# Response functions of hot QCD matter from 5-dimensional gravity

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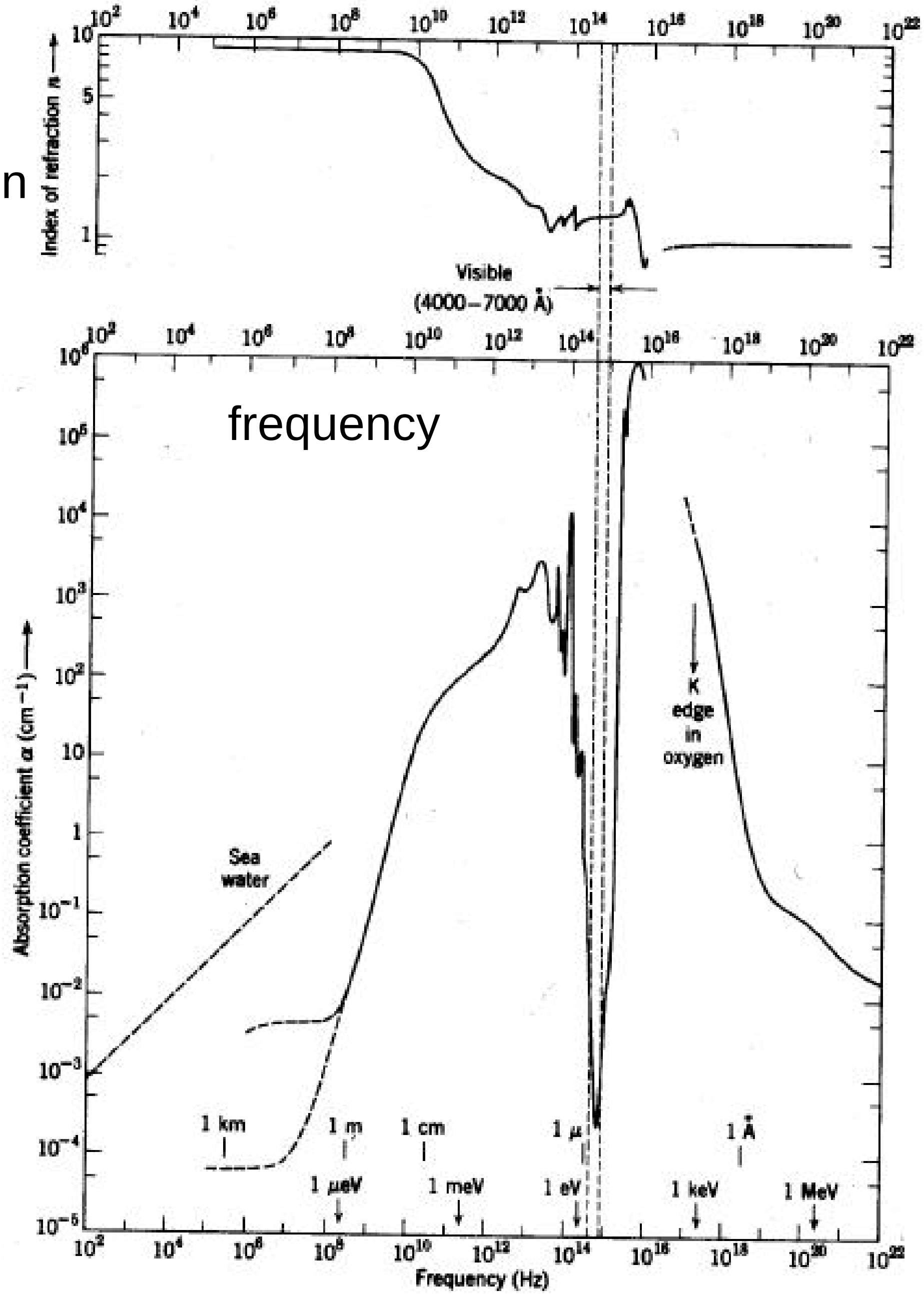
Kajantie-Vepsäläinen 1011.5570 PRD
Kajantie-Krssak-Vepsäläinen-Vuorinen, in preparation

# Response of water to elmag radiation:

n

Spectral function $\rho(\omega,\kappa)$

absorption




Huge number of different scales!

