The often-cited RS metric looks very similar so let us go through it, too:

<u>Randall - Sundrum</u> (Csaki hep-ph/0404096)

$$5d: \quad ds^2 = e^{-A(y)}\,\eta_{\mu\nu}\,dx^\mu dx^\nu - dy^2 \quad \text{solved from} \quad \boxed{S = -\int d^5x\left[\sqrt{g}\,(M_*^3 R + \Lambda) + \sqrt{-g_4}\,\delta(z)\sigma\right]}$$

branes

now $\eta = (+\,-\,-\,-)$! S^1/Z_2 "orbifold compactified"

$-\infty < y <$ (1) $y \to y + 2\pi R$ $\quad -2\pi R \quad -\pi R \quad 0 \quad \pi R \quad 2\pi R$ $\quad S^1$ (p. 53)

(2) $y \to -y$

transform to a conformally flat form

$$= e^{-A(z)}\left(\eta_{\mu\nu}\,dx^\mu dx^\nu - dz^2\right) \equiv \underbrace{e^{-A(z)}\,\eta_{MN}}_{\equiv\, g_{MN}}\,dx^M dx^N$$

$$\left[\sqrt{g_5} = e^{-2A(y)} = e^{-2A(z)}\,e^{-\frac{1}{2}A(z)}\right] dy = e^{-A(z)/2}\,dz$$

$$\text{Find}\begin{cases} e^{-A(z)} = \dfrac{1}{(k|z|+1)^2} = e^{-2k|y|} \qquad \dfrac{dy}{dz} = \dfrac{1}{k|z|+1} \qquad \tilde{\Lambda} \equiv -\dfrac{\Lambda}{2M_*^3} \\[2ex] k \equiv \sqrt{-\dfrac{\Lambda}{12M_*^3}} \equiv \sqrt{\dfrac{\tilde\Lambda}{6}} \end{cases}$$

"Inverse radius of AdS_5" $\to k$

$$M_*^3\left(R + \frac{\Lambda}{M_*^3}\right) \equiv M_*^3\,(R - 2\tilde\Lambda)$$

convention on p. 39

where 5d gravity action is

$$S = -\int d^5x\,\sqrt{+g_5}\,\left(M_*^3 R + \Lambda\right)$$

$\Lambda < 0$ is AdS in these conventions since $T_{00} = \frac{\Lambda}{2}$

$$ds^2 = g_{MN}\,dx^M dx^N$$

$$R_{MN} - \tfrac{1}{2} g_{MN} R = \frac{1}{M_*^3} T_{MN} = \frac{1}{M_*^3}\,\frac{1}{2}\Lambda\, g_{MN} \equiv -\tilde\Lambda\, g_{MN}$$

$\dim \Lambda = M^5 = \frac{en}{V_4}$

(note that dim'lly

$$-\frac{1}{16\pi G_4}\int d^4x\,\sqrt{-g}\;R \quad \Rightarrow \quad -\frac{1}{16\pi G_{4+m}}\int d^{4+m}x\,\sqrt{\pm g_{4+m}}\;R_{4+m}$$

$M^2 \quad \frac{1}{M^4} \quad 1 \quad M^2$ in any d !! $\qquad M^{2+m} \quad \frac{1}{M^{4+m}} \quad 1 \quad M^2$

$\dim \frac{\Lambda}{M_*^3} = \dim R = M^2$

$$S_{4+m} = -M_*^{2+m}\int d^{4+m}x\,\sqrt{\pm g_{4+m}}\;R_{4+m}$$

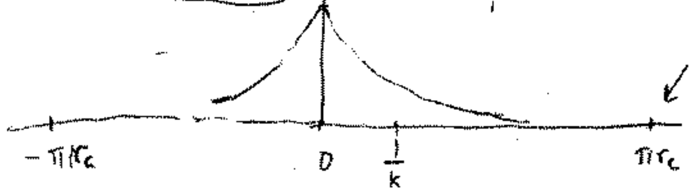
$$\frac{1}{M_*^3} = 16\pi G_5 \Leftarrow \quad \Rightarrow \quad -M_*^3\int d^5x\,\sqrt{+g_5}\;R_5$$

$$\boxed{ds^2 = e^{-2k|y|}\,\eta_{\mu\nu}\,dx^\mu dx^\nu - dy^2 = \frac{1}{(k|z|+1)^2}\left(\eta_{\mu\nu}\,dx^\mu dx^\nu - dz^2\right)}$$

$$= k^2 r^2 (dt^2 - d\vec{x}^2) - \frac{1}{k^2 r^2}\,dr^2$$

$r = \frac{1}{k}e^{-k|y|}$

$(x^\mu, y) = (x^\mu, -y)$



used to impose bound. cond; then $r_c \to \infty$

Redo previous page:

