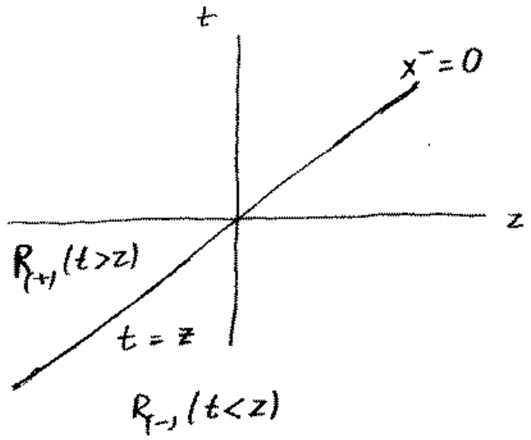
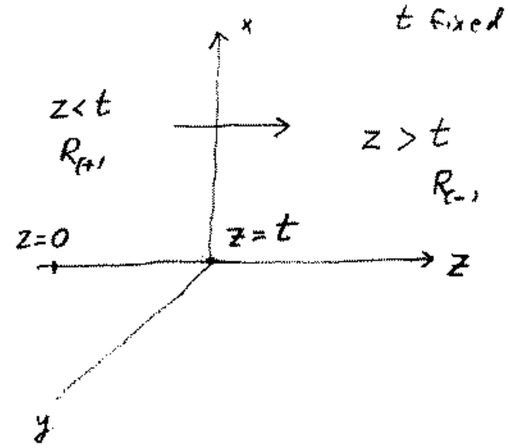
"Aichelburg-Sexl shock waves"

Gravitational field of a particle moving with

$$E = c\frac{M_{Pl}}{m}\cdot M_{Pl} = c\frac{1}{Gm}$$ so this is >> than M_{Pl}





Answer (A-Sexl, 1971, Dray-'t Hooft NPB253 (85) 173)

Two flat regions $z>t$ and $z<t$ joined so that on the plane $z=t$

$$\begin{cases} x_{(+)} = x_{(-)} \quad y_{(+)} = y_{(-)} \quad \text{or} \quad \underset{\sim}{x}_{(+)} = \underset{\sim}{x}_{(-)} \\ z_{(+)} = z_{(-)} - 2GE\log(\underset{\sim}{x}^2/C) \\ t_{(+)} = t_{(-)} - 2GE\log(\underset{\sim}{x}^2/C) \end{cases} \qquad x^-_{(+)} = \frac{t-z}{\sqrt{2}} = x^-_{(-)}$$

or: in d dim

$$ds^2 = 2\,dx^+dx^- + (dx^-)^2\phi(\underset{\sim}{x})\,\delta(x^-) + d\underset{\sim}{x}^2$$

$$\Delta_{d-2}\,\phi(\underset{\sim}{x}) = -16\pi GE\,\delta^{d-2}(\underset{\sim}{x})$$

$$\underset{\sim}{\partial}^2\phi(\underset{\sim}{x}) = -16\pi GE\,\delta^2(\underset{\sim}{x})$$

$\Rightarrow$

I want to back to gravity-gauge theory holography so let us for once work out <u>signs & conventions</u> carefully: (AdS written above "gravity")

$$\eta_{\mu\nu} = + - - - \quad \Rightarrow \quad \eta_{\mu\nu} = - + + + \qquad \eta_{\mu\nu}\eta^{\mu\nu} = \eta^\mu_\mu = d$$

(R)

$$\Gamma \sim g\partial g \quad \Rightarrow \quad \Gamma$$

$$R^\mu{}_{\nu\alpha\beta} \sim \partial\Gamma + \Gamma^2 \quad \Rightarrow \quad R^\mu{}_{\nu\alpha\beta}$$

$$R_{\mu\nu\alpha\beta} \sim g_{\mu\beta} R^\beta{}_{\nu\alpha\beta} \sim \partial^2 g - g\Gamma^2 \Rightarrow -R_{\mu\nu\alpha\beta}$$

$$R_{\mu\nu} \sim g^{\alpha\beta} R_{\mu\alpha\nu\beta} \quad \Rightarrow \quad R_{\mu\nu}$$

$$R = g^{\mu\nu} R_{\mu\nu} \quad \Rightarrow \quad -R$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} \Rightarrow R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} \; ; \quad : \Lambda \Rightarrow +\Lambda$$

keep Λ unchanged just $g \to -g$

$$-\frac{1}{16\pi G}\int d^dx\sqrt{g}\,(R-2\Lambda) \Rightarrow -\frac{1}{16\pi G}\int d^dx \sqrt{g}\,(-R-2\Lambda)$$