# Mathematics of response functions

$$G_R(t) = \langle\, i\,[A(t), B(0)]\,\theta(t)\,\rangle_T$$

$$\rho(\omega) = \mathrm{Im} G_R(\omega)$$

$$G_R(\omega) = \int_{-\infty}^{\infty} \frac{d\omega'}{\pi} \frac{\rho(\omega')}{\omega' - \omega - i\epsilon} \sim \int_0^{\infty} dt\, e^{i(\omega + i\epsilon)t} \ldots$$

$$\omega + i\epsilon \rightarrow i\omega_n \equiv i2\pi n T$$

$$G_\beta(\tau) = T \sum_{n=-\infty}^{\infty} e^{-i\omega_n \tau} \int_{-\infty}^{\infty} \frac{d\omega'}{\pi} \frac{\rho(\omega')}{\omega' - i\omega_n}$$

$$0 < \tau < \beta = \hbar / T$$

$$G(\tau, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{x}\cdot\mathbf{k}} \int_0^{\infty} \frac{d\omega}{\pi} \rho(\omega, \mathbf{k}) \frac{\cosh(\frac{1}{2}\,\beta - \tau)\omega}{\sinh \frac{1}{2}\,\beta\omega}$$

# Spectral function of $T_{12}$ (t,**x**)

$$G(\tau, \mathbf{k}=0; T) = \langle \int d^3x \, T_{12}(\tau, \mathbf{x}) T_{12}(0, \mathbf{0}) \rangle_T$$

$$\rho(\omega, \mathbf{k}=0) = \eta \, \omega + \ldots$$

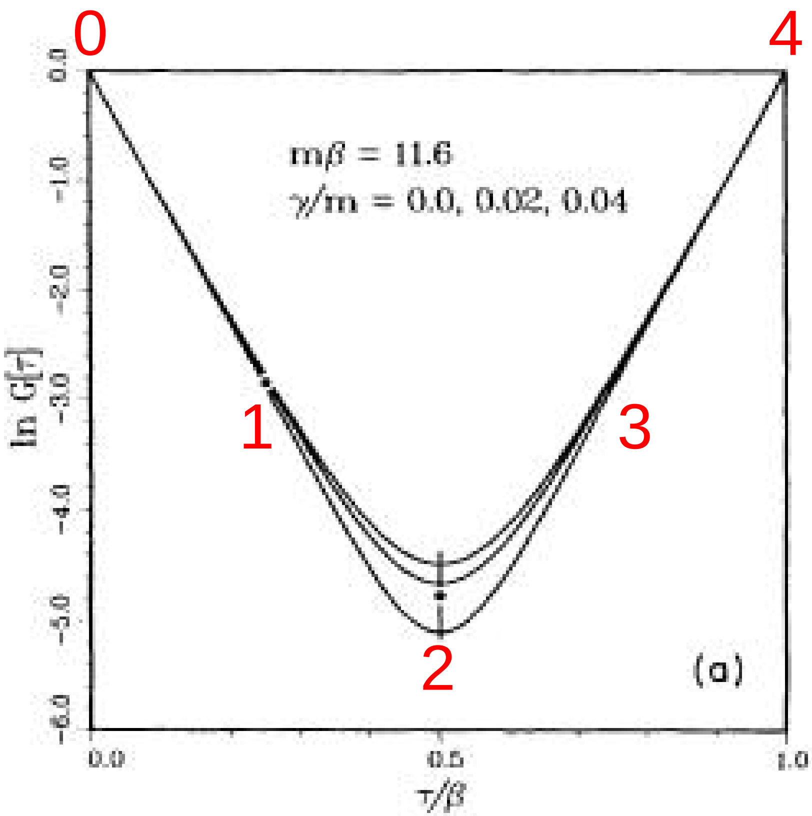
$$\frac{\eta}{s} \approx \frac{p\tau_c}{s} \approx \frac{T^4 \tau_c}{T^3} = T\tau_c \gtrsim \hbar \qquad \text{uncertainty principle}$$

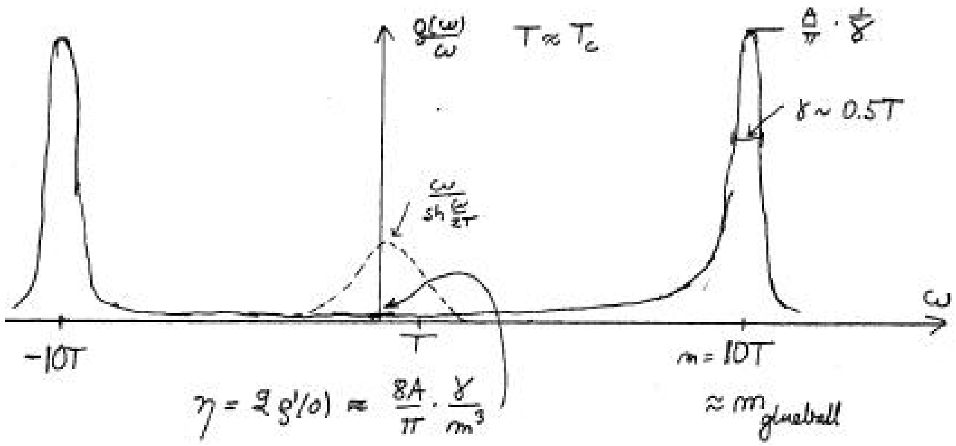
$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi} \qquad \approx \frac{10^{-5}}{10^{27}} \gg \frac{10^{-35}}{4\pi}$$

