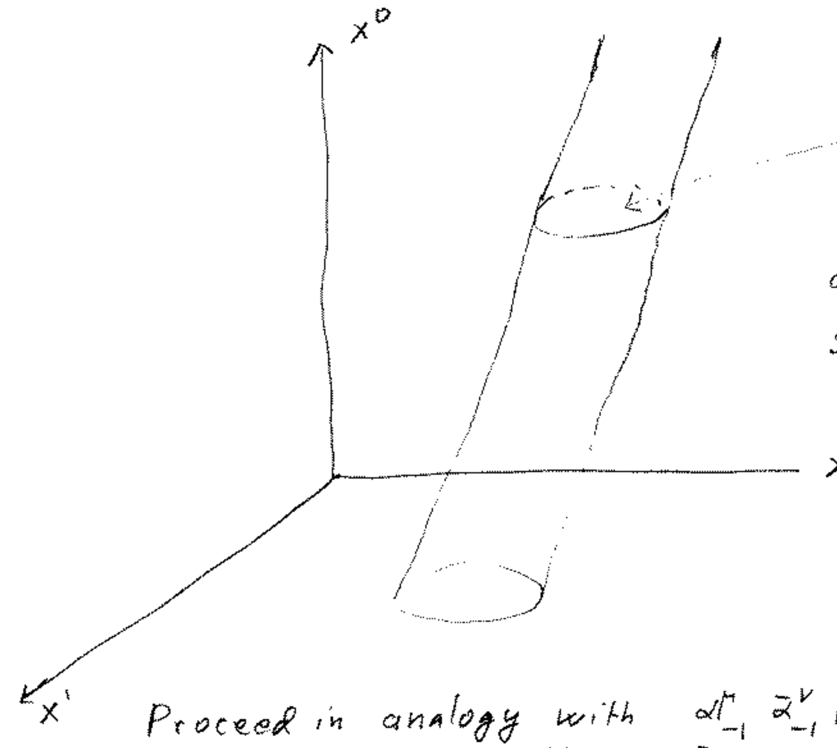
6. Superstrings in space-time, supergravities



find massless states; interactions of which are described by supergravities

string $\to$ point, $\alpha' \sim \frac{1}{T} \to 0$

$x^{2,3,\underbrace{4,\dots,9}}$ what to do with these?

$\eta_{\mu\nu} \to G_{\mu\nu}(x)$!!

Proceed in analogy with $\alpha^\mu_{-1}\, \bar{\alpha}^\nu_{-1} |0\, p\rangle$ (p. 46, 50) and separate

<u>Closed</u>:

NS/NS: $$M^2 = \frac{4}{\ell_s^2}\Big[\sum_1^\infty(\underset{R}{\overset{\tau-\sigma}{\alpha_{-m}\cdot\alpha_m}} + \underset{L}{\overset{\tau+\sigma}{\bar{\alpha}_{-m}\cdot\bar{\alpha}_m}}) + \sum_{r=\frac{1}{2}}^\infty (r\, b_{-r}\cdot b_r + r\,\bar{b}_{-r}\cdot\bar{b}_r) \underbrace{- a_{NS} - \bar{a}_{NS}}_{-\frac{1}{2}\;-\;\frac{1}{2}}\Big] \qquad \boxed{a_{NS} = \tfrac{1}{2}\ !}$$

- $|0,p\rangle^R_{NS} \otimes |0,p\rangle^L_{NS}$ $\quad M^2 = -\frac{4}{\ell_s^2}$ out!
- $b^\mu_{-\frac{1}{2}}|0,p\rangle^R_{NS} \otimes b^\nu_{-\frac{1}{2}}|0,p\rangle^L_{NS}$ $\quad M^2 = 0 \Rightarrow \underset{35}{G_{\mu\nu}},\ \underset{28}{B_{\mu\nu}},\ \underset{1}{\phi}$

  ($b^i_{-\frac{1}{2}}\,\bar{b}^j_{-\frac{1}{2}}\,|0, p^+ p\rangle_{NS}$ in light cone gauge)
- $\alpha^\mu_{-1}\,\bar{\alpha}^\nu_{-1}\,|0,p\rangle_{NS}$ $\quad M^2 = \frac{4}{\ell_s^2} = \frac{2}{\alpha'}$ out!

  bosonic graviton is now massive!