The following should be correct:

$\eta = + - - -$

$$S = \int d^4x \sqrt{-g} \left\{ -\frac{1}{16\pi G}(R - 2\Lambda) + \mathcal{L}_m(g_{\mu\nu}) \right\}$$

$\delta S / \delta g^{\mu\nu} = 0$

$$\Rightarrow R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G \left(2\frac{\partial \mathcal{L}_m}{\partial g^{\mu\nu}} - g_{\mu\nu}\mathcal{L}_m \right)$$

$$\mathcal{L}_m = 0, \quad g^{\mu\nu} \cdot (\ ) = R - \frac{1}{2}\cdot 4R + 4\Lambda = 0 \Rightarrow R = 4\Lambda$$

$$\boxed{R_{\mu\nu} = \Lambda g_{\mu\nu} \qquad R = 4\Lambda} \quad \boxed{+ - - -}$$

or if $4 \to d$: $\quad R - \frac{1}{2} d R + d\Lambda = (1 - \frac{d}{2})R + d\Lambda \Rightarrow R = \frac{d}{\frac{d}{2}-1}\Lambda = \frac{2d}{d-2}\Lambda$

(p. 39)

$$R_{\mu\nu} - \frac{d}{d-2}\Lambda g_{\mu\nu} + \Lambda g_{\mu\nu} = R_{\mu\nu} + \left(1 - \frac{d}{d-2}\right)\Lambda g_{\mu\nu}$$

$$\boxed{R_{\mu\nu} = \frac{2}{d-2}\Lambda g_{\mu\nu}, \quad R = \frac{2d}{d-2}\Lambda} \quad \boxed{+ - - -}$$

$$\begin{cases} AdS_d: \Lambda = \frac{(d-1)(d-2)}{2R^2_{radius}} > 0 \\ dS: \Lambda < 0 \end{cases}$$

$$R_{\mu\nu\alpha\beta} = \frac{R}{d(d-1)}(g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha})$$

so if we start from

$$S = -\int d^5x \sqrt{g}\, M_*^3 \left(R + \overbrace{\frac{\Lambda}{M_*^3}}^{\equiv -2\tilde{\Lambda}} \right)$$

we of course get $R_{MN} - \frac{1}{2} g_{MN} R = -\tilde{\Lambda} g_{MN} \equiv \frac{\Lambda}{2M_*^3} g_{MN}$

and either the dS or AdS metric. For $\tilde{\Lambda} > 0$, $\Lambda < 0 \Rightarrow AdS_{m+1}$

with $\tilde{\Lambda} = \frac{m(m-1)}{2R^2_{radius}} = \frac{6}{R^2_{rad}} = \frac{3}{10} R_{curvature}$ $\qquad R - 2\tilde{\Lambda} = \frac{10}{3}\tilde{\Lambda} - 2\tilde{\Lambda} = \frac{4}{3}\tilde{\Lambda}$!

$$ds^2 = \frac{r^2}{R^2}(dt^2 - d\bar{x}^2) + R^2\frac{dr^2}{r^2} = \frac{R^2}{\varrho^2}(dt^2 - d\bar{x}^2 - d\varrho^2)$$

In general, if $g_{\mu\nu} = e^{\phi(x)}\eta_{\mu\nu}$ then (derive by explicit computation!)

$$R_{\mu\nu} = (1-\tfrac{d}{2})\left(\partial_\mu\partial_\nu\phi - \tfrac{1}{2}\partial_\mu\phi\,\partial_\nu\phi\right) + \tfrac{1}{2}\eta_{\mu\nu}\left[(1-\tfrac{d}{2})\,\partial_\alpha\phi\,\partial^\alpha\phi - \partial_\alpha\partial^\alpha\phi\right]$$