air

Karsch-Wyld PRD 1987 SU(3) $4*8^3$

$$\beta = N_t a$$

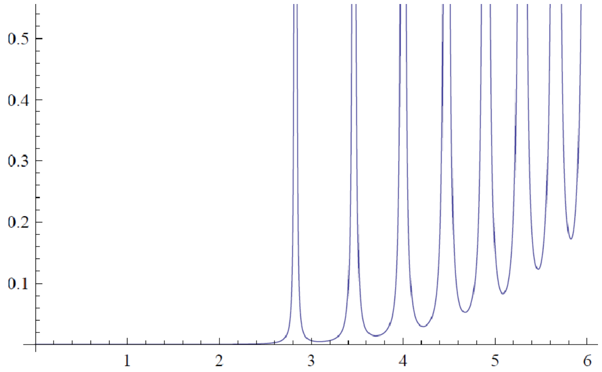



$$G(\tfrac{1}{2}\beta) = \int_0^\infty \frac{d\omega}{\pi} \frac{\rho(\omega)}{\sinh(\frac{1}{2}\beta\omega)}$$

# Vacuum spectral function

(from 5d gravity)




$$\rho_{\text{vac}}(\omega) = \frac{\pi}{32}\omega^2(\omega^2 - 4\Lambda^2)\sum_{m=0}^{\infty}\delta\left(m + 2 - \frac{\omega^2}{4\Lambda^2}\right)$$

$\delta$ function poles at glueball masses $2\Lambda\sqrt{2+m}$, $m = 0, 1, 2, ..$

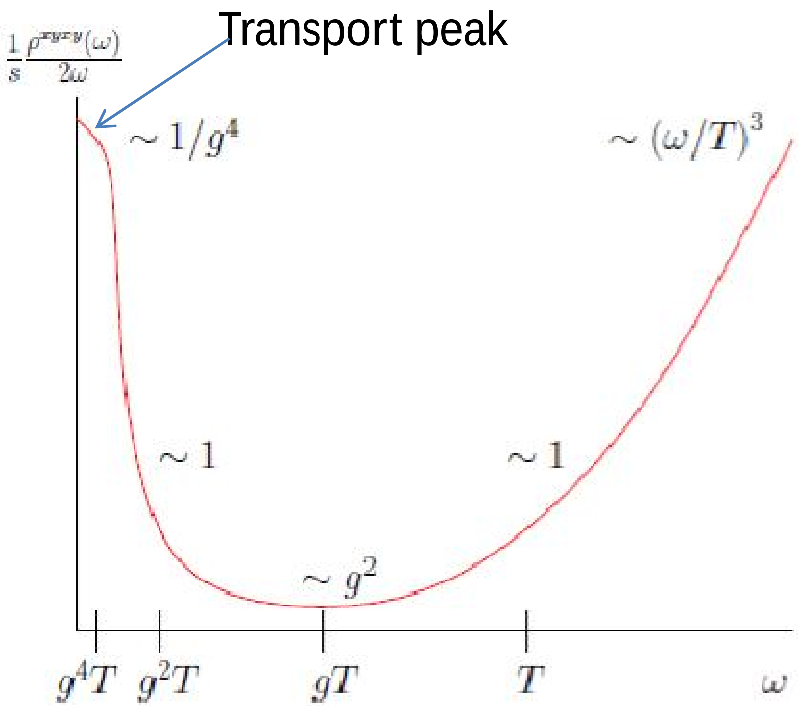
In vacuum $T_{12}$ just excites glueballs

What happens at finite T? How is viscosity created?

Perturbation theory, weak coupling

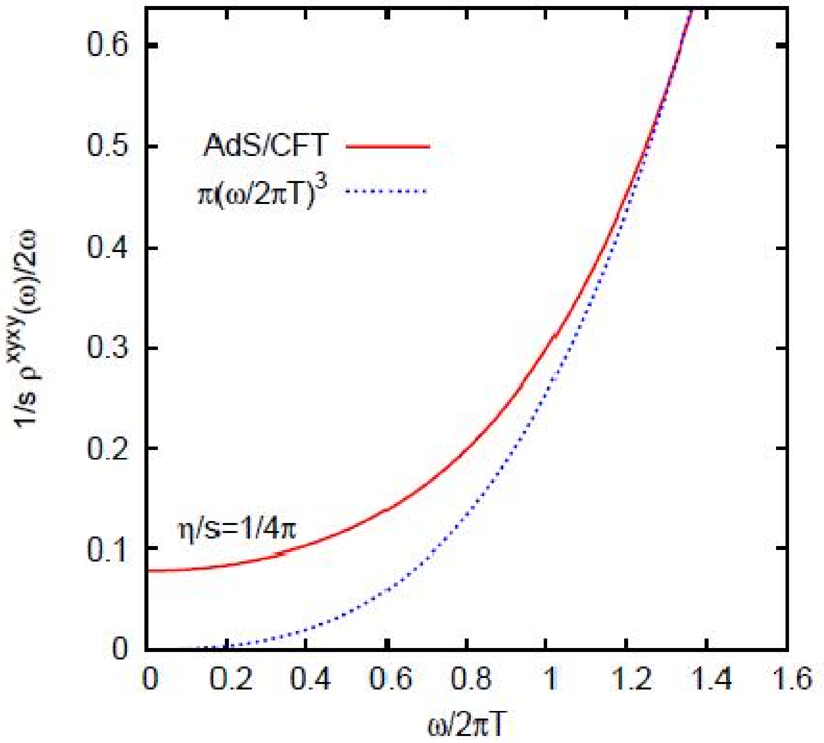
Aarts-Resco hep-lat/0110145 Meyer 0907.4095