(T)

$$T_{\mu\nu} = (\varepsilon + p) u_\mu u_\nu - p\, g_{\mu\nu} \quad \Rightarrow \quad T_{\mu\nu} = (\varepsilon + p) u_\mu u_\nu + p\, g_{\mu\nu}$$

$$\underset{u^\mu = (1,\vec{0})}{=} \begin{pmatrix} \varepsilon & & & 0 \\ & p & & \\ & & p & \\ 0 & & & p \end{pmatrix} \qquad \underset{u^\mu = (1,0)}{=} \begin{pmatrix} \varepsilon & & & 0 \\ & p & & \\ & & p & \\ 0 & & & p \end{pmatrix}$$

$u^0 = u_0$ $\qquad$ $u^0 = -u_0$

$u^2 = 1$ $\qquad$ $u^2 = -1$

$$= -p_{vac}\, g_{\mu\nu} = \varepsilon_{vac}\, g_{\mu\nu} \qquad = +p\, g_{\mu\nu} = -\varepsilon_{vac}\, g_{\mu\nu}$$

$\varepsilon = -p \equiv \varepsilon_{vac}$ $\qquad$ $\varepsilon = -p = \varepsilon_{vac}$

Always $T_{00} = \varepsilon_{vac}$ but $T_{\mu\nu} = \begin{cases} +\varepsilon_{vac}\, g_{\mu\nu} & + - - - \\ -\varepsilon_{vac}\, g_{\mu\nu} & - + + + \end{cases}$

(Onestone) $\quad R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \Rightarrow R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$

no $T_{\mu\nu}$ only Λ: $\quad R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\Lambda g_{\mu\nu} = \text{"}8\pi G \varepsilon_{vac} g_{\mu\nu}\text{"} \qquad R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \Lambda g_{\mu\nu} = \text{"}-8\pi G\varepsilon_{vac} g_{\mu\nu}\text{"}$

$g^{\mu\nu}\cdot$ $\quad R - \frac{1}{2} d R = -\Lambda d$

$g^{\mu\nu} g_{\mu\nu} = d$

$$\boxed{R = \frac{2d}{d-2}\Lambda \qquad R_{\mu\nu} = \frac{2\Lambda}{d-2} g_{\mu\nu}} \; \boxed{+ - - - -} \qquad \int d^4x\sqrt{g}\,(R+2\Lambda) \qquad \boxed{R = \frac{2d}{d-2}(-\Lambda) \qquad R_{\mu\nu} = \frac{2(-\Lambda)}{d-2} g_{\mu\nu}} \; \boxed{- + + \cdots +}$$

$$\Lambda = -8\pi G \varepsilon_{vac} \begin{cases} < 0 & \text{de Sitter, "usual" } \varepsilon_{vac} > 0 \\ > 0 & \text{Anti deSitter} \end{cases}$$

$-\int d^4x \sqrt{g}\,(R - 2\Lambda)$

Further sources of confusion:

(1) sign of def of $R^{\mu}{}_{\nu\alpha\beta}$ arbitrary

(2) $R_{\mu\nu} = R_{\mu\alpha\nu\beta} = -\underbrace{R_{\alpha\mu\nu\beta}}_{\text{can also be defined as } R_{\mu\nu}}$

<u>Examples of AdS:</u>

AdS_{m+1} $\quad ds^2 = \underbrace{b^2 \frac{4}{(1-x^2)^2}\eta_{\mu\nu}}_{g_{\mu\nu}} dx^\mu dx^\nu \qquad x^2 \equiv \eta_{\mu\nu}x^\mu x^\nu$

