Hot QCD matter: lattice, perturbation theory and AdS/QCD.

Keijo Kajantie

University of Helsinki

CERN, 1 July 2009

This talk is about doing the integral:

$$Z(T,V) = e^{p(T)\frac{V}{T}} = \int \mathcal{D}[A\bar{\psi}\psi] e^{-\int_0^{1/T} d\tau d^3x \, \mathcal{L}_{\rm QCD}}$$

$$\mathcal{L}_{\rm QCD} = \frac{1}{4g^2} \sum_{a=1}^{N_c^2-1} F_{\mu\nu}^a F_{\mu\nu}^a + \sum_{i=1}^{N_f} \bar{\psi}_i [\gamma_\mu D_\mu + m_i] \psi_i$$

Lattice: $N_t \cdot N_s^3$ $\qquad U_\mu(x) = e^{igaA_\mu(x)}$

$$\frac{1}{T} = N_t a \ll N_s a = V^{1/3}$$

Derivatives of Z are expectation values;

Trace of en-mom tensor:

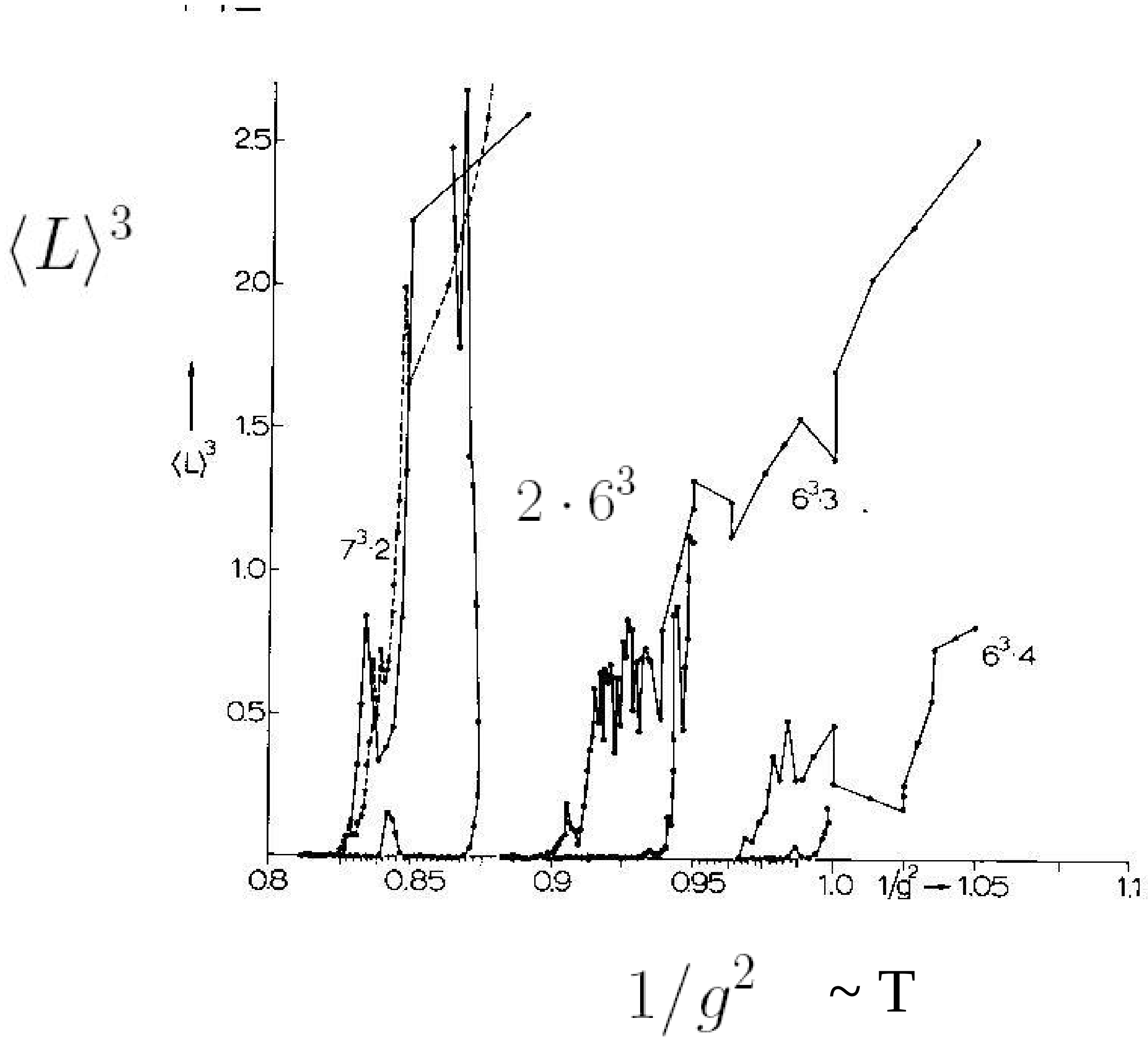
$$\frac{-1}{VT^3}\, a\frac{d\log Z}{da} = \frac{\epsilon - 3p}{T^4} = T\frac{\partial}{\partial T}\frac{p(T)}{T^4}$$

Determine this and by integration $p(T)$

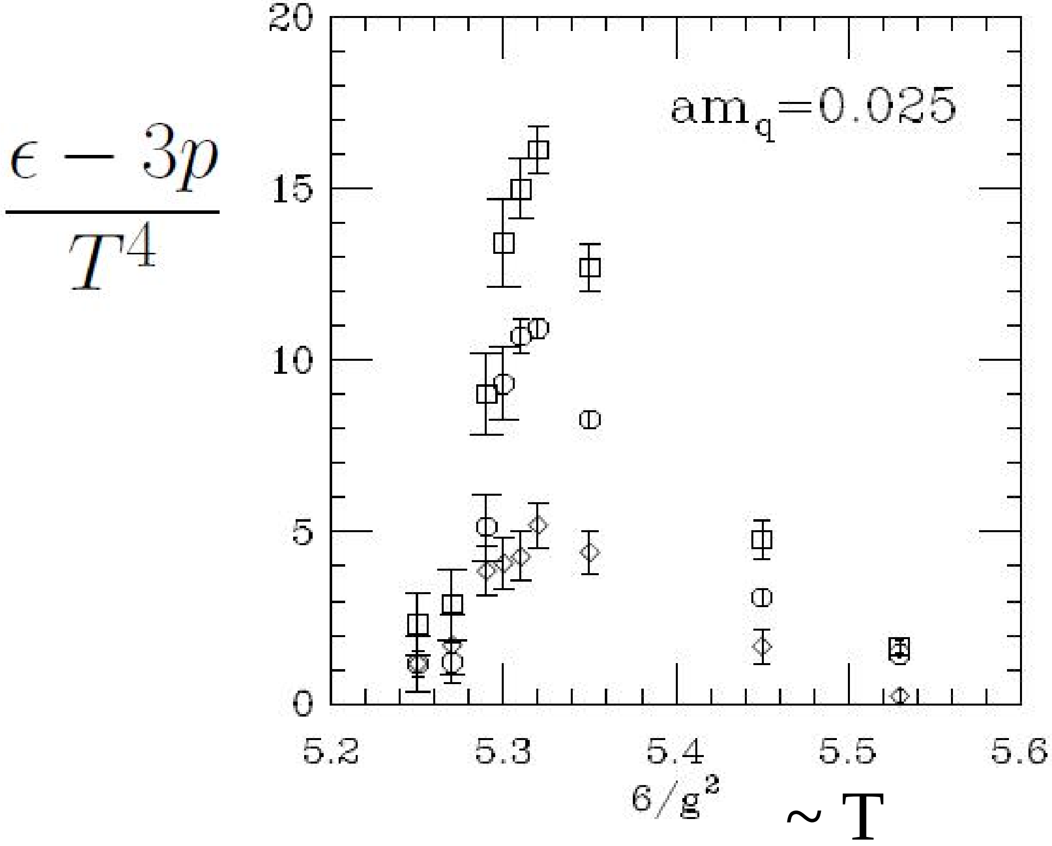
$$s(T) = p'(T), \quad \epsilon(T) = Ts - p$$