Schafer-Teaney 0904.3107




Strong coupling, gauge/gravity duality, conformal theory

Teaney ph/0602044




We shall work out strong coupling nonconformal: Tc, glueballs

# Algorithm for computing $\rho(\omega)$ from 5dim gravity

1. Choose action leading to asymptotically AdS background

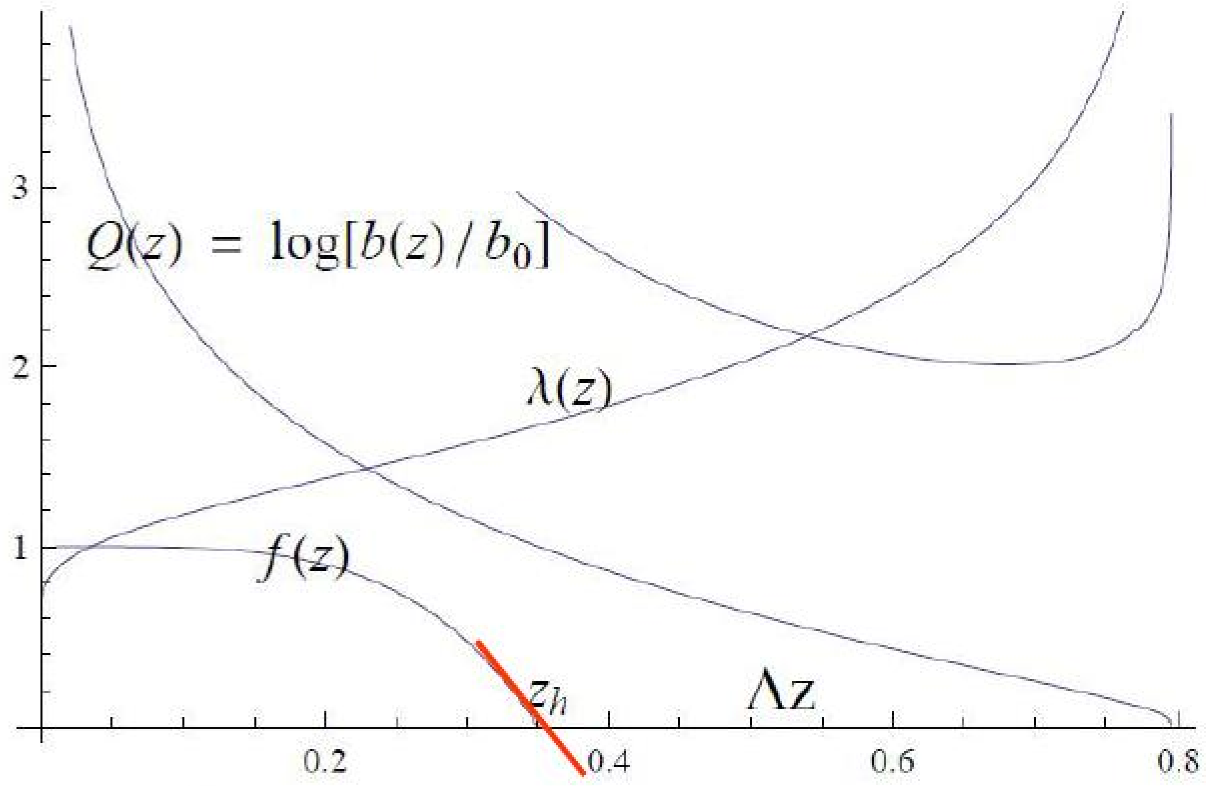
$$S = \frac{1}{16\pi G_5}\int d^5x\,\sqrt{-g}\,\left[R - \tfrac{4}{3}\,(\partial_\mu\phi)^2 + V(\phi)\right] \quad \lambda(z) = e^{\phi(z)}$$

$$ds^2 = b^2(z)\left[-f(z)dt^2 + d\mathbf{x}^2 + \frac{dz^2}{f(z)}\right]$$

$$b(z) \to \frac{\mathcal{L}}{z} \qquad f(z_h) = 0 \qquad s = \frac{b^3(z_h)}{4G_5}$$

