) = X(\tau,\sigma) + n\, 2\pi R$$



$$\Rightarrow \quad M^2 = \frac{2}{\alpha'}\left(N_L + N_R - 2\right) + \underbrace{\left(\frac{m}{R}\right)^2}_{\text{momentum mode}} + \underbrace{\left(\frac{n}{\alpha'}R\right)^2}_{\text{winding mode}} \qquad N_L - N_R = nm$$

$$N_R = \sum_{0}^{24}\sum_{1}^{\infty} \alpha^\mu_{-m}\cdot\alpha_{m\mu} + \sum^{\infty}\alpha^{25}_{-m}\alpha^{25}_m$$

KK: $g^5_{MN}(x^\mu, x^4) = \{ g_{\mu\nu}(x^\mu),\ A_\mu(x^\mu) = g^5_{\mu 4}(x^\mu),\ g_{44}(x^\mu)\}$ $\quad x^4 \to x^4 + \lambda(x^\mu)$, $A_\mu \to A_\mu + \partial_\mu\lambda$

One winding has the energy $\quad T\cdot 2\pi R \equiv \frac{R}{\alpha'} \quad \left(2\pi T = \frac{1}{\alpha'}\right)$ $\quad \frac{\text{energy}}{\text{length}}$

$$\text{T-duality:} \quad M(m, n, R) = M\Big(n, m, \underbrace{\frac{\alpha'}{R}}_{\text{radius of T-dual theory}}\Big)$$

$$\Rightarrow R \geq \sqrt{\alpha'} = l_s/\sqrt{2}$$

Now work out how T-duality: $\frac{R}{\sqrt{\alpha'}} \to \frac{\sqrt{\alpha'}}{R}$ affects closed & open string Fourier series:

For compact closed coordinate

$$X = x + \overset{2\alpha'}{\widetilde{l_s^2}} p \underset{\frac{1}{2}(\sigma_+ + \sigma_-)}{\underset{\downarrow}{\tau}} + 2mR \underset{\frac{1}{2}(\sigma_+ - \sigma_-)}{\underset{\downarrow}{\sigma}} + S(\sigma_+) + S(\sigma_-) \equiv [X_R(\sigma_-) + X_L(\sigma_+)]$$

$$\begin{cases} X_L = \frac{1}{2}x + (\alpha' p + mR)\sigma_+ + S(\sigma_+) \\ X_R = \frac{1}{2}x + \underbrace{(\alpha' p - mR)\sigma_-}_{} + \underbrace{S(\sigma_-)}_{} \end{cases} \qquad l_s = \sqrt{2\alpha'}$$

$$+\alpha'\left(\frac{m}{R} - \frac{mR}{\alpha'}\right)\sigma_- \qquad + i l_s \sum_{n\neq 0} \frac{\alpha_n}{n} e^{-in(\tau-\sigma)}$$

$$= \sqrt{\alpha'}\left(\frac{m}{R/\sqrt{\alpha'}} - m\frac{R}{\sqrt{\alpha'}}\right)$$

T-duality: $R \to \frac{\alpha'}{R}$, $m \leftrightarrow m$

compact coord only:

$$\begin{cases} \partial_+ X = \alpha'\left(\frac{m}{R} + \frac{mR}{\alpha'}\right) + l_s \sum_{n\neq 0} \tilde{\alpha}_n e^{-in(\tau+\sigma)} \to +\partial_+ X \quad (\to +\tilde{\alpha}_n) \\ \partial_- X = \alpha'\left(\frac{m}{R} - \frac{mR}{\alpha'}\right) + l_s \sum \alpha_n e^{-in(\tau-\sigma)} \to -\partial_- X \end{cases} \qquad X \equiv X^{25}$$

require this! $\to -\alpha_n$

$$\Rightarrow \boxed{\begin{aligned} X_L &\to X_L(\tau+\sigma) \\ X_R &\to -X_R(\tau-\sigma) \end{aligned}} \boxed{\begin{aligned} &X^\mu \to X^\mu \\ &\text{for } \mu = 0,1,\dots,24 \end{aligned}}$$

Sign ch for R movers

Open string has no winding modes since ends are free.