Bag model: $p = a_q T^4 - B$

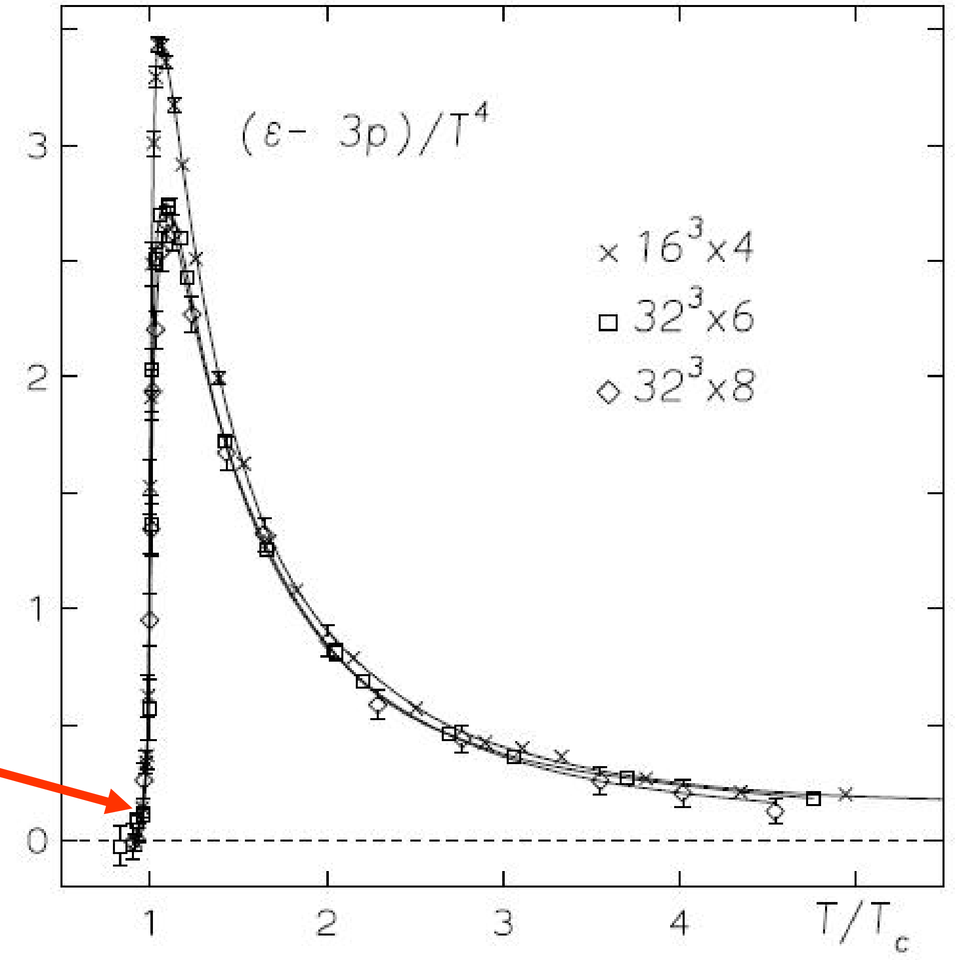
1982, SU(3) : Kajantie-Montonen-Pietarinen



1994, SU(3)+(N_f = 2): Blum- Gottlieb-Kärkkäinen-Toussaint

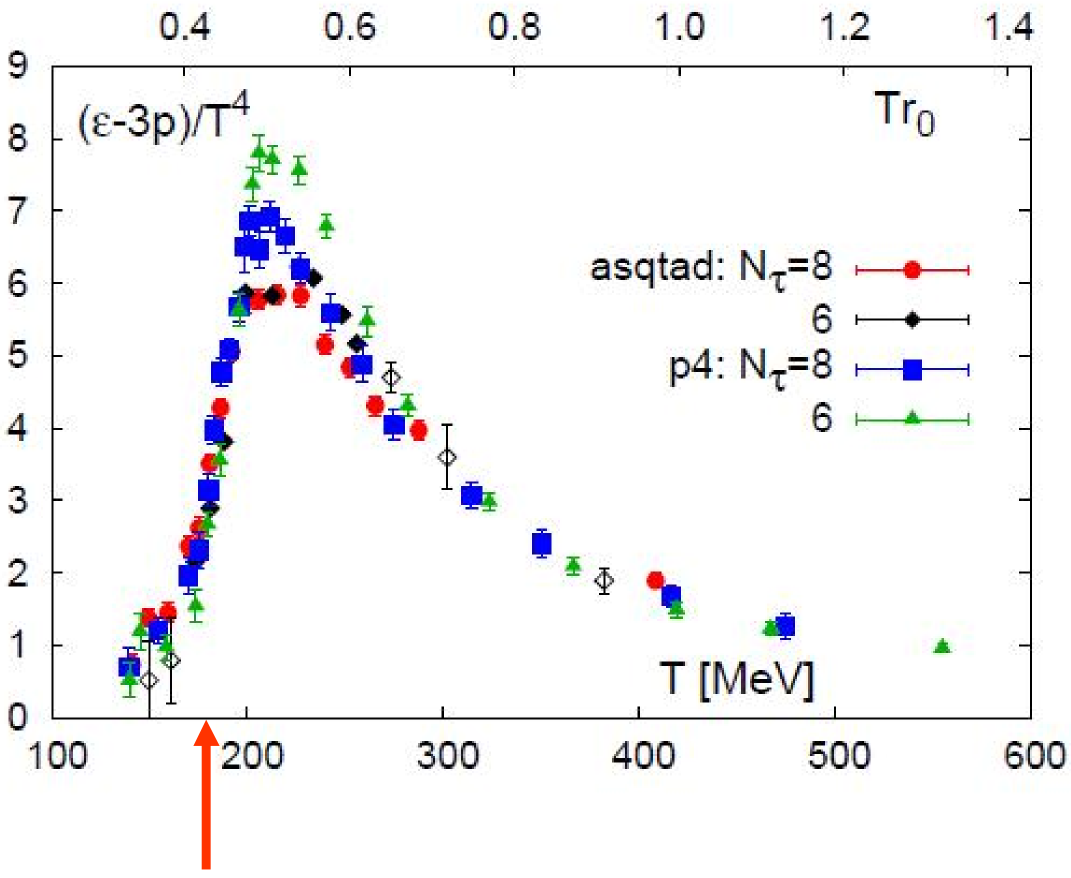


1996, SU(3): Boyd-Engels-Karsch-Laermann…



Weak 1st order transition

2009: SU(3)+(N_f = 2+1) 0903.4379, 23 authors

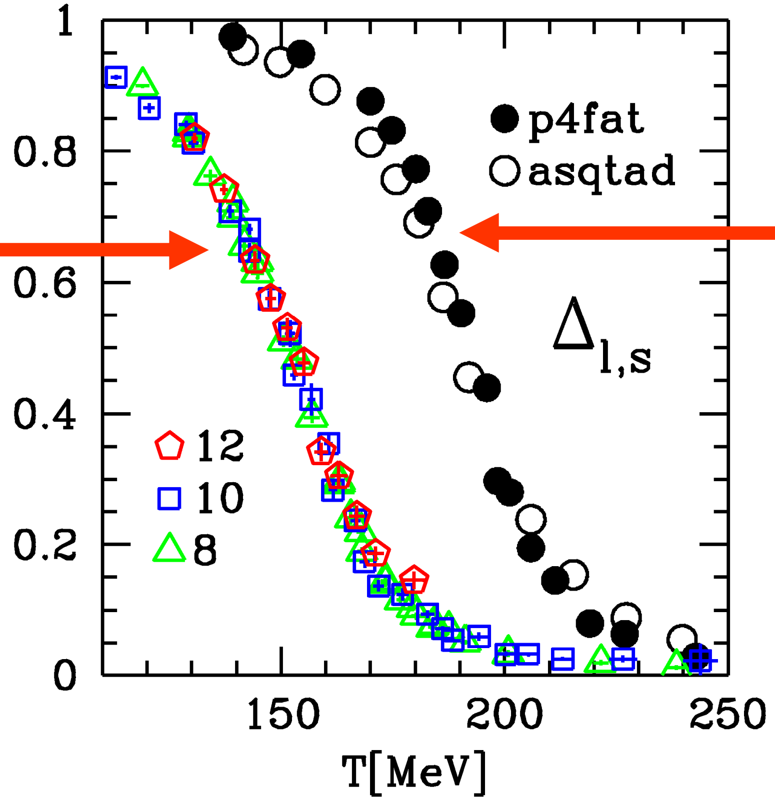


$$6 \cdot 24^3$$

T_c Continuous transition!!