$(= R_\mu{}^\alpha{}_{\nu\alpha})$

$$\Rightarrow R = (1-\tfrac{d}{2})\left(\partial^2\phi - \tfrac{1}{2}\partial_\alpha\phi\,\partial^\alpha\phi\right) + \tfrac{d}{2}\left[(1-\tfrac{d}{2})\,\partial_\alpha\phi\,\partial^\alpha\phi - \partial^2\phi\right]$$

$$= \partial^2\phi\left[1-\tfrac{d}{2}-\tfrac{d}{2}\right] + \partial_\alpha\phi\,\partial^\alpha\phi\left(-\tfrac{1}{2}(1-\tfrac{d}{2}) + \tfrac{d}{2}(1-\tfrac{d}{2})\right)$$

$$= \partial^2\phi\,(1-d) + (1-\tfrac{d}{2})\,\tfrac{d-1}{2}\,\partial_\alpha\phi\,\partial^\alpha\phi$$

$$G_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2}\eta_{\mu\nu}R = (1-\tfrac{d}{2})\left(\partial_\mu\partial_\nu\phi - \tfrac{1}{2}\partial_\mu\phi\,\partial_\nu\phi\right)$$

$$+\tfrac{1}{2}\eta_{\mu\nu}\left[(1-\tfrac{d}{2})(\partial_\alpha\phi)^2 - \partial^2\phi + (1-\tfrac{d}{2})\tfrac{1-d}{2}(\partial_\alpha\phi)^2 - (1-d)\partial^2\phi\right]$$

$$= (1-\tfrac{d}{2})\left[\tfrac{3-d}{2}(\partial_\alpha\phi)^2 - 2\,\partial^2\phi\right]$$

$$(1-\tfrac{d}{2})(\partial_\alpha\phi)^2(1-\tfrac{d}{2}+\tfrac{1}{2}) - \partial^2\phi\,(1+1-d)$$

$$\boxed{G_{\mu\nu} = (1-\tfrac{d}{2})\left\{\partial_\mu\partial_\nu\phi - \tfrac{1}{2}\partial_\mu\phi\,\partial_\nu\phi + \tfrac{1}{2}\eta_{\mu\nu}\left[\tfrac{3-d}{2}(\partial_\alpha\phi)^2 - 2\,\partial^2\phi\right]\right\}}$$

Now replace $e^{\phi(x)} \Rightarrow e^{-A(z)}$ $\quad d=5$

$$G_{MN} = (1-\tfrac{d}{2})\left[-\partial_M\partial_N A - \tfrac{1}{2}\partial_M A\,\partial_N A + \eta_{MN}\left(\tfrac{3-d}{4}\partial_M A\,\partial^M A + \tfrac{4-d}{4}\partial^2 A\right)\right]$$

$$\Rightarrow \begin{cases} G_{55} = -\tfrac{3}{2}\left[-A'' - \tfrac{1}{2}(A')^2 - \left(+\tfrac{1}{2}(A')^2 - A''\right)\right] = \tfrac{3}{2}[A'(z)]^2 \\ G_{\mu\nu} = -\tfrac{3}{2}\eta_{\mu\nu}\left[\tfrac{1}{2}(A')^2 - A''\right]\end{cases}$$

Csaki has here 1!

<u>55-component</u>: $G_{55} = \tfrac{3}{2}(A'(z))^2 = \dfrac{\Lambda}{2M_*^3}\overbrace{(-)e^{-A(z)}}^{g_{55}}$ $\Rightarrow$ need $\Lambda < 0$ $\Rightarrow$ <u>AdS</u>

$\tilde{\Lambda} = \dfrac{-\Lambda}{2M_*^3} > 0$

$$\Rightarrow \frac{dA}{dz} = \sqrt{-\frac{\Lambda}{3M_*^3}}\;e^{-\frac{1}{2}A(z)} \equiv 2k e^{-\frac{1}{2}A(z)}$$