$\qquad - \underbrace{+ + \cdots +}_{m}$

$$\underset{\text{compute}}{\Rightarrow} \quad R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{m(m-1)}{2b^2} g_{\mu\nu} \quad =$$

$$R = -\frac{m(m+1)}{b^2} \underset{p.32}{=} \frac{2(m+1)}{(m-1)}(-\Lambda) \quad \Rightarrow \Lambda = \frac{m(m-1)}{2b^2} > 0$$

$$ds^2 = \frac{b^2}{r^2}\left(-dt^2 + dx_1^2 + \cdots + dx_{m-1}^2 + dr^2\right) \equiv \frac{b^2}{x_m^2} \underbrace{\eta_{\mu\nu}}_{- \underbrace{+ \cdots +}_{m}} dx^\mu dx^\nu$$

$$\Rightarrow R_{\mu\nu} = -\frac{m}{b^2} g_{\mu\nu} = -\frac{m}{(x^m)^2}\eta_{\mu\nu}$$

factor one coord out

In AdS/CFT one often meets (p.44 with $r_0 = 0$ (extremal), $r \ll R$ (throat approx.))

$$ds^2 = \frac{r^2}{R^2} \underset{-+++}{\eta_{\mu\nu}} dx^\mu dx^\nu + \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5^2 \qquad \frac{1}{\sqrt{1+\frac{R^4}{r^4}}} \sim \frac{r^2}{R^2}$$

$$= \frac{r^2}{R^2}\Big(\eta_{\mu\nu}dx^\mu dx^\nu + \underbrace{\left(\frac{R^2}{r^2}dr\right)^2}_{d\varrho^2} + \frac{R^4}{r^2} d\Omega_5^2\Big)$$

$$\varrho = \frac{R^2}{r} \quad d\varrho = -\frac{R^2}{r^2}dr$$

$$= \underbrace{\frac{R^2}{\varrho^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + d\varrho^2\right)}_{\text{as above; } AdS_5} + R^2 d\Omega_5^2$$

AdS by embedding

$\boxed{AdS_3}$ $\quad AdS_{m+1}$:

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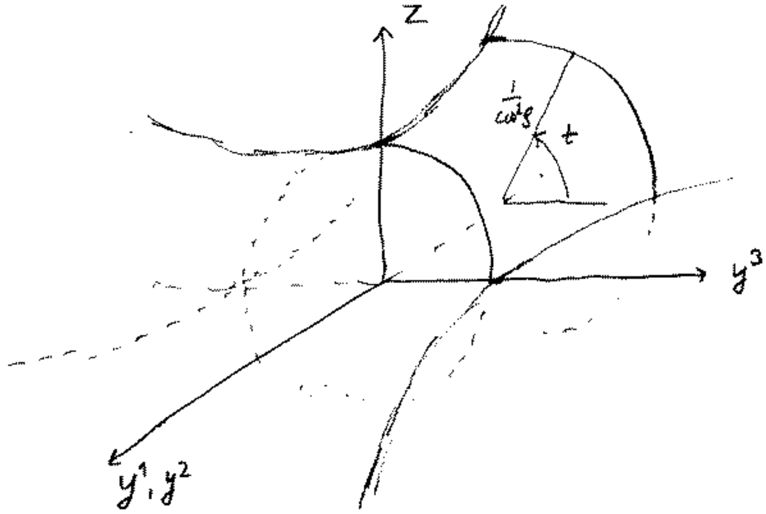
$$\boxed{\begin{array}{l}\text{Surface} \quad z^2 - y_1^2 - y_2^2 + y_3^2 = b^2 \\ \text{in the space} \quad ds^2 = dz^2 - dy_1^2 - dy_2^2 + dy_3^2\end{array}}$$