Controversy about the value of T_c :

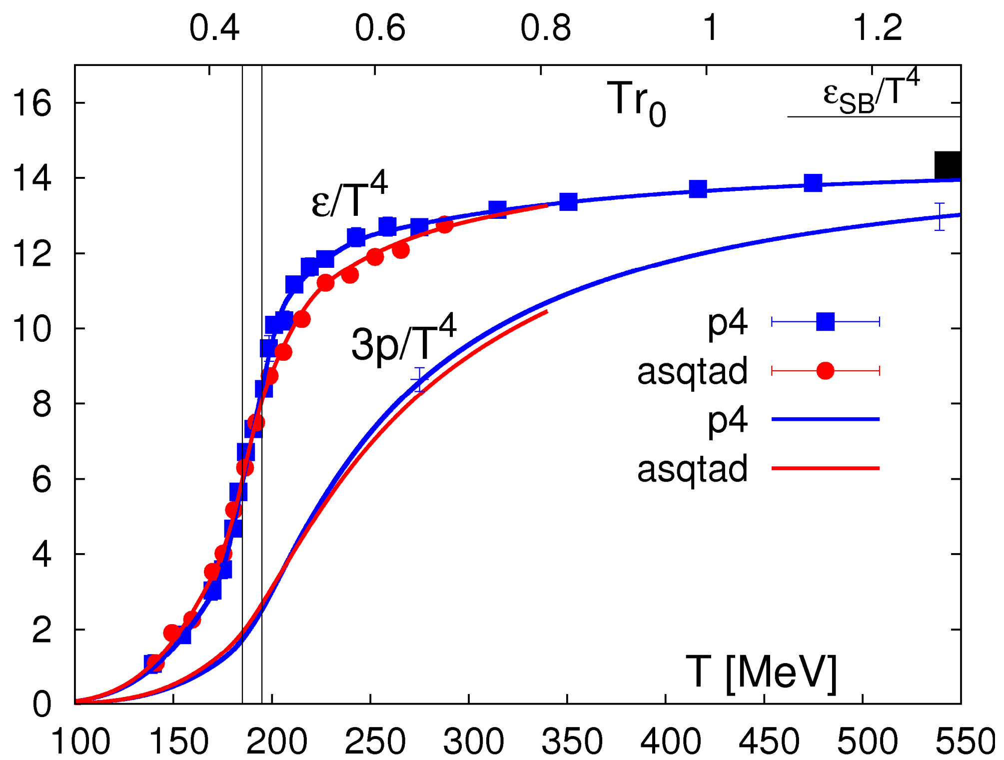
Budapest-Wuppertal



Hot QCD, USA-Bielefeld

Cosmological effects from a cross-over?

Integrate from $\epsilon - 3p$

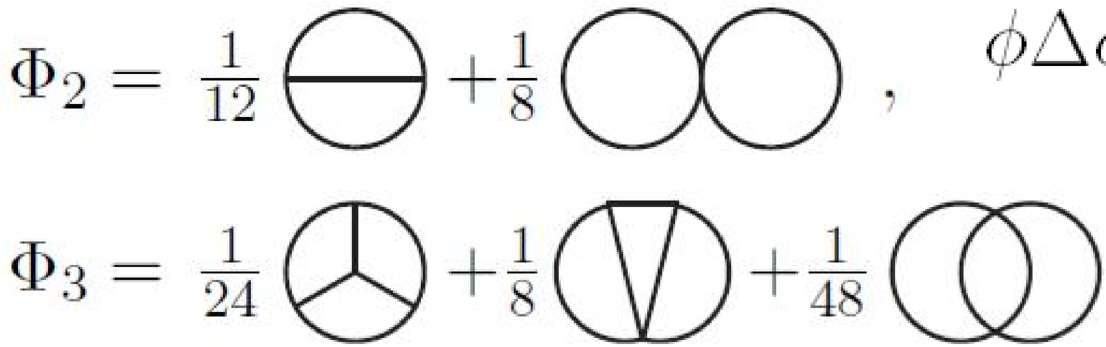


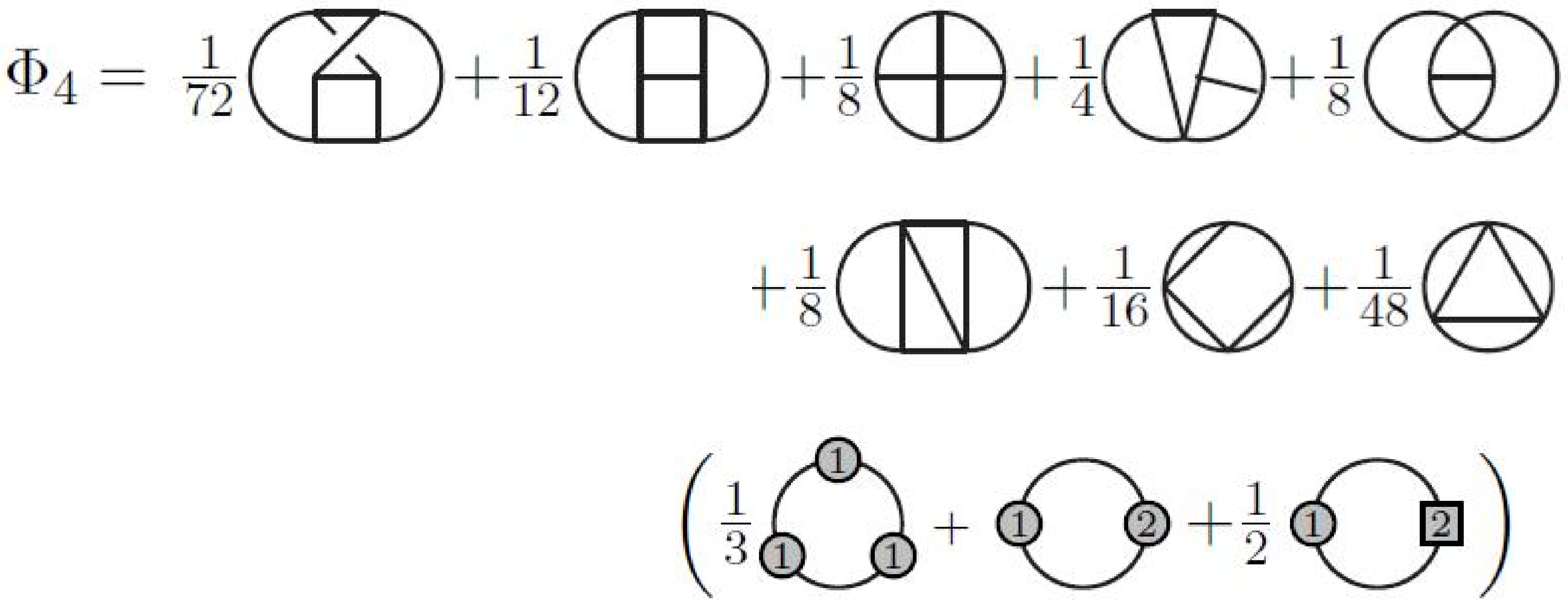
RHIC

LHC

2. Perturbation theory: expanding in g get diagrams of type:

$$\phi\Delta\phi + g\phi^3 + \lambda\phi^4$$





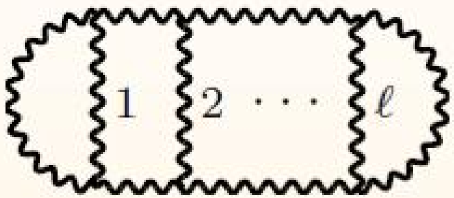
IR divergences at finite T lead to an expansion of the form:

g = standard 2-loop MSbar running coupling

$$c_{\rm SB} + c_2 g^2 + c_3 g^3 + (c_4' \log g + c_4) g^4 + c_5 g^5 + (c_6' \log g + c_6) g^6 + c_7 g^7 + ..$$

c_2 Shuryak 78, c_3 Kapusta 79, c_4' Toimela 83, c_4 Arnold-Zhai 94,
c_5 Zhai-Kastening, Braaten-Nieto 95, c_6' Kajantie-Laine-Rummukainen-Schröder 03

The Linde term c_6 is non-perturbative: all loops contribute!



$$\sim \left(T \sum_n \int d^3p\right)^{\ell+1} \frac{(gp)^{2\ell}}{[(2\pi nT)^2 + p^2 + \Pi(2\pi nT, p)]^{3\ell}}$$

(ℓ+1) loops, 2ℓ vert, 3ℓ propags

$$\sim T^{\ell+1} g^{2\ell} m^{3(\ell+1)+2\ell-6\ell} = g^6 T^4 \left(\frac{g^2 T}{m}\right)^{\ell-3}$$

$$n = 0, \quad \Pi(0,0) = m^2$$

We want to compute c_6 !

Computation has to be organised according to the pattern

$$\frac{g^2(T)N_c}{(4\pi)^2}$$

$$\mathrm{QCD} \equiv \text{4d YM} + \text{quarks};\ |\mathbf{k}| \sim g^2T, gT, 2\pi T$$

$\Downarrow$ perturbation theory (1)

$$\frac{\sqrt{g^2(T)N_c}}{4\pi}$$

$$\mathrm{EQCD} \equiv \text{3d YM} + A_0;\ |\mathbf{k}| \sim g^2T, gT \qquad \int d^3x\,\left[\tfrac{1}{4}F_{ij}^2 + (D_iA_0)^2 + m^2A_0^2 + ..\right]$$

$\Downarrow$ perturbation theory (2)

$$\mathrm{MQCD} \equiv \text{3d YM};\ |\mathbf{k}| \sim g^2T \qquad \int d^3x \tfrac{1}{4}F_{ij}^2$$

Get expansion of type

πT	$1 + g_{(1)}^2$		$+g_{(1)}^4 \ln$		$+g_{(1)}^6(\ln + [\text{pert}]_1) + \ldots$
E: gT		$+g_{(2)}^3$	$+g_{(2)}^4 \ln$	$+g_{(2)}^5$	$+g_{(2)}^6(\ln + [\text{pert}]_2) + \ldots$
M: g^2 T					$+g_{(3)}^6(\ln + [\text{non-pert}]) +$

All but the last term on first line is known!