$dA\, e^{\frac{1}{2}A} = \sqrt{\tfrac{2}{3}\tilde{\Lambda}}\;dz \qquad k = \sqrt{\dfrac{-\Lambda}{12M_*^3}} = \sqrt{\dfrac{\tilde{\Lambda}}{6}}$

$$\frac{1}{2}\int_{A(0)}^{A(z)} dA\, e^{\frac{1}{2}A} = \left(e^{\frac{1}{2}A(z)} - e^{\frac{1}{2}A(0)}\right) = \sqrt{\frac{\tilde{\Lambda}}{6}}\;z \equiv kz \qquad \text{Fix } A(0)=0$$

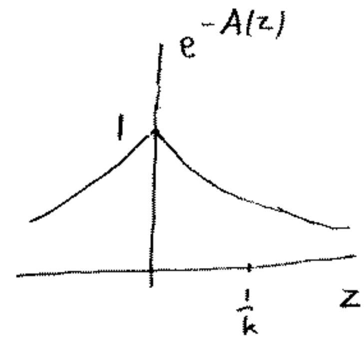
$dy = \frac{1}{2k}A'(z)\,dz \quad \Rightarrow \boxed{e^{A(z)} = (kz+1)^2} \quad \Rightarrow (k|z|+1)^2$

$\Downarrow$

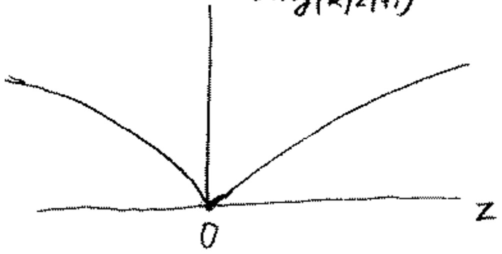
$y = \frac{1}{2k}A \Rightarrow e^{-A(z)} = e^{-2k|y|} \qquad A(z) = 2\log(k|z|+1) = 2k|y|$

μν-comps $\quad G_{\mu\nu} = \underbrace{\frac{\Lambda}{2M_*^3}}_{-\tilde{\Lambda} \equiv -6k^2} \underbrace{g_{\mu\nu}}_{e^{-A(z)}\eta_{\mu\nu}}$

$$\frac{3}{2}\left(\frac{1}{2}(A')^2 - A''\right) = -\tilde{\Lambda}\, \underbrace{e^{-A(z)}}_{= \frac{1}{(k|z|+1)^2}}$$



$A(z) = 2\log(k|z|+1)$



$$\frac{d|z|}{dz} = 2\theta(z) - 1 \qquad \theta'(z) = \delta(z)$$

$$A'(z) = 2k\frac{2\theta(z)-1}{k|z|+1}$$

$$A''(z) = 2k\ \frac{2\delta(z)\left(k|z|+1\right) - (2\theta(z)-1)\ k(2\theta(z)-1)}{(k|z|+1)^2}$$

$$= \frac{-2k^2}{(k|z|+1)^2} + \frac{4k\,\delta(z)}{k|z|+1}$$

$$\frac{1}{2}(A')^2 - A'' = \frac{2\cdot 2k^2}{(k|z|+1)^2} - \frac{4k\,\delta(z)}{k|z|+1}$$

$$G_{\mu\nu} = -6k^2\frac{1}{(k|z|+1)^2}\eta_{\mu\nu} = -\frac{3}{2}\eta_{\mu\nu}\left(\frac{1}{2}(A')^2 - A''\right)$$

$$-\eta_{\mu\nu}\Bigg[\underbrace{\frac{6k^2}{(k|z|+1)^2}}_{\text{cancel}} - \underbrace{\frac{6k\,\delta(z)}{k|z|+1}}_{\exists\ \text{uncancelled}}\Bigg] = -\eta_{\mu\nu}\frac{6k^2}{(k|z|+1)^2}$$