IHQCD, Kiritsis et al: solve numerically

Model: $b(z) = \dfrac{\mathcal{L}}{z}\exp(-\frac{1}{3}\,\Lambda^2 z^2)$ $\lambda(z), f(z)$ analytic



2. Write down the linearised equation of fluctuations of $g_{12}$ in this background

Cosmologists, wake up!!

$$g_{12} = b^2(z)[1 + h(t, x^3, z)] \qquad \frac{1}{2} h_{12} T^{12}$$

K=(ω,**k**)

$$\boxed{\ddot{h}_K(z) + \frac{d}{dz}\log(b^3 f) \cdot \dot{h}_K(z) + \left(\frac{\omega^2}{f^2} - \frac{\mathbf{k}^2}{f}\right) h_K = 0}$$

$$f(z) = \dot{f}_h(z - z_h) + .. \qquad h_K = (z - z_h)^p \Rightarrow p^2 + \frac{\omega^2}{\dot{f}_h^2} = 0$$

3. Solve the equation with the boundary condition: falling into BH

$$h_K(z \to z_h) = (z - z_h)^{i\omega/\dot{f}_h}[1 + d_1(z - z_h) + ..]$$

4. Then simply

$$\rho(\omega, k) = \frac{1}{4\pi} s(T) \frac{\omega}{|h_K(0)|^2}$$

$$K = (\omega, k)$$

Backtrack a bit: Fundamental formula of gauge/gravity duality is

Gubser-Klebanov-Polyakov 98, 4866 citations

$$\langle \exp\left[i \int d^4x \, h_0 \, T^{12}\right] \rangle = \exp\left[i S_{\text{grav}}[h(x,z)]\right]$$

$$h(x, z \to 0) = h_0(x)$$

Generating functional of correlators of $T_{12}$

$S_{\text{grav}}$ at EOM depends only on boundary terms

Remember Noether, Wronskians:

$$S_{\text{grav}} = \frac{1}{16\pi G_5}\int d^4x dz\, \partial^\mu h \partial_\mu h$$

$$= -\underbrace{\partial_\mu \partial^\mu h}_{=0} \cdot h + \partial_z \underbrace{\left[\partial^z h \cdot h\right]}_{\text{remains!}} + \ldots$$

$$\Rightarrow G(K) = \frac{1}{16\pi G_5} f b^3 \dot{h}_K h_{-K} \qquad h_K(0) = 1$$