But take the plane wave expansion

$$X^{\mu=\text{compact}}_{\text{open}} = x + l_s^2 \overset{=\frac{m}{R}}{p} \underset{\frac{1}{2}(\sigma_+ + \sigma_-)}{\tau} + \frac{i l_s}{2} \sum_n \frac{1}{n} \alpha_n \left[e^{-in(\tau+\sigma)} + e^{-in(\tau-\sigma)}\right]$$

↑ one set of α^μ_n

$$= \frac{1}{2}x + \frac{1}{2}l_s^2 p\cdot(\tau+\sigma) + \frac{i l_s}{2}\sum_n \frac{1}{n}\alpha_n e^{-in(\tau+\sigma)} \quad = X_L(\tau+\sigma)$$

$$+\frac{1}{2}x + \frac{1}{2}l_s^2 p(\tau-\sigma) + \frac{i l_s}{2}\sum_n \frac{1}{n}\alpha_n e^{-in(\tau-\sigma)} \qquad + X_R(\tau-\sigma)$$

and do T-duality map:

$$X = X_L + X_R \xrightarrow{\text{T-dual}} X_L - X_R$$

$$= l_s^2 p \sigma + l_s \sum \frac{1}{n} \alpha_n \frac{i}{2} \left(e^{-in(\tau+\sigma)} - e^{-in\tau} \cdot e^{+in\sigma} \right)$$

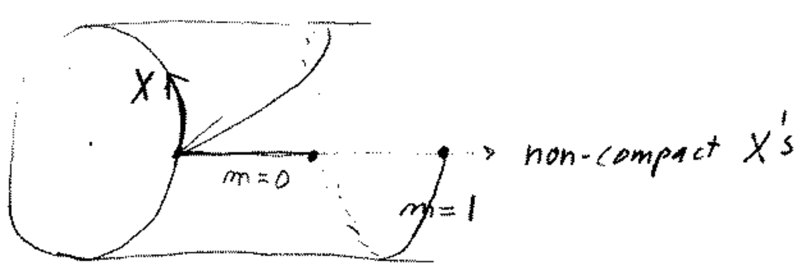
$$\boxed{\begin{array}{l} X = l_s^2 p \sigma + l_s \sum\limits_{m \neq 0} \frac{1}{m} \alpha_m \sin(m\sigma)\, e^{-im\tau} \\ X(\tau, 0) = 0 \quad X(\tau, \pi) = \pi l_s^2 p = \pi l_s^2 \frac{m}{R} = 2\pi\alpha' \frac{m}{R} \end{array}}$$

for compact coordinate

So one obtained an open string with fixed end points!
Compactify D-1-p spacelike dim's, do T-duality on all of them, strings will be stuck in a D-1-(D-1-p) = p-dimensional hyperplane, Dp brane

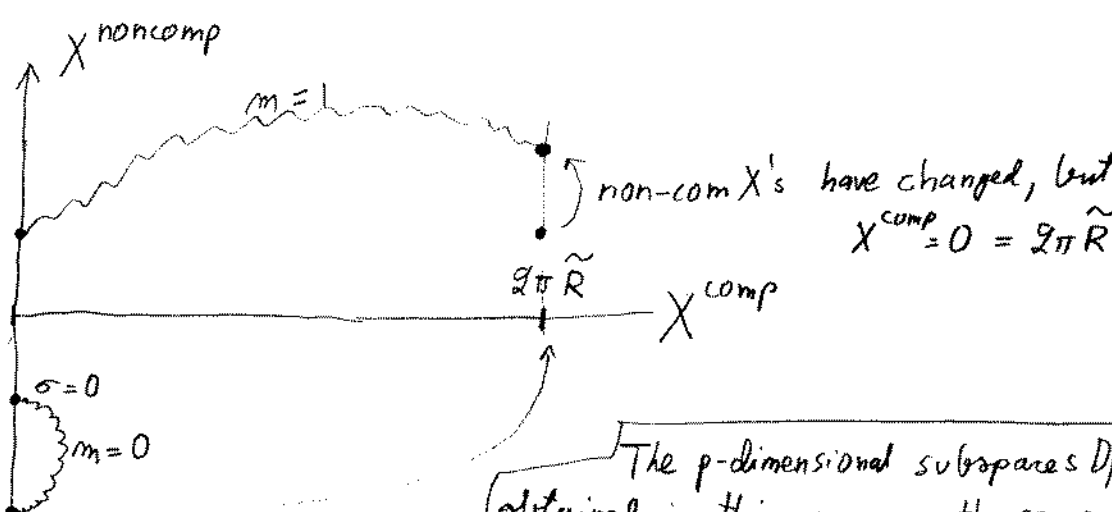
$$X = l_s^2 p \sigma = 2\alpha' \frac{m}{R} \cdot \sigma \equiv 2m\tilde{R}\, \sigma \qquad \tilde{R} = \frac{\alpha'}{R}$$

originally a momentum $\frac{m}{R}$ — ends up being winding after T-duality map



$$2m\tilde{R} \cdot \sigma + l_s \alpha_1 \sin\sigma\, e^{-i\tau}$$

Or:



For the superstring T-duality maps, say, IIA ⟺ IIB

The p-dimensional subspaces Dp obtained in this way are the same as the p branes obtained as soln's of low en closed string eff. theories

Proof = ?