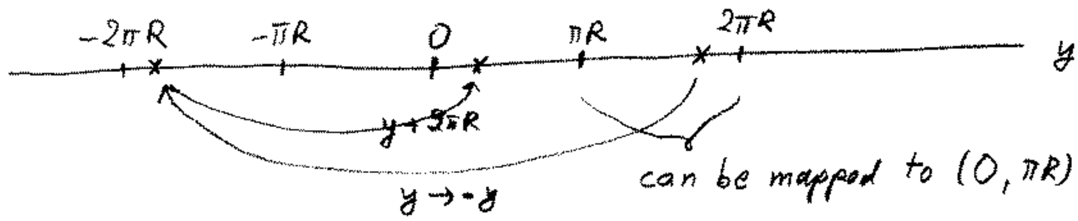
from bulk cosmological constant $\quad k = \sqrt{\frac{-\Lambda}{12M_*^3}}$

to cancel this, add to T_{MN} in $G_{MN} = \frac{1}{M_*^3}T_{MN}$ a term which only contributes for $\mu\nu$ and sits at $z=0$ and contributes
$$\frac{1}{M_*^3}T_{\mu\nu} = \eta_{\mu\nu}\frac{6k}{k|z|+1}\delta(z)$$

$$T_{\mu\nu} = \eta_{\mu\nu}\ 6kM_*^3\,\delta(z)$$

<u>S^1/Z_2 orbifold</u>: - Start from $-\infty < y < \infty$

- identify $y \to y + 2\pi R \Rightarrow S^1$
- -- " -- y and $-y$ $\quad S^1/Z_2$



can be mapped to $(0, \pi R)$

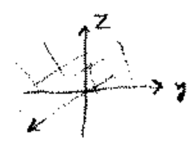
so that $2\pi R - y \to y$

fundamental domain is $(0, \pi R)$ with

2 fixed points, $y = 0$ and $y = \pi R$

<u>String, domain wall, vacuum en in 4d:</u>

Vilenkin PRD23(81)852



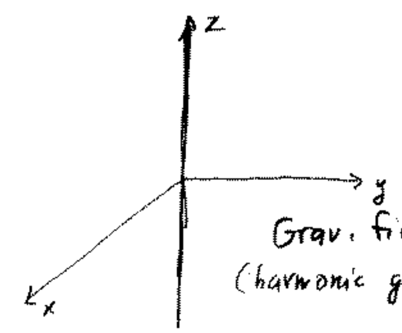
$$T_{\mu\nu} = \mu\, \delta^2(\underline{x}) \begin{pmatrix} 1 & & & \\ & 0 & & \\ & & 0 & \\ & & & -1 \end{pmatrix} \quad T_{\mu\nu} = \sigma\, \delta(x) \begin{pmatrix} 1 & & & \\ & 0 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \quad T_{\mu\nu}^{vac} = \varepsilon_{vac} \overbrace{\begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}}^{\eta_{\mu\nu}}$$

string tension

$\Downarrow$ weak field

$$ds^2 = (1 - 4\pi G\sigma |x|)\, \eta_{\mu\nu}\, dx^\mu dx^\nu$$

Exact sol'ns are time dependent (p.



Grav. field in weak field appro:
(harmonic gauge, p.31)

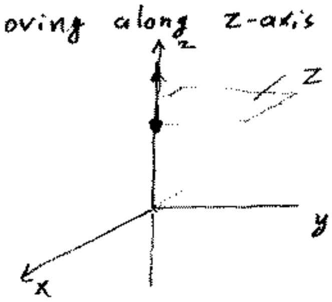
$$\Rightarrow ds^2 = dt^2 - dz^2 - \left(1 - 8\pi G\mu \log \frac{r}{r_0}\right)(dr^2 + r^2 d\varphi^2)$$

$$= dt^2 - dz^2 - dr'^2 - (1 - 8\pi G\mu) r'^2 d\varphi^2$$

$$- r'^2 d\varphi'^2$$

φ' changes from 0 to $(1 - 8\pi G\mu) 2\pi$

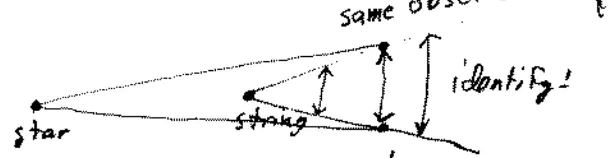
Compare here "Aichelburg-Sexl shock waves" (p.37) which had a part. with $E \sim \frac{1}{Gm}$ moving along z-axis



two flat regions with a singularity in ds^2 at $t = z$

conical space, angular deficiency

(this is what one tried to avoid when deriving T_H (p. 47))



this is stable!