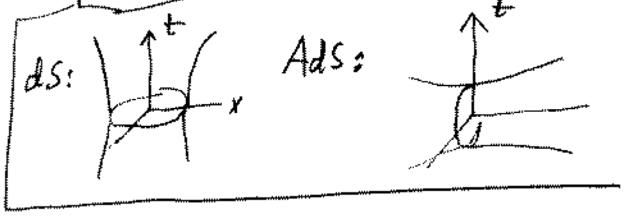
$$z^2 - y_1^2 - \cdots - y_m^2 + y_{m+1}^2 = b^2$$



AdS_{n+1}

$$\underbrace{z^2 - y_1^2 - \cdots - y_{m-1}^2}_{\text{become } x\Gamma} \quad \underbrace{- y_m^2 + z_{m+1}^2}_{\text{become } r} = b^2$$

$$ds^2 = \frac{b^2}{r^2}\left(-dt^2 + dx_1^2 + \cdots + dx_{m-1}^2 + dr^2\right)$$



1. Poincaré coordinates:

$t \quad x^1 \quad r$

$$\begin{cases} z = \dfrac{t}{r}\, b & -\infty < t, x^1 < +\infty \\ y^1 = \dfrac{x^1}{r}\, b & 0 < r < \infty \\ y^2 + y^3 = -\dfrac{b^2}{r} & \Rightarrow y^m + y^{m+1} \end{cases}$$

covers $\frac{1}{2}$ of AdS_3

constraint

$$\Rightarrow (y^3 - y^2)\frac{-b^2}{r} + \frac{b^2}{r^2}(t^2 - x_1^2) = b^2 \Rightarrow y^3 - y^2 = \frac{t^2 - x_1^2 - r^2}{r^2}$$

or $\quad y^{3\,(m+1)} = \dfrac{1}{2r}\left(-b^2 + t^2 - x_1^2 - r^2\right), \quad y^2 = \dfrac{1}{2r}\left(-b^2 - t^2 + x_1^2 + r^2\right)$

$(b=1)$ $\quad dz = \frac{1}{r}dt - \frac{t}{r^2}dr \qquad dy^1 = \frac{1}{r}dx^1 - \frac{x^1}{r^2}dr$

$$dy^2 = \frac{1}{r}(x^1 dx^1 - t\,dt) + \frac{1 - x_1^2 + r^2 + t^2}{2r^2}dr \qquad dy^3 = -dy^2 + \frac{dr}{r^2}$$

$$dy_3^2 - dy_2^2 = -\frac{2}{r^2}dr\,dy^2 + \frac{dr^2}{r^4}$$

$$\Box\phi = \frac{1}{\sqrt{g}}\partial_\mu\left(\sqrt{g}\,g^{\mu\alpha}\partial_\alpha\phi\right)$$

AdS_{m+1}

$$\Rightarrow \boxed{ds^2 = \frac{b^2}{r^2}\left(-dt^2 + dx_1^2 + dr^2\right)} \qquad \boxed{b^2\Box = -r^2\partial_t^2 + r^2\partial_r^2 - (m-1)r\partial_r + r^2\nabla^2_{S_{m-1}}}$$

$AdS_3 = AdS_{m+1}$

Laplacian on sphere, metric on sph

$$\nabla^2_{S_{m-1}} = \frac{1}{\sqrt{\gamma}}\partial_\alpha\left(\sqrt{\gamma}\,\gamma^{\alpha\beta}\partial_\beta\right)$$

2. Global coordinates

AdS_{n+1}:

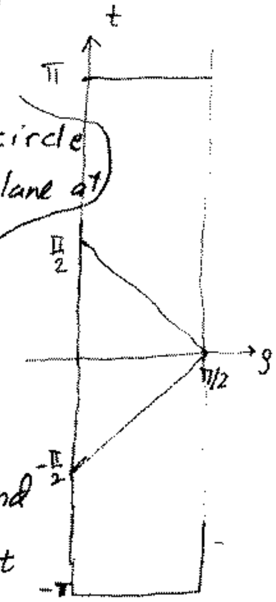
$$z^2 - (y_1^2 + \cdots + y_{n-1}^2) - y_n^2 + y_{n+1}^2 = b^2 = 1 \qquad R^{2,n}$$