The nonperturbative contribution,

Hietanen-Kajantie-Laine-Rummukainen-Schröder hep-lat/0412008

-free energy of 3d SU(N) gauge theory:

$$\frac{1}{\epsilon_{\rm UV}} - \frac{1}{\epsilon_{\rm IR}} = 0$$

$$\frac{1}{V} \ln \left[\int \mathcal{D} A_k \, \exp\left(- S_{\rm M} \right) \right]_{\overline{\rm MS}}$$

$$= \quad g_3^6 \frac{d_A N_c^3}{(4\pi)^4} \left[\left(\frac{43}{12} - \frac{157}{768} \pi^2 \right) \ln \frac{\bar\mu}{2 N_c g_3^2} - 0.2 \pm 0.8 \; \right]$$

Lattice and continuum are matched so that $\bar{\mu}$ is THE MSbar scale!

Needed 4-loop lattice perturbation theory in 3d, thought to be impossible, was solved with numerical stochastic lattice perturbation theory

DiRenzo, Laine, Miccio, Schröder, Torrero hep-ph/0605042

Contribution from the scale gT

-free energy of 3d SU(N) gauge + adjoint scalar theory:

KLRS hep-ph/0304048

$$\frac{1}{V}\ln\int\mathcal{D}A_k\mathcal{D}A_0\,\exp\Big(-S_{\rm E}\Big)$$

$$= \cdots + g_{\rm E}^6\,\frac{d_A N_c^3}{(4\pi)^4}\left[\left(\frac{43}{4}-\frac{491}{768}\pi^2\right)\ln\frac{\bar\mu}{2m(\bar\mu)}-1.391512\right]$$

$$1.391512 = \frac{311}{256}+\frac{43}{32}\ln 2 + \frac{11}{3}\ln^2 2-\frac{461}{9216}\pi^2+\frac{491}{1536}\pi^2\ln 2-\frac{1793}{512}\zeta(3)$$

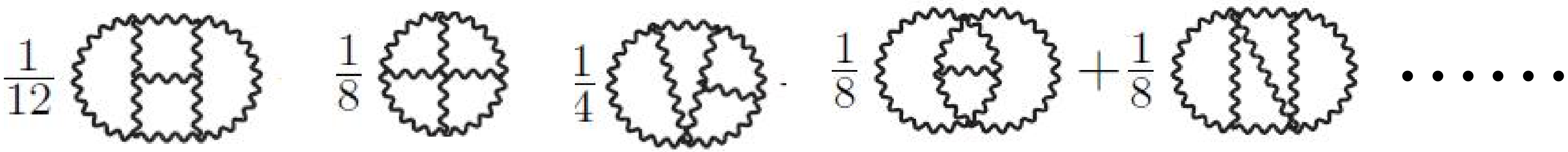
Again: $\bar\mu$ is THE MSbar scale!

Contribution from scale πT

$$1 \quad + g^2_{(1)} \qquad + g^4_{(1)} \ln \qquad + g^6_{(1)} (\ln + [\mathrm{pert}]_1)$$

is unknown!!

Have to do a 4loop sum-integral computation:

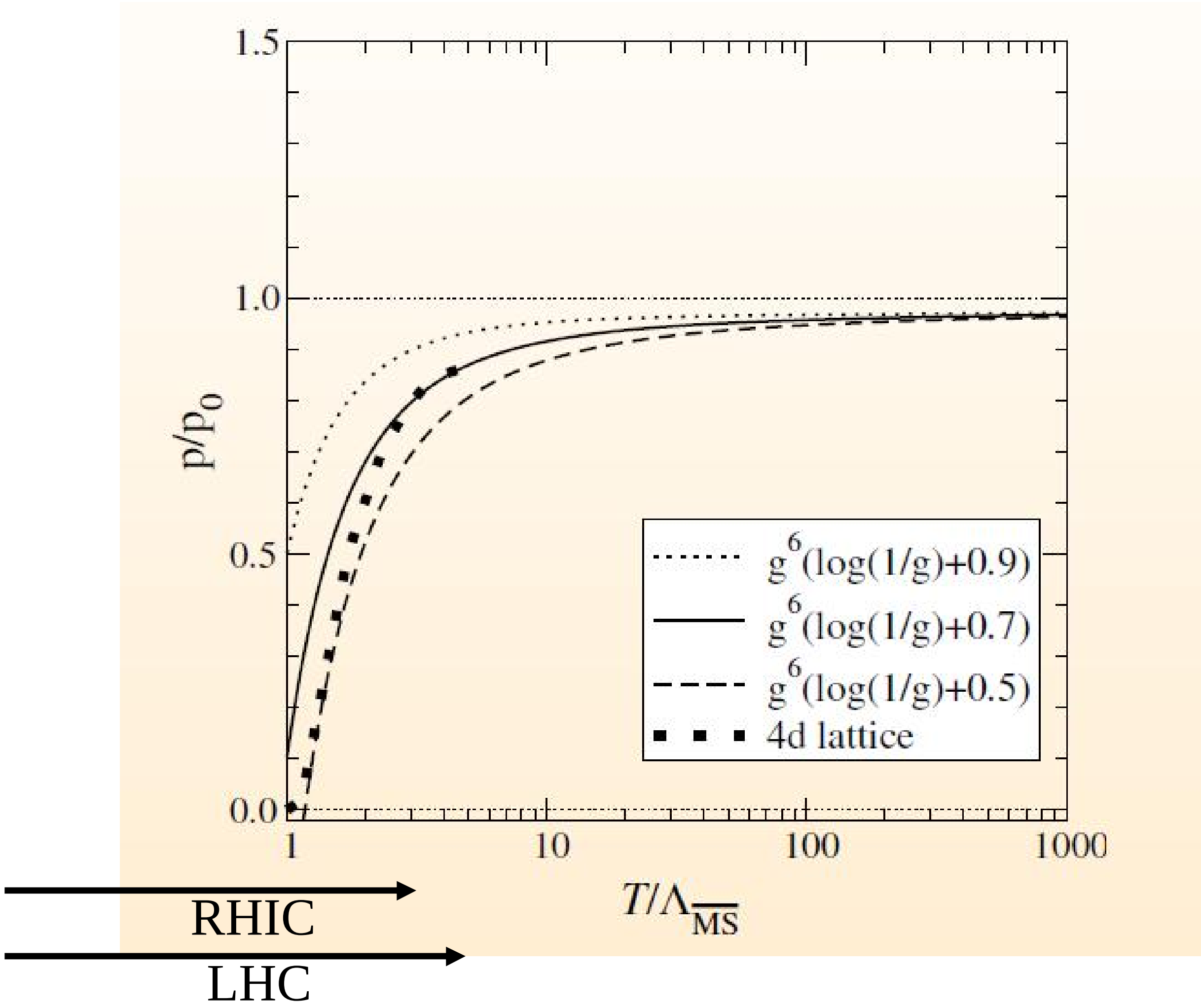


Strict MSbar, sums over n, integrals in 3-2ε dimension

Symbolic techniques not yet fully developed!

York Schröder

One can fit the constant to agree with data:



but how does this compare with the outcome of computation?

Conjecture: Laine2004 Including 1st non-pert order gives dominant effect

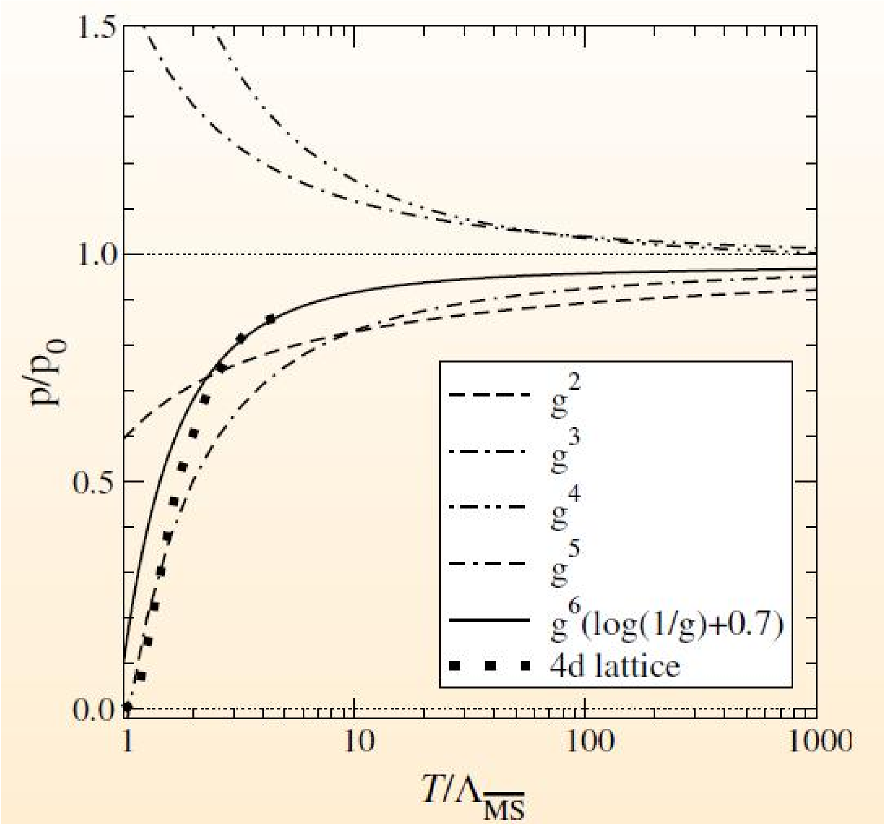
Goal:

Some day the number for c_6 should be out

But what then?