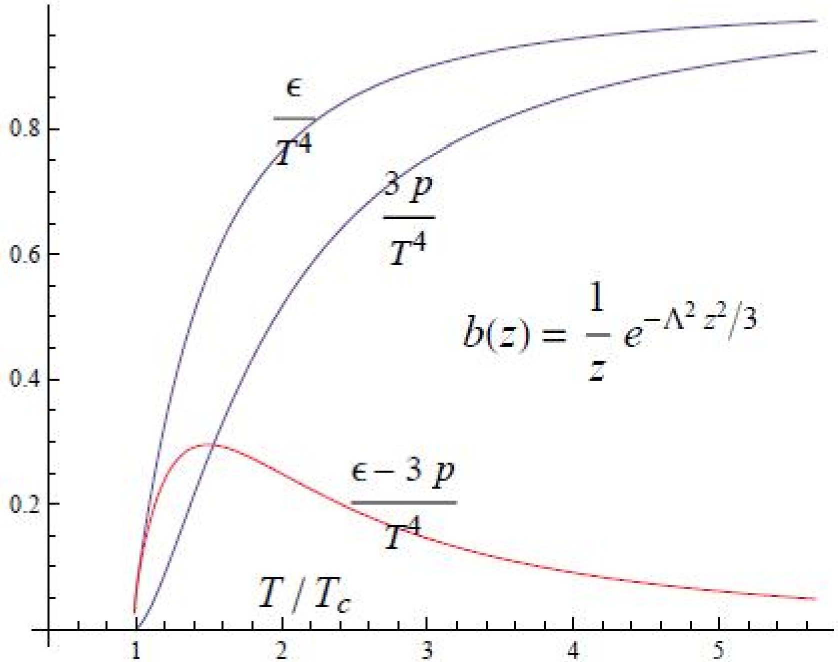
$$s = \frac{1}{4G_5} b^3(z_h)$$

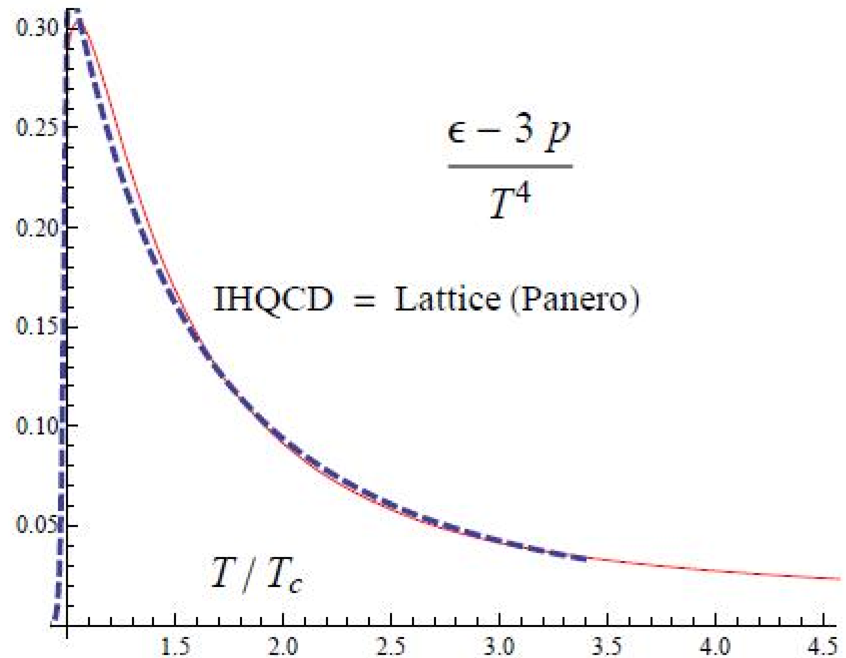
$$= \frac{\mathcal{L}^3}{4G_5}(\pi T)^3$$

In formula for ImG(K)
you **divide**
to correctly normalize!

Here is G(K) of quantum field theory from classical gravity!

# Check Model,IHQCD: thermodynamics, glueballs



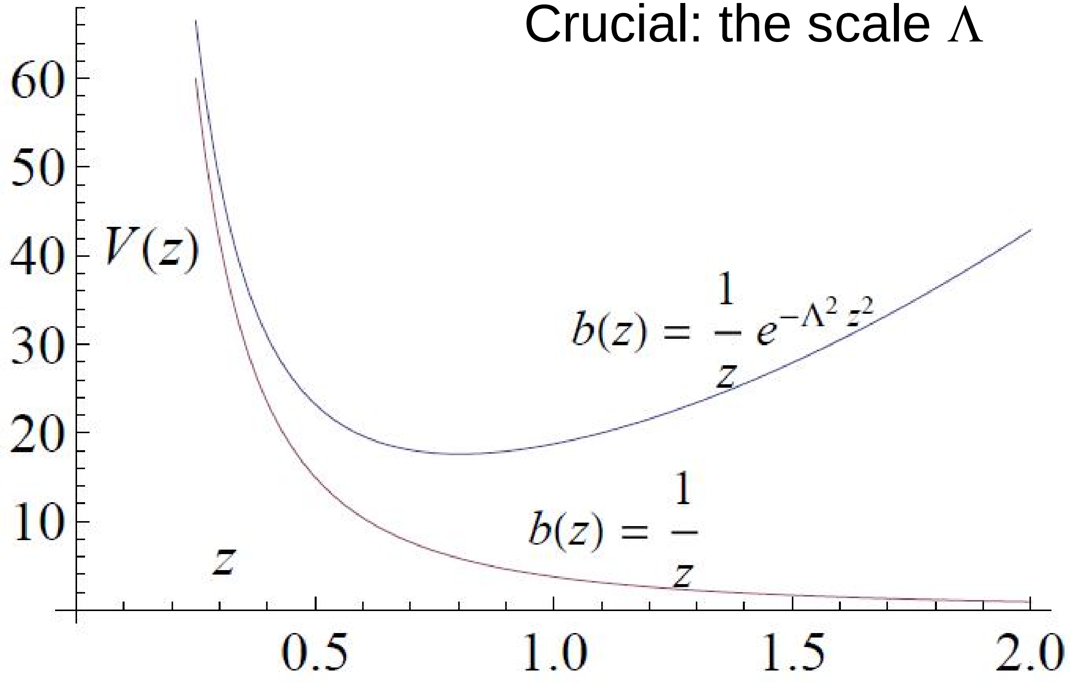

$$T_c = 0.4000\,\Lambda$$

$\frac{\mathcal{L}^3}{4G_5}$ from large T

# Glueballs; why is the model confining?

$$-\psi''(z) + V(z)\psi(z) = m^2\psi(z) \qquad V(z) = \frac{15}{4z^2} + \Lambda^4 z^2$$