$$ds^2 = \left(\frac{r}{r_0}\right)^{\frac{4(1-K)}{K^2+3}} dt^2 - \left(\frac{r}{r_0}\right)^{\frac{2(K^2-1)}{K^2+3}} dz^2 - dr^2 - \left(\frac{r}{r_0}\right)^{-\frac{2(K-1)^2}{K^2+3}} r^2 d\varphi$$

general cylindrical metric; matches to weak field if $K = 1 + O(G^2\mu^2)$

Now we are in 5d and had (p.50) a bulk vacuum energy

$$T_{MN} = \frac{\Lambda}{2} g_{MN} = \frac{\Lambda}{2} \begin{pmatrix} 1 & & & & \\ & -1 & & & \\ & & -1 & & \\ & & & -1 & \\ & & & & -1 \end{pmatrix} \qquad \Lambda < 0$$

$$\underbrace{\qquad\qquad}_{x^\mu} \ \underbrace{\quad}_{z,y}$$

a brane vacuum energy

$$T_{MN}^{\text{brane}} = 6kM_*^3\delta(z) \begin{pmatrix} 1 & & & & \\ & -1 & & & \\ & & -1 & & \\ & & & -1 & \\ & & & & 0 \end{pmatrix}$$

$\dim T_{MN} = \frac{\text{energy}}{V_4} = M^5$

$\dim kM_*^3 = \frac{\text{energy}}{V_3} = M^4$

= brane tension

Then $G_{MN} = \frac{1}{M_*^3} T_{MN} + 6k\,\delta(z) \begin{pmatrix} \eta_{\mu\nu} & \\ & 0 \end{pmatrix}$

and the new term cancels the term $+\eta_{\mu\nu} 6k\,\delta(z)$ in $G_{\mu\nu}$ on p. 52.

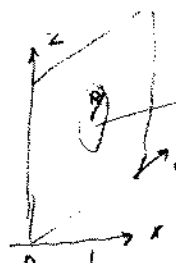
Taub PR 103(1956) 454 eq. 16.7)

Gravitational field of domain wall: $ds^2 = \frac{1}{\sqrt{1+K|x|}}(dt^2 - dx^2) - (1+K|x|)(dy^2+dz^2)$

planar symm., but $T_{\mu\nu} \neq (1\ 0\ -1\ -1)$ ..

$ds^2 = e^{-K|x|}\{dt^2 - dx^2 - e^{Kt}(dy^2+dz^2)\}$

$K = 4\pi G\sigma$



$M = \pi R^2 \sigma$

$r_s = 2GM > R$ if $2G \cdot \pi R^2 \sigma > R$

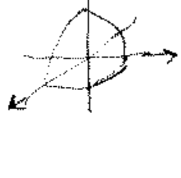
$\Rightarrow$ If $R > \frac{1}{2\pi G\sigma}$ wall collapses $\Rightarrow t$ dependence

String: $M = R \cdot \mu$ now $r_s > R \Rightarrow 2GM = 2GR\mu > R$

$\Rightarrow \mu > \frac{1}{G} \sim M_{Pl}^2$

no t dep. is needed! $\Leftarrow$ only a Planck scale string will collapse

"Vacuum" $M = \frac{4}{3}\pi R^3 \varepsilon_{vac}$



$2G \cdot \frac{4}{3}\pi R^3 \varepsilon_{vac} > R = \frac{8\pi G\,\varepsilon_{vac}}{3} R^2 > 1$ "unstable"

solutions with sph. symm. are Rob.-Walker,

$$\underset{0,\pm1}{\dot R^2 + k} = \underbrace{\frac{8\pi G\,\varepsilon_{vac}}{3}}_{\equiv \Lambda/3} R^2 \Rightarrow \begin{cases} e^{\pm\sqrt{\frac{\Lambda}{3}}t} & k=0 \\ \operatorname{sh}\sqrt{\frac{\Lambda}{3}}\,t & =\pm 1 \\ \sin\sqrt{\frac{-\Lambda}{3}}\,t & =-1 \end{cases}$$

Remember:

Einsteins static universe:

$$\frac{\ddot a}{a} = -\frac{4\pi G}{3}\underbrace{\left(\varepsilon + 3p + \underbrace{\varepsilon_{vac} + 3p_{vac}}_{-2\varepsilon_{vac}}\right)}_{=0 \text{ if } p \ll \varepsilon \text{ \& } \varepsilon = 2\varepsilon_{vac}}$$

$$\dot a^2 = \frac{8\pi G}{3}\underbrace{(\varepsilon + \varepsilon_{vac})}_{3\varepsilon_{vac}} a^2 - 1 = 0 \quad \text{if} \quad a^2 = \frac{1}{8\pi G\,\varepsilon_{vac}}$$

$$\varepsilon_{vac} = \frac{1}{8\pi G R_0^2}$$

Why were we interested in all this (RS)?

$(+ - - - \dots)$ ↓ has to be here for dim'l reasons

$$S = -\int d^{4+m}x \sqrt{g_{4+m}}\left[M_*^{m+2} R_{4+m} + \frac{1}{4g_{4+m}^2} F_{MN}F^{MN}\right]$$

mass dims: $\frac{1}{M^{4+m}} \quad 1 \quad \left[M^{m+2}\, M^2 + M^m\, M^4\right]$

$\frac{1}{g_d^2} = M^{d-4}$ $\quad g_3^2 = M$ etc

$g_5^2 = \frac{1}{M}$ "

Try $x^M = (x^\mu, y^k)$ $\quad ds^2 = \underbrace{g_{\mu\nu}(x)\,dx^\mu dx^\nu}_{4} - b^2(x)\, \underbrace{h_{kl}(y)\, dy^k dy^l}_{m \text{ dim}}$

$$g_{MN}^{(x,y)} = \left(\begin{array}{c|c} g(x) & 0 \\ \hline 0 & h(y) \end{array}\right)$$

$$\Rightarrow R_{4+m} = b(x)^m\left[R_4(x) - \frac{1}{b^2}R_h(y) + m(m-1)\, g^{\mu\nu}(x)\partial_\mu b\, \partial_\nu b\right]$$

$$= R_4 - R_h = R^\mu{}_\mu - R^k{}_k \quad \text{if } b = 1$$

Take y-space to be (flat &) compact

$m=1$: $-dy^2 \Rightarrow -r^2 d\varphi^2 \quad 0<\varphi<2\pi$

2: $-(dr^2 + r^2 d\varphi^2) \Rightarrow -r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$

$\vdots$

m: $\Rightarrow -r^2 d\Omega_m^2$

$\int d^k y \sim r^m \sim V_m$

$$\Rightarrow S = -\int d^4x\, \underbrace{r^m \sqrt{g_4}}\left[M_*^{m+2} \overset{M^2}{R_4} + \frac{1}{4g_{4+m}^2}\overset{M^4}{\left(F_{\mu\nu}^2 + F_{kl}^2\right)}\right]$$

may argue that $\frac{1}{g_{4+m}^2} = M_*^m = \frac{1}{r^m g_4^2}$

$M_* = M_{Pl}$! $\quad r = \frac{1}{M_*}\cdot\frac{1}{g_4^{2/m}}$

$$M_{Pl}^2 = r^m M_*^{m+2} = (r M_*)^m M_*^2$$

$$r = \left(\frac{M_{Pl}}{M_*}\right)^{\frac{2}{m}} \frac{1}{M_*} = 10^{\frac{32}{m}} \cdot 2\cdot 10^{-19} m \cong 10^{23} m \quad m=1$$

$10^{-3} m \quad m=2$

$10^{-9} m \quad m=3$

$\vdots$

Take $M_* = TeV = 10^{-16} M_{Pl}$

$1/M_* = 10^{-3}\, 0.2\,fm \sim 2\cdot 10^{-19} m$

Btw, why not higher invariants: (Rizzo in 2004 - 5)

$$S = \int d^5x \sqrt{g_5} \left\{ +2\Lambda - \frac{M_*^3}{2} R + \underbrace{\left[R^2 - 4 R_{MN} R^{MN} + R_{MNPQ} R^{MNPQ} \right]}_{\text{a topological invariant} \sim \text{Gauss-Bonnet}} M_* \right\}$$

a topological invariant ~ Gauss-Bonnet in 4d, (not in 5d! surface term in 4d)   very complicated R^3- R^4- etc analogous terms