At least: work out g^7 and show it is small….

Long ago, there was the picture of ”ideal quark-gluon gas”, but



-there is the confining magnetic sector

-pert theory converges slowly

-experiments!

↓

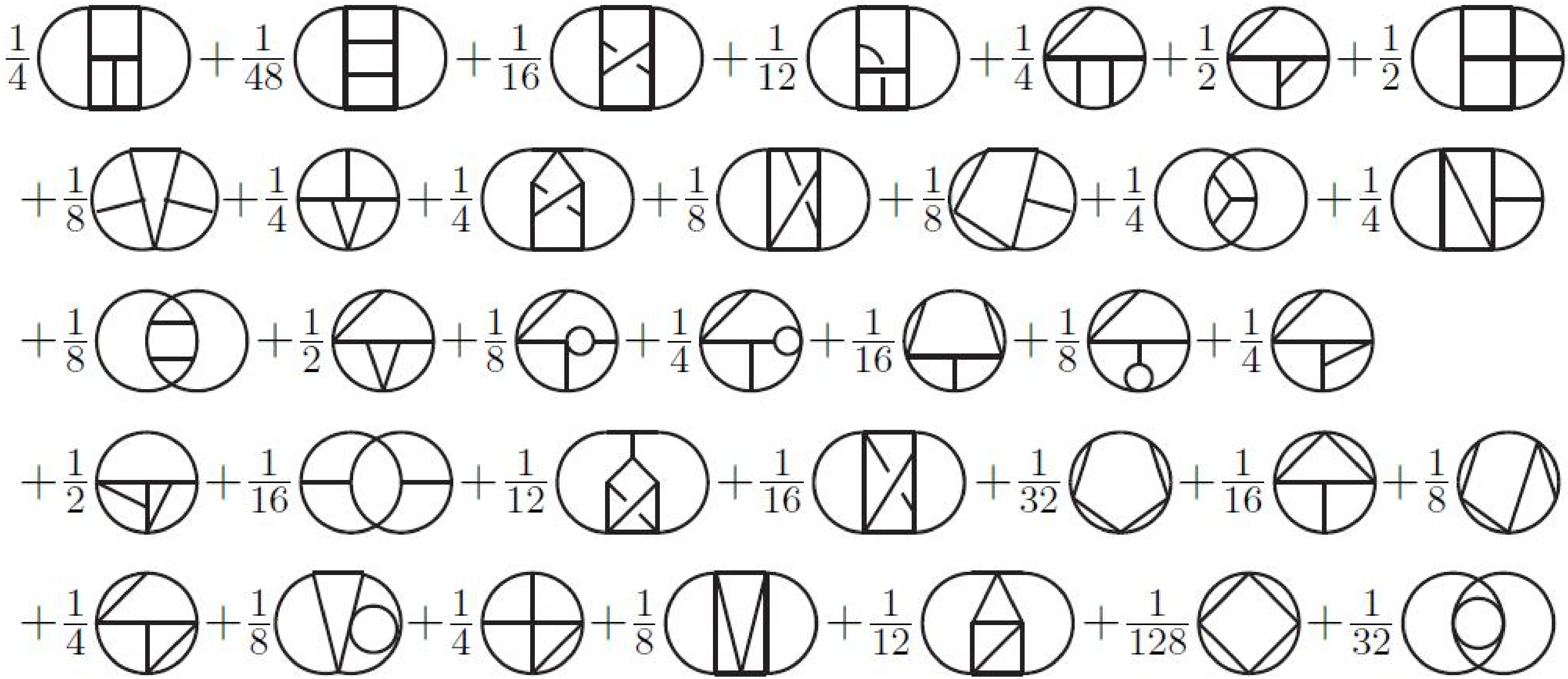
a strongly coupled system

↓

AdS/QCD

All topologically distinct 5-loop vacuum diags;

Kajantie-Laine-Schröder
hep-ph/0109100



Exercise in futility (mathematics): generalise to n loops

No wonder QCD matter becomes strongly interacting!

Add 5th dim z > 0, QCD lives at z=0

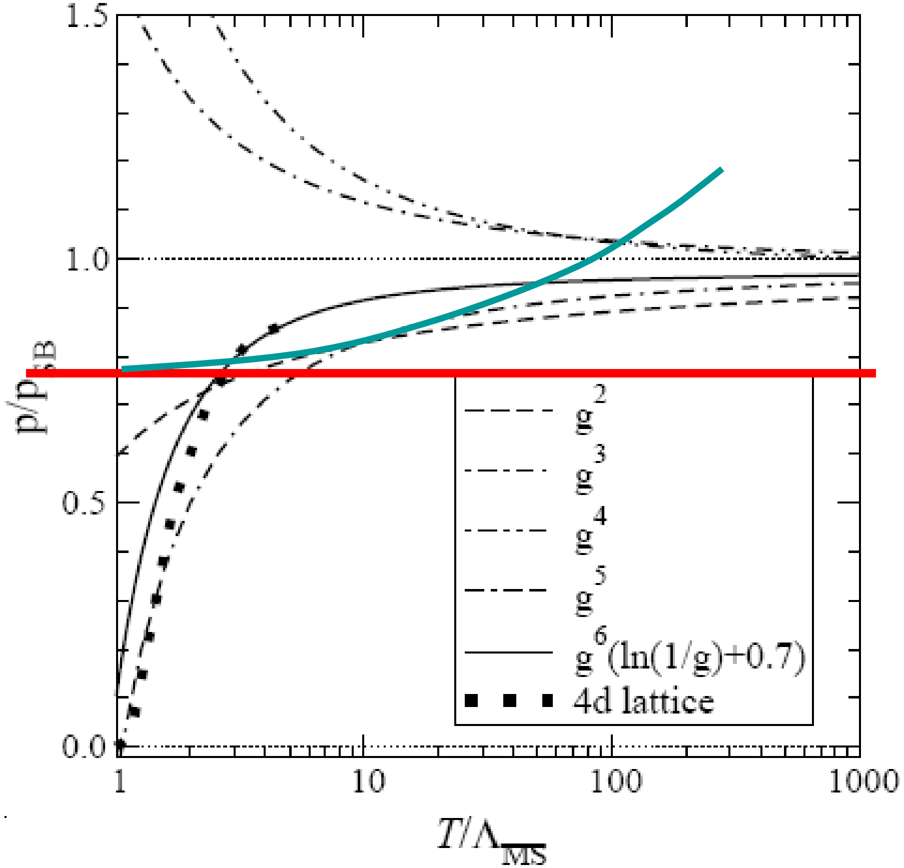
BH in 5d asymptotically (z→0) AdS_5

$$ds^2 = \frac{\mathcal{L}^2}{z^2}\left[-\left(1-\frac{z^4}{z_0^4}\right)dt^2 + d\mathbf{x}^2 + \frac{dz^2}{1-z^4/z_0^4}\right]$$

$$T_{\text{Hawk}} = \frac{1}{\pi z_0} \qquad S = \frac{A}{4G_5} \qquad = V_3 \cdot \frac{\pi^2 N_c^2}{2} T^3$$

⟶ The famous 3/4 :

p(T): lattice, perturbation theory, AdS/CFT



Bottom-up models for breaking conf inv

- Hard, $z < z_0$ ”bag model”, Soft, insert exp[c z^2]

- Dynamical, generate z-dep from Einstein for metric + scalar

Gürsoy-Kiritsis-Mazzanti-Nitti 0812.0792 etc, total of 330 pages

$$S = \frac{1}{16\pi G_5}\left\{\int d^5x\,\sqrt{-g}\left[R - \tfrac{4}{3}\left(\partial_\mu\phi\right)^2 + V(\phi)\right]\right\}$$

$$ds^2 = b^2(z)\left[-f(z)dt^2 + d\mathbf{x}^2 + \frac{dz^2}{f(z)}\right], \qquad \phi = \phi(z)$$

3 Einstein eqs + $g^2 = \lambda(z) = e^{\phi(z)}$, $\beta(\lambda) = b\dfrac{d\lambda}{db}$ → Four functions of z: b, f, φ, V(φ)

Dual of a theory with any beta function!