(2a) $z = \dfrac{\cos t}{\cos\rho}$ $\quad y^i = z_i \tan\rho \quad y^{n+1} = \dfrac{\sin t}{\cos\rho}$

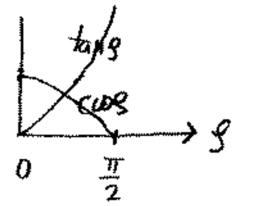
$\left(\begin{array}{c}\text{Compare Poincaré} \\ z = \frac{t}{r}\end{array}\right)$ $\quad \sum_1^n z_i^2 = 1 \Rightarrow z_i \in S^{n-1}$

Penrose = ?

satisfies constraint: $\dfrac{\cos^2 t}{\cos^2\rho} - \sum_1^n z_i^2 \tan^2\rho + \dfrac{\sin^2 t}{\cos^2\rho} = \dfrac{1}{\cos^2\rho} - \tan^2\rho = 1$

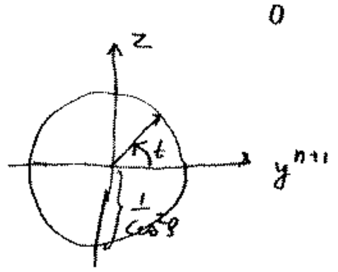


ρ: $\quad z^2 + y_{n+1}^2 = 1 + \bar{y}^2 = \dfrac{1}{\cos^2\rho}$ = radius of circle on z, y_{n+1} plane at fixed $\bar{y}$



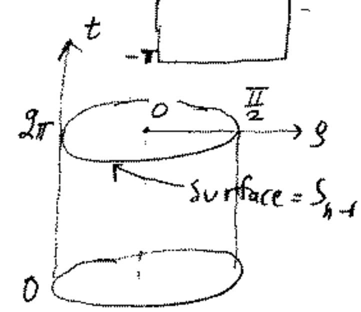
$\boxed{0 \le \rho \le \frac{\pi}{2}}$

t:



t is the angle around the circle

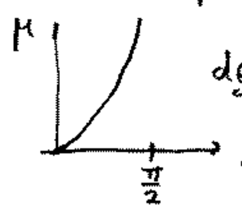
$\boxed{0 \le t < 2\pi}$



$= dz^2 - dy_1^2 - \cdots dy_n^2 + dy_{n+1}^2 =$

$$\Rightarrow \boxed{ds^2/b^2 = +\frac{1}{\cos^2\rho}\left(-dt^2 + d\rho^2\right) + \tan^2\rho \underbrace{d_\perp\Omega_{n-1}^2}_{d\varphi^2}}$$

(2b) Change radial variable ρ by $\tan\rho = \mathrm{sh}\mu \quad \dfrac{1}{\cos\rho} = \mathrm{ch}\mu$



$d\rho = \cos\rho\, d\mu = \dfrac{d\mu}{\mathrm{ch}\mu}$

$$\boxed{\frac{ds^2}{b^2} = -\mathrm{ch}^2\mu\, dt^2 + d\mu^2 + \mathrm{sh}^2\mu\, d_\perp\Omega_{n-1}^2}$$

$0 < \mu < \infty \qquad \underbrace{0 \le t < 2\pi}_{\text{periodic "time" !}}$

How do we get to AdS_5 etc ?

String theory (II B) $S = -\frac{T}{2}\int d^2\sigma \sqrt{-h}\Big\{h^{ab}G_{\mu\nu}(X)\,\partial_a X^\mu \partial_b X^\nu$

$T = \frac{1}{\pi l_s^2} = \frac{1}{2\pi\alpha'}$ $\qquad + \varepsilon^{ab}B_{\mu\nu}(x)\,\partial_a X^\mu \partial_b X^\nu + \frac{\alpha'}{2}R_h(X)\phi(X)\Big\}$