⋮

One finds that consistently states with even number of b_{-r}'s (or $\bar{b}_{-r}$'s) can be eliminated (meaning that if they are thrown out they do not reappear) = GSO projection

$$M^2 = \frac{4}{l_s^2}\Big[\underbrace{\sum_1^\infty(\alpha_{-m}\cdot\alpha_m + \bar\alpha_{-m}\cdot\bar\alpha_m)}_{m\,a_m^\dagger\cdot a_m} + \sum_1^\infty(m\,d_{-m}\cdot d_m + m\,\bar d_{-m}\cdot\bar d_m) - 0\Big]$$

$$\boxed{a_R = \bar a_R = 0\ !}$$

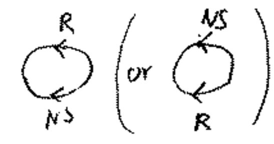
spinor index in 10 (or 8) dims

- $|A,p\rangle_R^R \otimes |B,p\rangle_R^L \qquad M^2 = 0$

bosonic states, "bispinors"

$$\underset{8}{C_\mu} + \underset{56}{C_{\mu\nu\lambda}} = 64 \quad \text{Type IIA}$$

$$\underset{1}{C_0} + \underset{28}{C_{\mu\nu}} + \underset{35}{C_{\mu\nu\lambda\kappa}} = 64 \quad \text{Type IIB}$$



$$M^2 = \frac{4}{l_s^2}\Big[N_R^\alpha + N_L^{\bar\alpha} + \sum_1^\infty m\,d_{-m}\cdot d_m + \sum_{\frac12}^\infty r\,\bar b_{-r}\cdot\bar b_r - \frac12\Big]$$

- $|A\,p\rangle_R^R \otimes |0\,p\rangle_{NS}^L \qquad M^2 = -\frac{2}{l_s^2}$ ⇐ tachyon again thrown out by GSO projection

- $\underbrace{|A\,p\rangle_R}_{\text{spin }\frac12} \otimes \underbrace{b^\mu_{-\frac12}}_{\text{spin }1}|0p\rangle_{NS}^L \qquad M^2 = 0$

$$\frac12 \times 1 = \frac12 + \frac32 = \underset{8}{\chi^a} + \underset{56}{\psi^{\mu a}}$$

Here one has to discuss the chiralities of $|A\rangle_R \otimes b^\mu_{-\frac12}|0\rangle_{NS}$

$\bar b^\mu_{-\frac12}|0\rangle_{NS} \otimes |A\rangle_R$

Opposite: type IIA

Same: type IIB

Ramond vacuum $|A\,0\rangle_R$ — NS vacuum has no spinor index!

$\{d_0^\mu, d_0^\nu\} = \eta^{\mu\nu} \Rightarrow \{\gamma^\mu,\gamma^\nu\} = -2\eta^{\mu\nu}$. # of dofs of a spinor in d:

$(\gamma^\mu = i\sqrt2\, d_0^\mu)$

zero mode,

$\psi(\sigma) = d_0 + \sum_{m\neq0} d_m$

like $X = x + l_s^2 p\,\tau + \sum_{m\neq 0}$

$$\begin{cases} 2^{d/2} & \text{Dirac spinor} \quad \gamma^\mu_{AB} \\ 2^{d/2-1} & \text{Majorana or Weyl} \\ 2^{d/2-2} & \text{" and "} \end{cases}$$

Spinor rep of $SO(1,d-1)$: $J_{\mu\nu} = \frac{i}{4}[\gamma_\mu,\gamma_\nu]$ should satisfy p. 52

$p_\mu \gamma^\mu_{AB}|B\,p\rangle_R = 0$

↑ one of the constraint eqs.

Now that we have massless states of the string how do they interact?

$\Rightarrow$ Strings in Background Fields

2d Einstein-Hilbert $\int d^2\sigma \sqrt{-h}\, R_h$ — string coupl. const p. 74

Add two new terms: dilaton

$$S = -\frac{T}{2}\int d^2\sigma \sqrt{-h}\left\{ h^{ab} \underbrace{G_{\mu\nu}(X)\partial_a X^\mu \partial_b X^\nu} + \varepsilon^{ab} B_{\mu\nu}(X)\partial_a X^\mu \partial_b X^\nu + \frac{\alpha'}{2} R_h(X)\phi(X) \right\}$$