With this model one has worked out:

- Full SU(3) bulk thermo, including phase transition

  Kiritsis et al 0812.0792, 0906.1890

- Spatial string tension as a function of T

Alanen-Kajantie-SuurUski, 0905.2032

Andreev-Zakharov2006

Spatial string tension σ(T)

Finite T QCD: 3d space + imaginary time 0 < τ < 1/T

Measure string tension in the 3d spatial sector for varying T, get σ(T)

But can also measure σ in the 3d spatial sector without any 4th dim, string tension in 3d SU(3) Yang-Mills

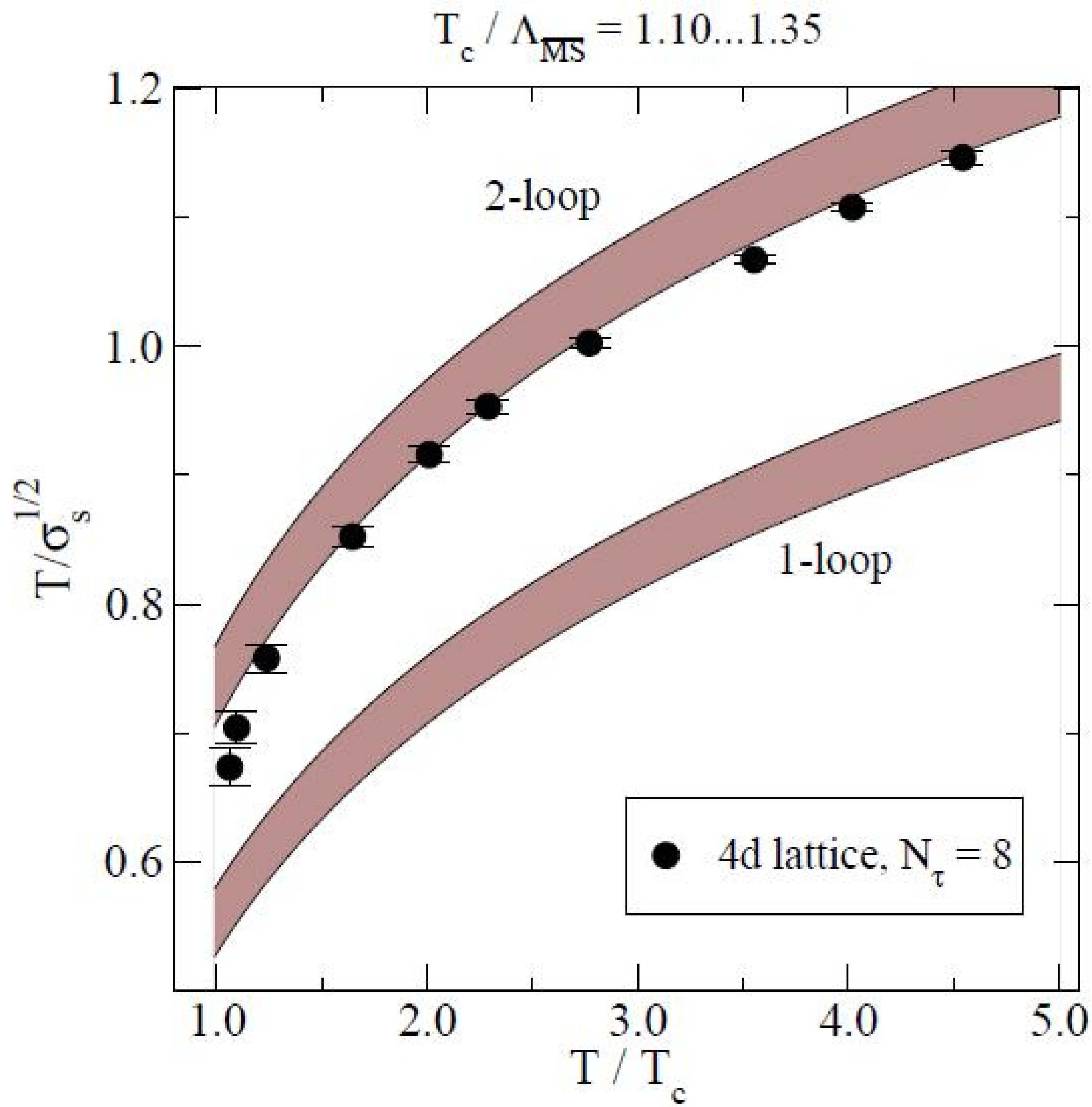
non-pert (→ 0.553(1))   pert (→ $g^2(T)$)

$$\sqrt{\sigma_s} = 0.553(1) g_M^2 \qquad g_M^2 = g^2(T)T$$

$$\frac{T}{\sqrt{\sigma_s}} = 1.81 \frac{1}{g^2(T)} = 0.25 \left[\log\frac{T}{\Lambda_\sigma} + \frac{51}{121}\log\left(2\log\frac{T}{\Lambda_\sigma}\right)\right]$$

$$\Lambda_\sigma = ? = T_c/7.753$$

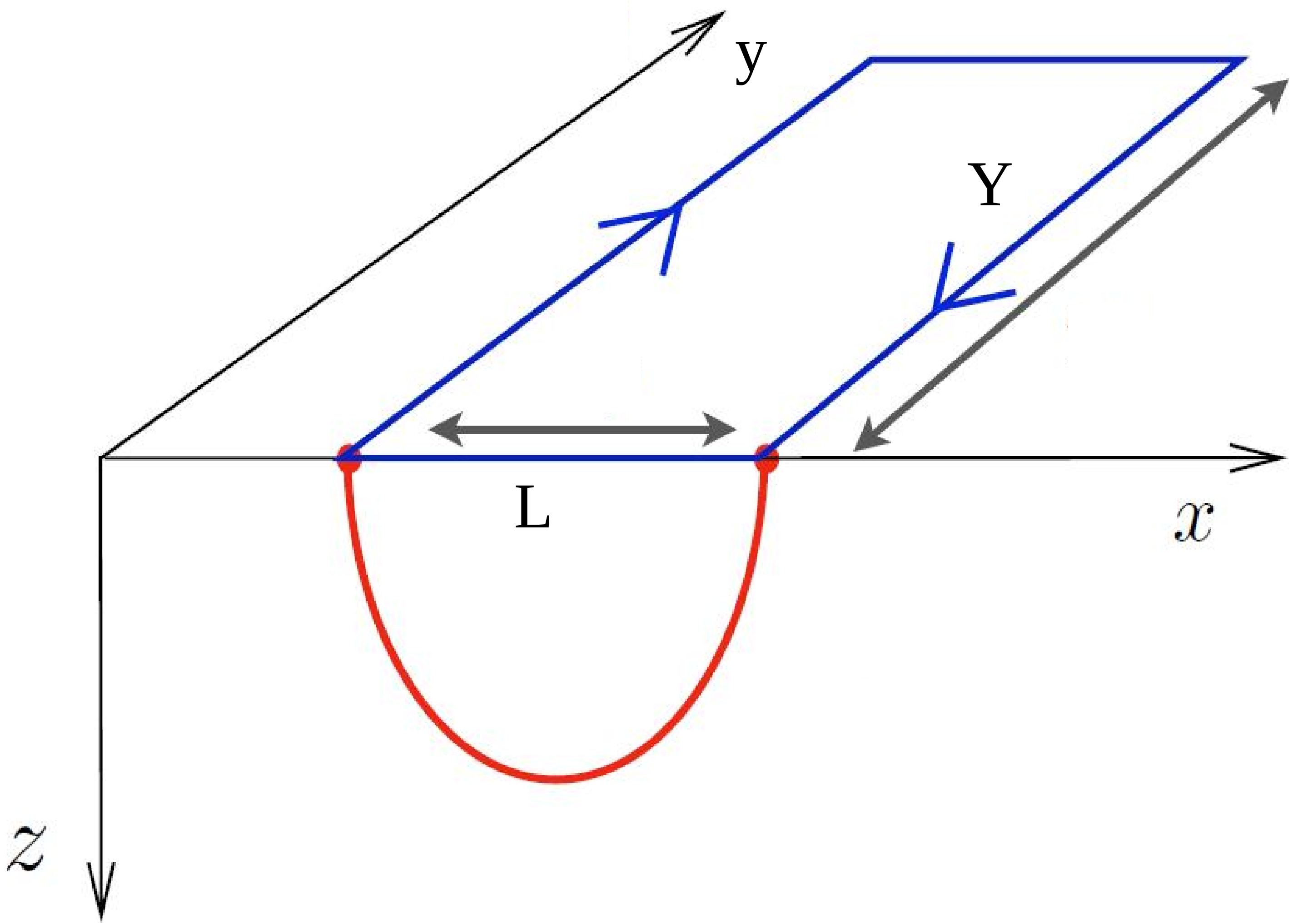
Hot QCD to 2 loops, Laine-Schröder hep-ph/0503061



Reproducible
well defined

3 loop ?
Lattice cont ?