$\Downarrow$ small α'

only massless excitations, spin 2, 1, 0

$\chi = \frac{1}{4\pi}\int d^2\sigma\sqrt{-h}\,R$
$=$ Euler $= 2-2\cdot$genus

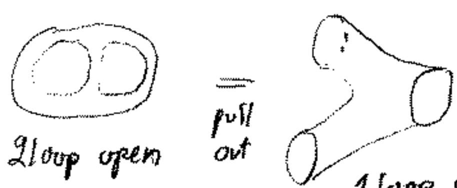
Sugra $S = \frac{1}{(2\pi)^7}\frac{1}{\alpha'^4 g_s^2}\int d^{10}x\sqrt{-g_E}\left[R_E - \frac{1}{2\cdot 5!}F_5^2 - \dots\right]$ $\longrightarrow \int_{S^5} F_5 = N$

(NOTE $\frac{1}{16\pi G_{10}}$ is outside the whole expression!)

"handle exp. parameter", $g_{open}^2 \sim g_{closed} \sim g_s \sim e^{\langle\phi\rangle}$

$16\pi G_{10} = (2\pi)^7\alpha'^4 g_s^2$

is computed by comparing



2loop open $=$ pull out 1loop closed

 and 

string amp projected to graviton

ggg vertex in weak field sugra

$\Downarrow$ classical gravity solutions if the relevant scale $\gg l_{Pl}^{10d} \sim (G_{10})^{\frac{1}{8}} \sim \alpha'^{\frac{1}{2}} g_s^{\frac{1}{4}} \sim g_s^{\frac{1}{4}} l_s$

p-brane solutions :

$D=10=$	1	$+\ p$	$+\ 1$	$+\ d-1$
	t	$\vec{x}$	r	S_{d-1}

$S = \frac{1}{(2\pi)^7}\frac{1}{\alpha'^4}\int d^{10}x\sqrt{-g}\left[e^{-2\phi}\left(R + 4(\nabla\phi)^2\right) - \frac{2}{(8-p)!}F_{p+2}^2\right]$ $\qquad \left(\dots - \frac{1}{2}\sum_n \frac{1}{n!}F_n^2\right)$

Solve Einstein $\left(D=10, \begin{cases} p=3 \\ m=5 \end{cases} \text{permits } \phi = 0 \right)$ $\left(\text{also } D=11, \; p=2 \text{ or } 5\right)$

$16\pi G_{10}$ is not visible here

$$\begin{cases} R_{\mu\nu} = \frac{1}{2\cdot 5!}\left(5 F_{\mu\alpha\beta\gamma\delta} F_\nu{}^{\alpha\beta\gamma\delta} - \frac{1}{2}\eta_{\mu\nu} F_5^2\right) \\ \partial_\mu\left(\sqrt{g}\, F\Gamma^{\alpha\beta\gamma\delta}\right) = 0 \end{cases}$$

Ansatz: $\begin{cases} ds^2 = -B^2 dt^2 + C^2 d\vec{x}^2 + F^2 dr^2 + G^2 r^2 d\Omega_5^2 \\ F_{0ijkr} = \varepsilon_{ijk}\, k(r) \end{cases}$ fns of r

$p=3$

$g_{\mu\nu}$: $$ds^2 = \frac{1}{\sqrt{1+\frac{R^4}{r^4}}}\left[-\left(1-\frac{r_0^4}{r^4}\right)dt^2 + dx^2 + dy^2 + dz^2\right] + \sqrt{1+\frac{R^4}{r^4}}\left[\frac{1}{1-\frac{r_0^4}{r^4}}dr^2 + r^2 d\Omega_5^2\right]$$

(This is NOT AdS! 10d Mink. for $r\to\infty$!)