Crucial: the scale $\Lambda$



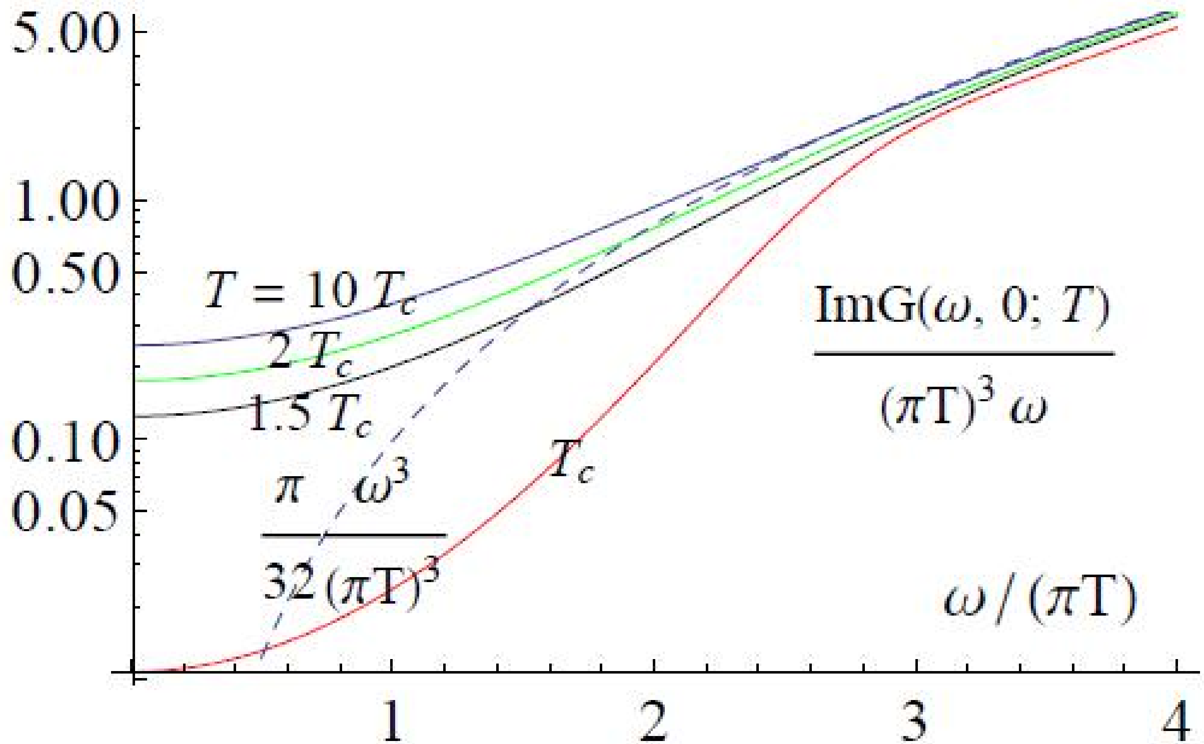

$$m^2 = 4\Lambda^2(2+n)$$
$$= (5T_c)^2(2+n)$$

These are poles of G(K), f(z)=1

# Results: $\rho(\omega, 0; T)$

(energy scales $\pi T \quad 2\Lambda$)

Model



# Reminder:

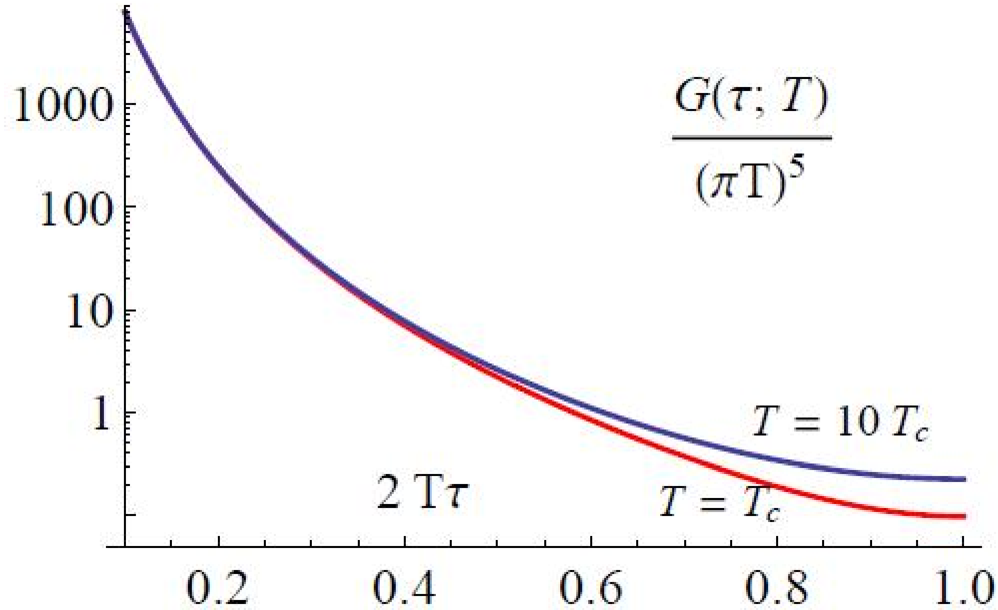
dims: 5 -3 4 4

$$G(\tau, \mathbf{k}=0; T) = \langle \int d^3x \, T_{12}(\tau, \mathbf{x}) T_{12}(0, \mathbf{0}) \rangle_T$$