---

Bulk + brane theories:

we had with R-S (p. 50):

$$S = - \int d^5x \Big[\underbrace{\sqrt{g_5} \left(M_*^3 R + \Lambda \right)}_{\text{bulk}} + \underbrace{\sqrt{-g_4}\, \sigma\, \delta(z)}_{\text{brane at } z=0} \Big]$$

orbifold compactified so $\exists$ singularity at $z=0$ ($|z|$)

$$\Downarrow \qquad \equiv M_*^{m+2} (R_{4+m} - 2\tilde{\Lambda}) \qquad \text{3-brane} \qquad (x^0, x^1, x^2, x^3, \overbrace{x^4 + \cdots x^{3+m}}^{m \text{ extra dims}})$$

$$- \int d^{4+m}x \sqrt{g_{4+m}} \left(M_*^{m+2} R_{4+m} + \Lambda \right)$$

$$+ \int d^4x \sqrt{-g_4} \Big\{ \sigma - \underbrace{R_4}_{\text{graviton}} + \underbrace{\tfrac{1}{2} g^{\mu\nu}_{(4)} D_\mu \phi D_\nu (\phi) - V(\phi)}_{\text{scalar}} - \underbrace{\tfrac{1}{4} F^{\mu\nu} F_{\mu\nu}}_{\text{vector}} + \cdots \Big\}$$

If $Y^M(x^0, \bar{x})$ is the position of the brane in the $4+m$ dim space with metric $g_{MN}(x^\mu, y^k)$ then on the brane

$$ds^2 = g_{MN}\, dx^M dx^N = \underbrace{g_{MN}(Y) \frac{\partial Y^M}{\partial x^\mu} \frac{\partial Y^N}{\partial x^\nu}}_{= g_{\mu\nu}(x) \text{ induced metric}} dx^\mu dx^\nu$$

With RS we had $Y^0 = x^0, \ldots, Y^3 = x^3$

Effect on SM fields :

$$e^{-2k|y|}\eta_{\mu\nu}\,dx^\mu dx^\nu - dy^2 \qquad g_{\mu\nu} = e^{-2kb}\eta_{\mu\nu}$$

$$S_{\phi(y=b)} = \int d^4x\,\sqrt{-g_4}\,\left[\tfrac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - \tfrac{1}{4}\lambda(\phi^2-v^2)^2\right]$$

$|y| \Rightarrow b$

$$e^{-4kb}\left[\tfrac{1}{2}e^{2kb}\,\partial^\mu\phi\,\partial_\mu\phi - \tfrac{1}{4}\lambda(\phi^2-v^2)^2\right]$$

assume SM fields are living at $y = b$

$$e^{-2kb}\,\partial^\mu\phi\,\partial_\mu\phi$$

$$\tilde{\phi} = e^{-kb}\phi$$

$$\int d^4x\left[\frac{1}{2}(\partial_\mu\tilde{\varphi})^2 - \frac{1}{4}\lambda\left(e^{-2kb}\left(e^{2kb}\tilde{\phi} - v^2\right)\right)^2\right]$$

$$= \int d^4x\left\{\frac{1}{2}(\partial_\mu\tilde{\varphi})^2 - \frac{1}{4}\left[\tilde{\phi} - \underbrace{(e^{-kb}v)^2}\right]^2\right\}$$

affects the hierachy problem

can make

$$e^{-kb}v \approx 246\ \text{GeV}$$

for $v \approx M_{Pl}$ for "reasonable"

$kb \approx 17\ln 10 \approx 39$

radius of AdS $= \frac{1}{k} \approx \frac{b}{39}$.

But let us step still one step backward (and sideways) and consider Einstein gravity in 5d without extra branes but keeping g_{M4} = basic KKN extension. For pedagogical exposition, see hep-ph/0210992 & 0412109 by Rawndal et al. (IIB) This is a good prototype of 10d susy string theories $\Rightarrow$ various compactifications in 10 sugra $\Rightarrow$ 4d theories.