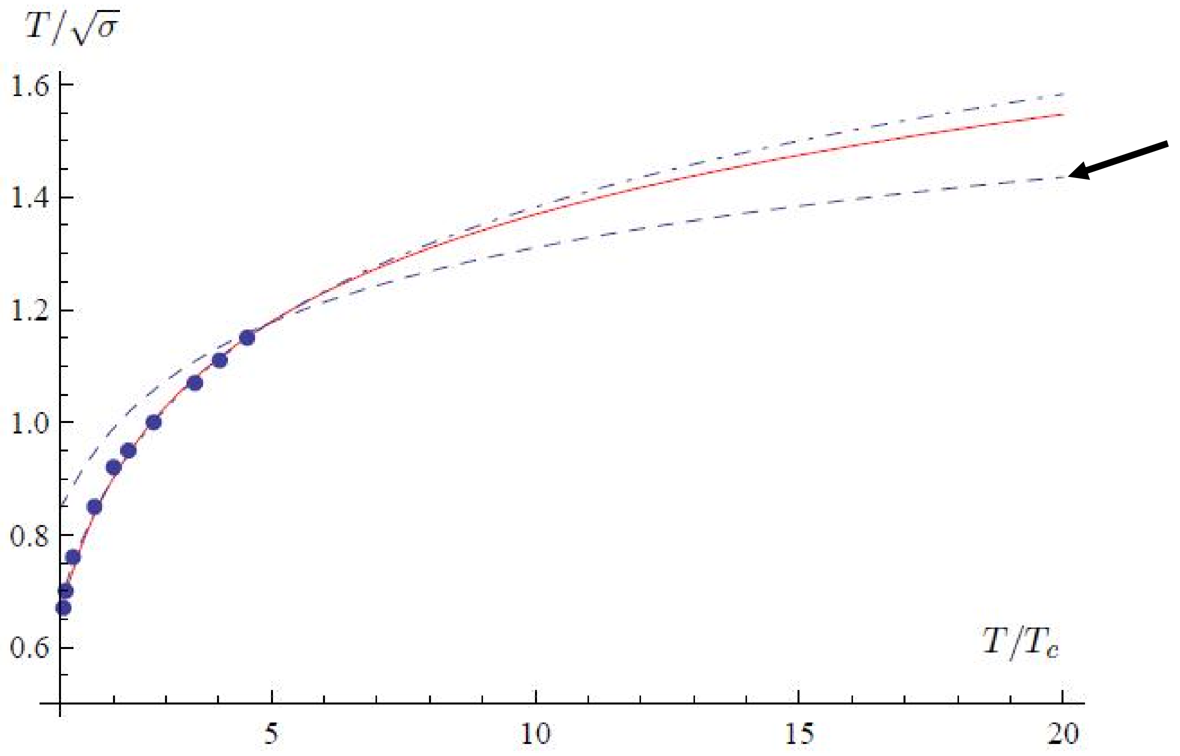
<Wilson loop> : value of extremal action of string sheet hanging from the loop to 5th dim



Andreev-Zakharov2006, Alanen-Kajantie-SuurUski, 0905.2032

$$\frac{T}{\sqrt{\sigma_s}} = \underbrace{\sqrt{\frac{2}{\pi\sqrt{g^2 N_c}}}}_{\text{conformal}} \underbrace{\left(1+\frac{4}{9\log(\pi T/T_c)}\right)\log^{2/3}\frac{\pi T}{T_c}}_{\text{conf inv breaking}}$$

simplified for presentation!



Analytic appro

Numerically
a still better fit
F.Nitti

Modified AdS can fit, but is this more than a fit?

Conclusions

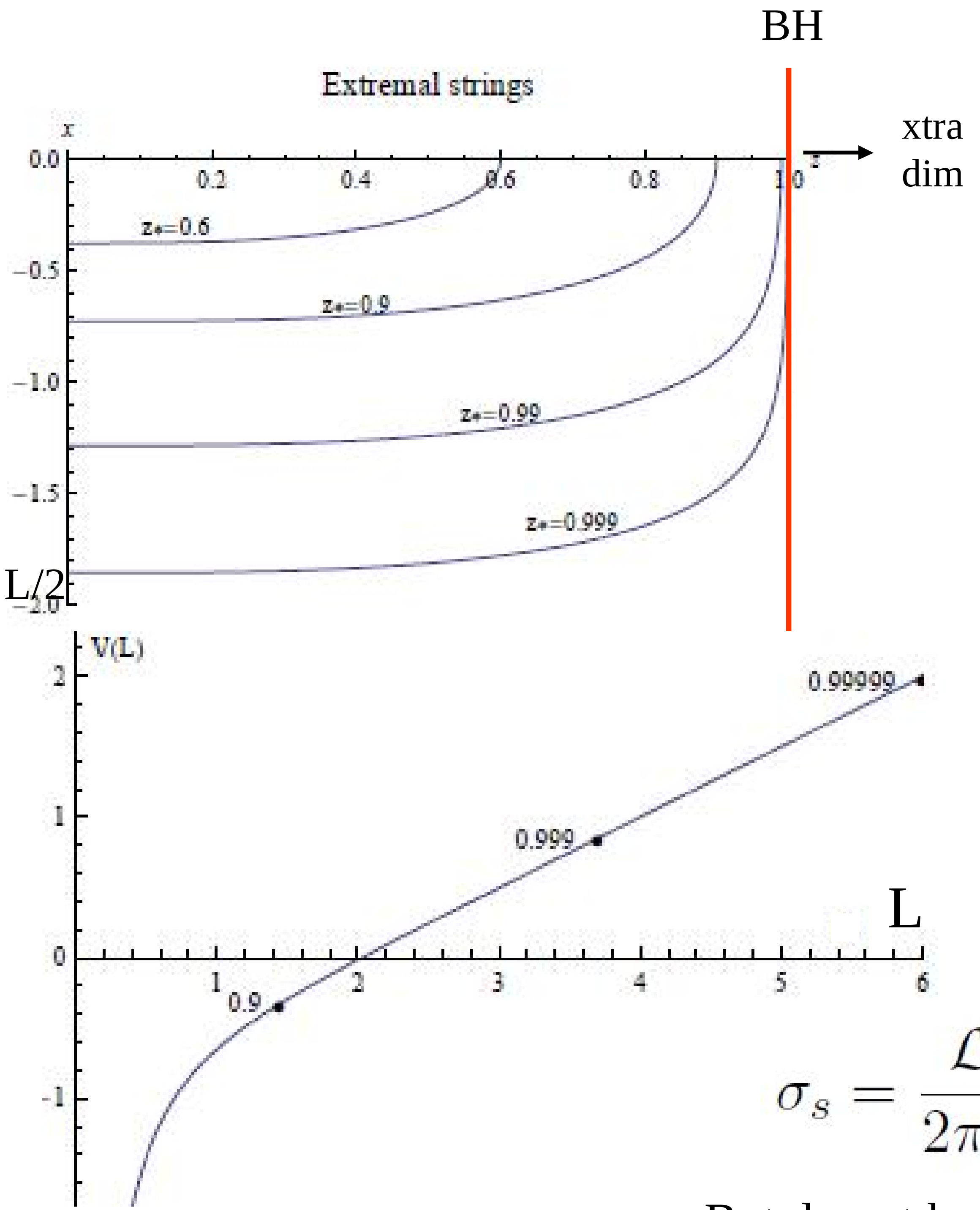
The determination of QCD EOS $p(T, \mu; m_q)$ is a long-term project which goes on and on

Experiments give convolutions of p(T,..), hard to measure with error bars

The coefficient in $p/T^4 = 1 + \dots + c_6 g^6 + ..$, $g = g_{MSbar}$, depends on the confining magnetic sector, but can be determined

AdS/QCD has brought in lots of new ideas but is so far a phenomenological approach

Overflow slides:

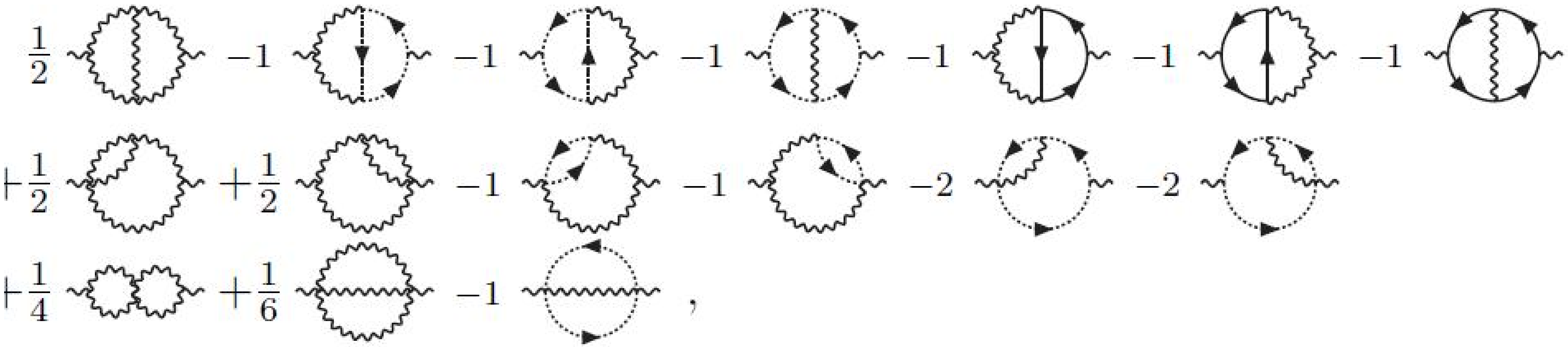


To get V(L) at large L the extr string must hang deep close to black hole

$$\sigma_s = \frac{\mathcal{L}^2}{2\pi\alpha'}(\pi T)^2 = \sqrt{g^2 N_c}\,\frac{\pi}{2}T^2$$