Terminology:

Non-extremal black 3-brane

Extremal: $r_0 = 0$ (brane ground state)

Near-extremal: $r_0 \ll R$

Throat approximation: $r \ll R$

$\approx$ Near horizon limit: $r \simeq r_0 \ll R$

any p:

$$\left(1+\frac{R^{7-p}}{r^{7-p}}\right)^{\frac{p-7}{8}} \Leftrightarrow \frac{1}{\sqrt{1+\frac{R^4}{r^4}}}$$

$$\left(1-\frac{r_0^{7-p}}{r^{7-p}}\right)$$

$F_{\mu\nu\alpha\beta\gamma}$: $$F_{0ijkr} = \varepsilon_{ijk}\frac{-4R^2\sqrt{r_0^2+R^2}}{\left(1+\frac{R^4}{r^4}\right)^2 r^5}$$

3-brane couples to 4-form $A \Rightarrow$ 5-form $F = dA$

$$Q = \frac{1}{\Omega_5}\int_{S_5}\tilde{F}_5 = 4\cdot R^4$$

Lengthy arguments (Lyng-Petersen, hep-th/9902131) lead to

$(r_0=0)$ $\quad R^4 = 4\pi g_s N \alpha'^2$ $\quad$ ("stack of N 3-branes"

$\frac{1}{4\pi}g_{YM}^2 = g_s$ $\quad$ to have massless quanta, put branes on top of each other)

Range of validity:

- classical sol's relevant if $R \gg l_{Planck}^{10d}$

  $4\pi g_s N\alpha'^2 \gg \alpha'^2 g_s \Rightarrow N \gg 1$ (class. gravity)

- small α': $R^4 = 4\pi g_s N \underbrace{\alpha'^2}_{\text{large} \Leftarrow \text{small}}$ $\quad$ also $g_s N \sim \underbrace{N g_{YM}^2}_{\text{'t Hooft parameter}} \gg 1$ (Strongly coupled field th.)

So this is kind of charged BH (Reissner-Nordström)

$$\int d^4x \sqrt{-g} \left\{ -\frac{1}{16\pi G}(R - 2\Lambda) - \frac{1}{4} g_{\mu\alpha} g_{\nu\beta} F^{\mu\nu} F^{\alpha\beta} \right\}$$

↑ $16\pi G$ sits only here

$$\Rightarrow \begin{cases} R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G \underbrace{T_{\mu\nu}^{\text{elmag}}}_{-F_{\mu\beta} F_{\nu\alpha} g^{\beta\alpha} + \frac{1}{4} g_{\mu\nu} F^2} \\ g^{\mu\nu} \nabla_\mu F_{\nu\alpha} = 0 \end{cases}$$

Symmetries: $ds^2 = -e^{2\alpha(r,t)} dt^2 + e^{2\beta(r,t)} dr^2 + r^2 d\Omega_2^2$

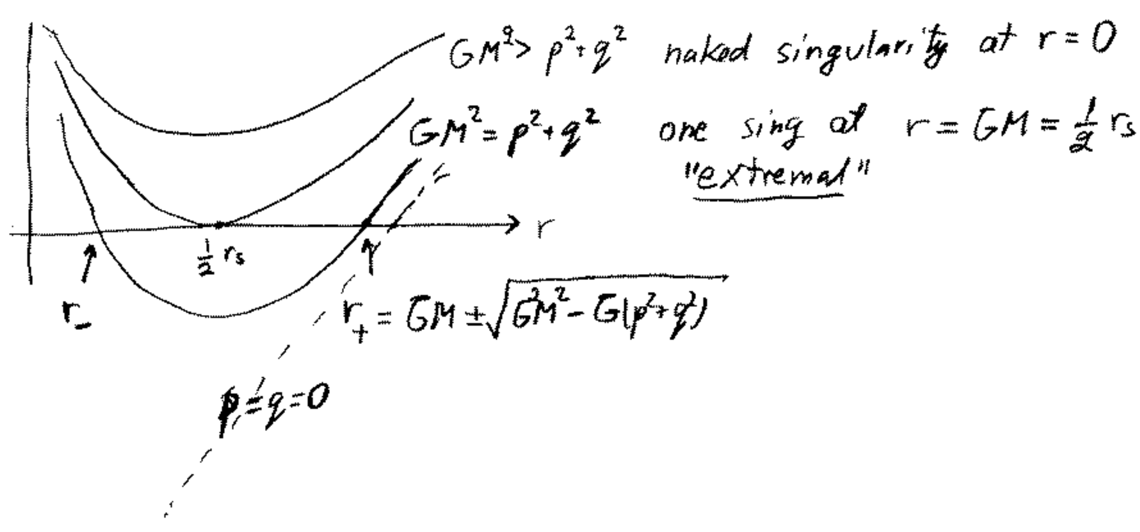
$(\mu = t\ r\ \theta\ \varphi)$ $\quad F_{tr} = f(t,r)$