let X_0 = some const BG

$\dim T X^2 = 1$

$\dim X = \frac{1}{\sqrt{T}} \sim \sqrt{\alpha'}$

$$= \left[G_{\mu\nu}(X_0) + (X-X_0)^\alpha \partial_\alpha G_{\mu\nu}(X_0) + \frac{1}{2!}(\;)^\alpha(\;)^\beta \partial_\alpha\partial_\beta G_{\mu\nu} + \dots \right] \partial_a X^\mu \partial_b X^\nu$$

$$\approx \left[1 + \underline{X\cdot\partial} + X^2\partial^2 + \dots \right](\partial X)^2$$

$$= \# \frac{1}{\sqrt{T}}\cdot\frac{1}{\text{Radius}} = \# \frac{\alpha'}{\text{Radius}} \quad \text{perturbative for small } \alpha'$$

$\Rightarrow$ Interactions! $\Rightarrow$ "Non-linear σ model"

or

$$= \left[\eta_{\mu\nu} + \varepsilon_{\mu\nu}^{(k)} e^{ik\cdot X} + \dots \right] \partial_a X^\mu \partial_b X^\nu$$

$$= \left[\eta_{\mu\nu} + \varepsilon_{\mu\nu}\left(1 + ik\cdot X - \frac{1}{2} k_\alpha k_\beta X^\mu X^\nu + \dots\right) \right] \partial_a X^\mu \partial_b X^\nu$$

$\Rightarrow$ graviton vertex operator : $\varepsilon_{\mu\nu}(k)\, \partial_a X^\mu \partial^a X^\nu e^{ik\cdot X}$:

The above action is classically Lorentz-reparametrisation-Weyl invariant but quantisation destroys these unless $d=26$ even if $G_{\mu\nu}(X) = \eta_{\mu\nu}$, $B_{\mu\nu} = \phi = 0$. When $G_{\mu\nu} = G_{\mu\nu}(X)$ this theory is a 2d interacting field theory, all the couplings start to "run":

$$\beta^G_{\mu\nu} = \frac{\partial G_{\mu\nu}}{\partial \log\Lambda} \leftarrow \text{regulator}$$

and invariance is true only if $\beta^G_{\mu\nu} = 0 = \beta^B_{\mu\nu} = \beta^\phi_{\mu\nu}$

These eqs generalise "$d=26$" to non-constant $G_{\mu\nu}$.

Parenthesis: String coupling g_s

$$e^{-S} = e^{-\frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{-h}\,\{\cdots\cdots + \alpha' R\Phi\}} \sim \left[e^{-\langle\phi\rangle}\right]^{2-2\tilde{g}} \sim (g_s^2)^{\tilde{g}-1}$$

$$\chi = \frac{1}{4\pi}\int d^2\sigma\sqrt{-h}\,R = \text{Euler number} = 2 - 2\tilde{g}$$ genus

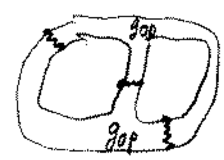
$\exists$ a potential for dilaton $\Rightarrow \exists\langle\phi\rangle$. Then $g_s = e^{\langle\phi\rangle}$

Why is this "string coupling"?

$\left[\begin{array}{l}\phi \to \phi - c \\ \Rightarrow g_s \to e^c g_s\end{array}\right.$

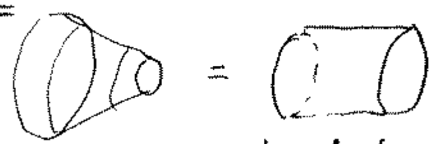
open string loop

2loop open string



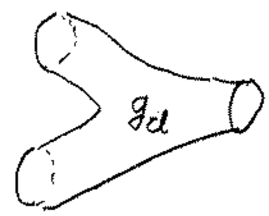
(pull out internal circle

$=$



closed string propag

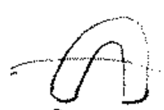
$\Downarrow$ pull out



closed string vertex

$\Rightarrow g_{open}^2 = g_{cl}$

Adding one handle costs g_s^2 (since $\tilde{g} \to \tilde{g}+1$) or g_{cl}^2:



$$\Rightarrow \boxed{g_{open}^2 \cong g_{cl} \cong g_s \cong e^{\langle\phi\rangle}}$$

$\left\{\begin{array}{lll} g_s^2 & \text{is} & \text{handle exp. parameter} \\ \frac{1}{N_c^2} & & \text{— " —} \end{array}\right.$

constants appear!