But do not know the parameters!

$$g_{\mathrm{M}}^2 = g_{\mathrm{E}}^2 \left[1 - \frac{1}{48} \frac{g_{\mathrm{E}}^2 C_A}{\pi m_{\mathrm{E}}} - \frac{17}{4608} \left(\frac{g_{\mathrm{E}}^2 C_A}{\pi m_{\mathrm{E}}} \right)^2 \right]$$



$$g_{\mathrm{E}}^2/T \;=\; g^2(\bar{\mu}_{\mathrm{op}}) + \frac{g^6(\bar{\mu}_{\mathrm{op}})}{(4\pi)^4} \frac{1}{198} N_c^2 [3547 - 220\zeta(3)]$$

$$\bar{\mu}_{\mathrm{op}} = 4\pi e^{-\gamma_{\mathrm{E}} - 1/22} T = 6.742\, T$$

Get a quantitative 2-loop prediction:

Laine-Schröder
hep-ph/0503061

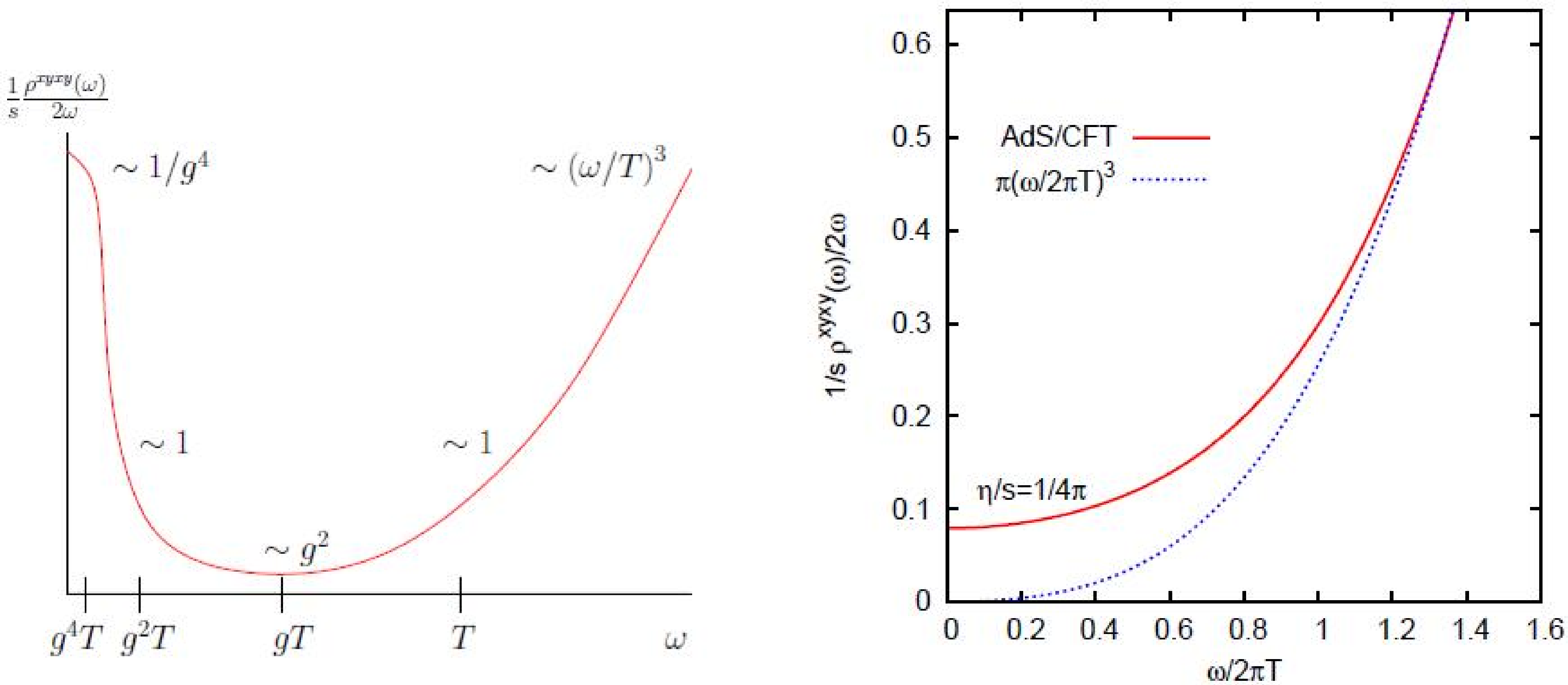
Dilemma

The heart of QCD is running coupling, Λ_{QCD}, breaking of conformal invariance, confinement

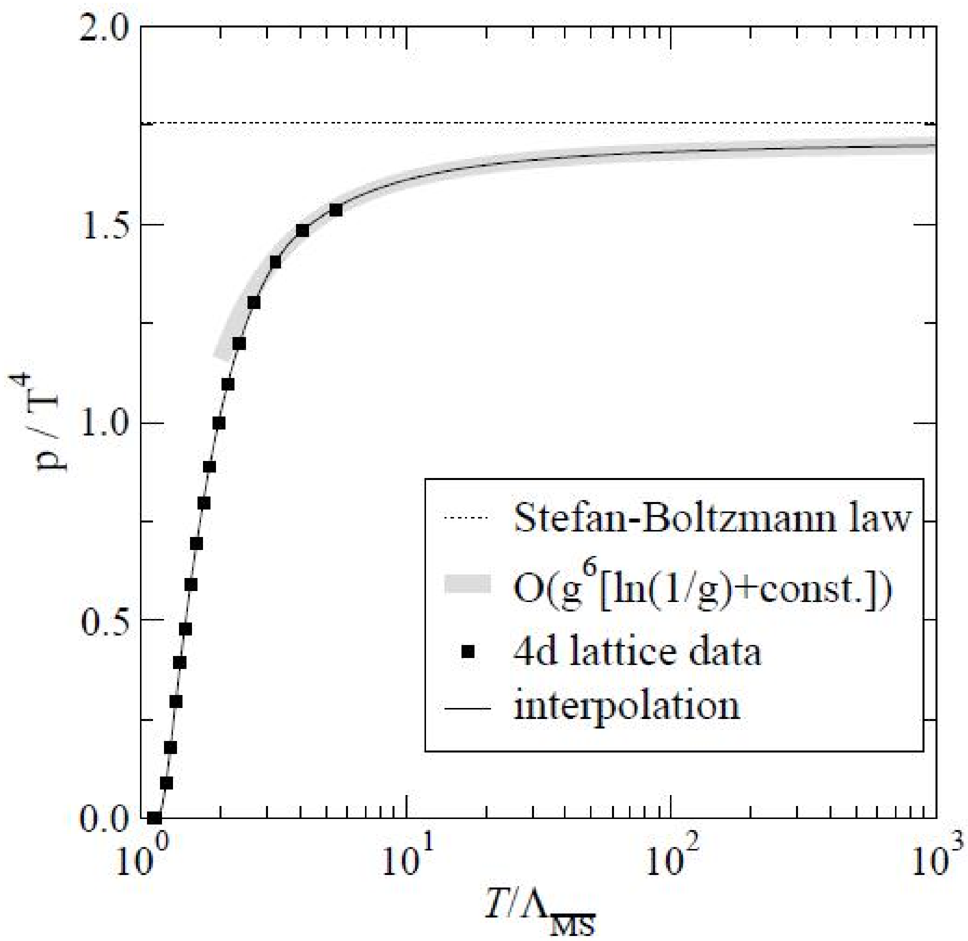
The heart of gauge/gravity duality, AdS/CFT, is conformal invariance

Can one find the gravity dual of QCD, i.e., some metric + other fields in > 4d space, with which one could reproduce QCD results?

Lots of models!!


$\frac{1}{s}\frac{\rho^{xyxy}(\omega)}{2\omega}$
$\sim 1/g^4$
$\sim (\omega/T)^3$
~ 1
~ 1
$\sim g^2$
g^4T
g^2T
gT
T
ω
AdS/CFT
$\pi(\omega/2\pi T)^3$
$\eta/s=1/4\pi$
$1/s\ \rho^{xyxy}(\omega)/2\omega$
$\omega/2\pi T$
0
0.1
0.2
0.3
0.4
0.5
0.6
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6

One can fit the constant to agree with data:



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