$\begin{cases} E^i \sim F^{0i} \\ B_i \sim \frac{1}{2}\varepsilon_{ijk} F^{jk} \end{cases}$ $\quad F_{\theta\varphi} = g(t,r) \sin\theta \qquad B^r = \varepsilon^{0r\theta\varphi} F_{\theta\varphi}$

$$ds^2 = -\left[1 - \frac{2GM}{r} + \frac{G(p^2+q^2)}{r^2}\right] dt^2 + \frac{1}{1 - \frac{2GM}{r} + \frac{G(p^2+q^2)}{r^2}} dr^2 + r^2 d\Omega^2$$

$\begin{cases} q = \text{total electric charge} = \text{dimless} \\ p = \text{" magn. "} = \text{"} \end{cases}$ $\left(\frac{G}{r^2} = \frac{1}{(M_{Pl}\, r)^2} = \text{dimless} \right)$

[ ]

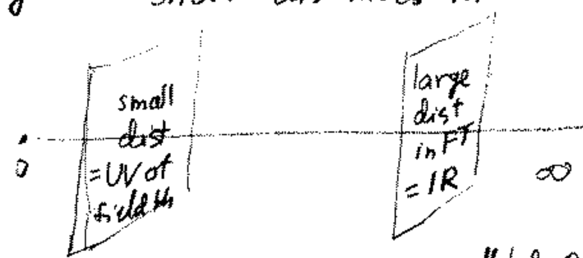


To get AdS $\Big|$ in Poincaré coord's one takes

1) $r_0 = 0$: "extremal"
2) $r \ll R$: "throat appro" Inv. under $\begin{cases} x^\mu \to \lambda x^\mu \\ r \to \frac{1}{\lambda} r \end{cases}$
3) inverts $r \to \frac{1}{r}$: $\varrho = \frac{R^2}{r}$ " $\begin{cases} x^\mu \to \lambda x^\mu \\ \varrho \to \lambda \varrho \end{cases}$

$$\Rightarrow ds^2 = \underbrace{\frac{R^2}{\varrho^2}\left(\eta_{\mu\nu}\, dx^\mu dx^\nu + d\varrho^2\right)}_{AdS_5 \text{ in Poincaré coord's (p. 41)}} + R^2 d\Omega_5^2 = AdS_5 \times S_5$$

Now argue: short distances in field theory $\Leftrightarrow$ large dist. in gravity



$\to \varrho = \frac{R^2}{r}$ $\qquad ds_{Bulk} = \frac{1}{\varrho}\, ds_{Field\ th}$

$ds_{Field\ th} = \varrho\ ds_{bulk}$ "4d QFT lives on the boundary at $\varrho = 0$"

Introducing QCD scale breaking by hand (Brodsky, hep-th/0501022)

$0 \quad r_0 = \Lambda_{QCD} R^2 \quad \infty \to r$

cut away from AdS small r, large ϱ $\left(\text{include } \varrho < \frac{1}{\Lambda_{QCD}} = \varrho_0 \sim 1\,\text{fm}\right)$

include $r > \Lambda_{QCD} R^2$

To get hadron masses write $\phi(x,\varrho) = e^{-ip\cdot x} f(\varrho)$, $f(\varrho_0) = 0$ $\quad (-\partial_t^2 + \vec{\partial}^2 = p^2 = M^2)$

$$R^2 \Box_{AdS} \phi = \left[\varrho^2 M^2 + \varrho^2 \partial_\varrho^2 - 2\varrho\partial_\varrho \underbrace{- L(4+L)}_{\text{eigenvalues of Laplacian on sphere}}\right] f(\varrho) = 0$$

(see p. 41)

one argues that this cut-off is better than the one from bag model