$$\left\{\begin{array}{l} g_{open}^2 = \dfrac{2\pi^{D/2}}{2^{(10-D)/4}}\, g_{cl} \\ g_{cl} = (2\pi)^2 g_s \end{array}\right.$$

The equations $\beta^{G}_{\mu\nu}=0$, $\beta^{B}_{\mu\nu}=0$, $\beta^{\phi}_{\mu\nu}=0$ are the eqs. of motion following from the field theory action

strings appear as point particles

$$(\alpha')^{d-2} S = \int d^d x\, \underbrace{\sqrt{G}\, e^{-\phi}\big[R}_{\Downarrow} + (\nabla\phi)^2 - \frac{1}{12}H^2 + \frac{D-26}{3}\Big] + O(\alpha')$$

$$\sqrt{G}\, R_E \quad \text{by } G^{E}_{\mu\nu} = e^{-\frac{2\phi}{d-2}} G_{\mu\nu}$$

$$\left\{\begin{array}{lll} \frac{1}{\alpha'}\beta^{B}_{\mu\nu} = & \nabla^{\alpha}\big[e^{-\phi}\overbrace{H_{\mu\nu\alpha}}^{\partial_\mu B_{\nu\alpha}+\partial_\nu B_{\alpha\mu}+\partial_\alpha B_{\mu\nu},\ H=dB}\big] = 0 & \text{"Maxwell"} \\ 3\frac{1}{\alpha'}\beta^{\phi}_{\mu\nu} = & \frac{d-26}{3\alpha'} + (\nabla\phi)^2 - 2\Box\phi - R + \frac{1}{12}H^{2'} = 0 & \text{"scalar"} \\ \frac{1}{\alpha'}\beta^{G}_{\mu\nu} = & R_{\mu\nu} - \frac{1}{4}H_{\mu\alpha\beta}H_{\nu}{}^{\alpha\beta} + \nabla_\mu\nabla_\nu\phi = 0 & \text{"gravity"} \end{array}\right.$$

Conformal transformation to Einstein frame:

$$\boxed{\begin{array}{l}\tilde{G}_{\mu\nu} = e^{2\Omega} G_{\mu\nu} \\ \Rightarrow \tilde{R} = e^{-2\Omega}\big[R - 2(d-1)\nabla^2\Omega - (d-2)(d-1)\partial_\mu\Omega\,\partial^\mu\Omega\big]\end{array}}$$

Try $\tilde{G}_{\mu\nu} = e^{\sigma\phi} G_{\mu\nu}$

$$\Rightarrow \sqrt{G}\, e^{-\phi} R = \sqrt{\tilde{G}}\, e^{-\underbrace{[\sigma(d-2)+2]}_{\text{require}=0}\frac{\phi}{2}}\Big\{\tilde{R} + \sigma(d-1)\frac{1}{\sqrt{\tilde{g}}}\partial_\mu(\sqrt{\tilde{g}}\,\partial^\mu\phi) - \frac{1}{4}\sigma^2(d-1)(d-2)\,\partial_\mu\phi\,\partial^\mu\phi\Big\}$$

$$\sigma = -\frac{2}{d-2}$$

Gives the "Einstein frame": $\sqrt{G}\, e^{-\phi} = \sqrt{\tilde{G}}\, e^{\frac{2}{d-2}\phi}$

$$(\alpha')^{d-2} S = \int d^d x \sqrt{G}\Big[R - \frac{1}{d-2}(\nabla\phi)^2 - e^{-\frac{4\phi}{d-2}}\frac{1}{12}H^2 + e^{\frac{2}{d-2}\phi}\frac{d-26}{3}\Big]$$

Cf <u>Brans - Dicke</u>

$$\int d^4x\, \sqrt{g}\Big[f(\phi)R + \frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi - V(\phi)\Big]$$

or <u>Reissner- Nordström</u>:

$$\int d^4x\, \sqrt{-g}\Big\{+\frac{1}{16\pi G}(R+2\Lambda) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}\Big\}$$

<u>Type IIA sugra</u>: (non-chiral) $\qquad F_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu \equiv F_2$

$$S = \underbrace{\frac{1}{2\kappa^2}}_{\equiv \frac{1}{16\pi G_{10}}} \int d^{10}x \sqrt{-G} \left\{ R - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{12} e^{-\phi} H_3^2 - \frac{1}{4} e^{\frac{3}{2}\phi} F_2^2 - \frac{1}{48} e^{\frac{1}{2}\phi} F_4^2 \right.$$

$$\left. - \frac{1}{2304} \frac{1}{\sqrt{-G}} \varepsilon^{\mu_0 \cdots \mu_9} B_{\mu_0\mu_1} \underbrace{F^4_{\mu_2 \cdots \mu_5}}_{} F^4_{\mu_6 \cdots \mu_9} \right\} + \text{fermions}$$

$B_2 \wedge F_4 \wedge F_4 \qquad$ field strength constructed from the R-R 3-form $C_{\mu\nu\lambda} \equiv F_4$

Fields

$\begin{cases} \text{R: periodic BC for } \psi \\ \text{NS: anti " " " } \psi \end{cases}$

	Fields	# of physical dofs	
NS-NS	ϕ	1	
	$B_{\mu\nu}$	28	← couples to a string
	$G_{\mu\nu}$	35	
R-R bispinors	C_μ	8	← couples to point
	$C_{\mu\nu\lambda}$	56	← " " 2d surface
NS-R	χ_a $s=\frac{1}{2}$	8	dilatino
	ψ^μ_a $s=\frac{3}{2}$	56	gravitino
R-NS	χ'_a	8	
	$\psi^{\mu\prime}_a$	56	

$128_b + 128_f$ dofs

$8^2 = 36 - 1 + 28 + 1 \qquad d = 10 \to 8$

$2^2 = 3 - 1 + 1 + 1 \qquad d = 4 \to 2$

= D branes carry RR charges $\binom{8}{3} = 56$

dim $= p$, p even for IIA

Type II B sugra (chiral, more physical)

$$S_{IIB} = \frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{-G} \left\{ e^{-2\phi} \left[R + 4(\nabla\phi)^2 - \frac{1}{2} H^2 \right] \right.$$

$$\left. - \frac{1}{2}\left(F_1^2 + \tilde{F}_3^2 + \tilde{F}_5^2\right) \right\} - \frac{1}{32\pi G_{10}} \int C_4 \wedge H_3 \wedge F_3 + \text{ferm's}$$

RR 1- 3- 5- forms $(C_4 \wedge H_3)_{\mu_1 \cdots \mu_7} = (C_4)_{[\mu_1 \cdots \mu_4} (H_3)_{\mu_5 \mu_6 \mu_7]}$

States;

NS-NS	ϕ	1
	$B_{\mu\nu}$	28
	$R_{\mu\nu}$	35
R-R	C_0	1
$C_2 \to$	$C_{\mu\nu}$	28
$C_4 \to$	$C_{\mu\nu\lambda\kappa}$	35

+ fermion states as for IIA

$H \equiv H_3 = dB$

$F_1 = dC_0$

$F_3 = dC_2 \qquad \tilde{F}_3 = F_3 - C_0 \wedge H_3$

$\longrightarrow F_5 = dC_4 \qquad \tilde{F}_5 = F_5 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3$

self-dual!
$* \tilde{F}_5 = \tilde{F}_5$

A general prototype action could be:

$$S = \frac{1}{2\kappa_d^2} \int d^d x \sqrt{g} \left\{ R - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{2} \sum_{m=1} \frac{1}{m!} e^{a_n \phi} \underbrace{F_m^2}_{F_{\mu_1 \cdots \mu_m} F^{\mu_1 \cdots \mu_m}} + \cdots \right\}$$

$a_n = -\frac{m-5}{2}$

Lyng-Petersen hep-th) 9909131 ch. 3.1

$$\begin{cases} d = 10 \\ d = 11 \quad a_m = 0, \ \phi = 0 \quad \text{M-theory!} \end{cases}$$

p-brane: source of charge for the p+1 form RR gauge field C $\Rightarrow$ $m = p+2$ form F

3-brane: source of $C_{\mu\nu\lambda\kappa} \Rightarrow F_{\mu\nu\lambda\kappa\rho} \cong \partial_{[\mu} C_{\nu\lambda\kappa\rho]}$

$$d = 1 + p + \underbrace{d-(p+1)}_{\text{dims transverse to the p-brane}}$$

$$z^\mu = (t, x^i, y^a) = (z^0, x^1, \ldots, x^p, \underbrace{y^1, \ldots y^{d-(p+1)}}_{\text{transverse}})$$

$p=3$: $10 = 4 + 6$