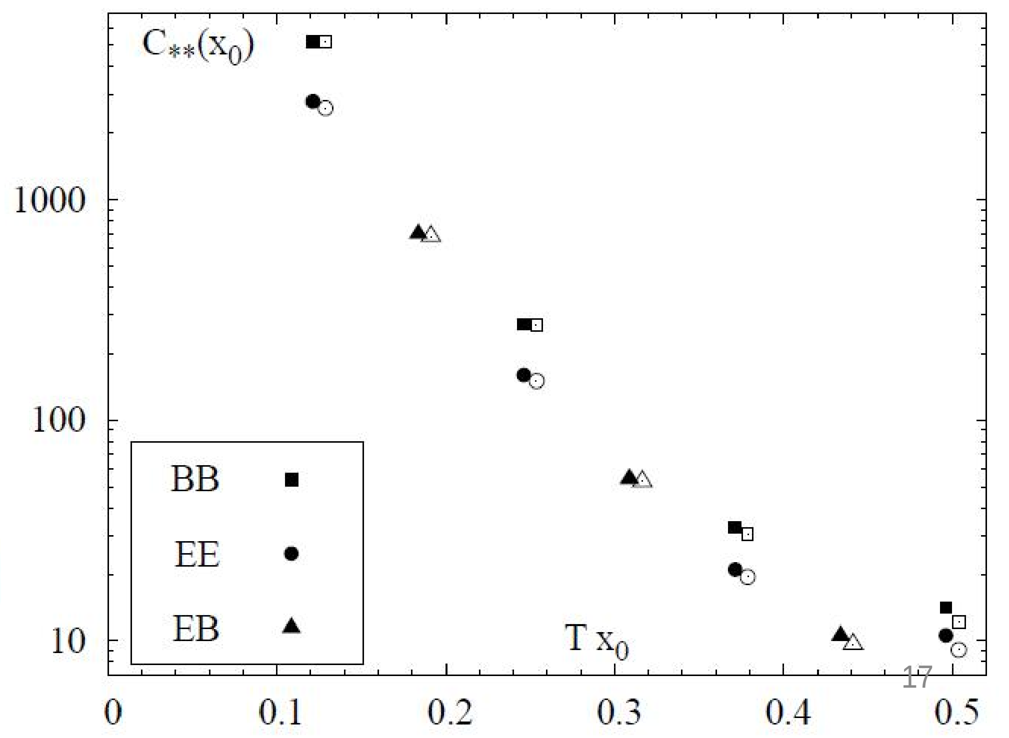
$$G(\tau, \mathbf{k}) = \int_0^\infty \frac{d\omega}{\pi} \rho(\omega, \mathbf{k}) \frac{\cosh(\frac{1}{2}\beta - \tau)\omega}{\sinh \frac{1}{2}\beta\omega}$$

5 1 4

Model




Meyer 0704.1801

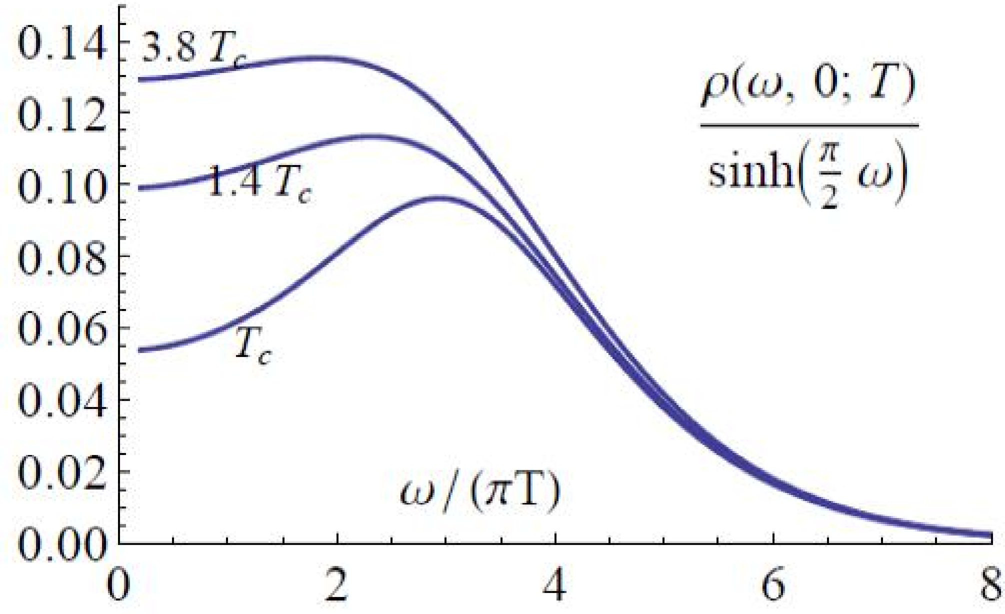



Filled 1.65Tc, open 1.24Tc

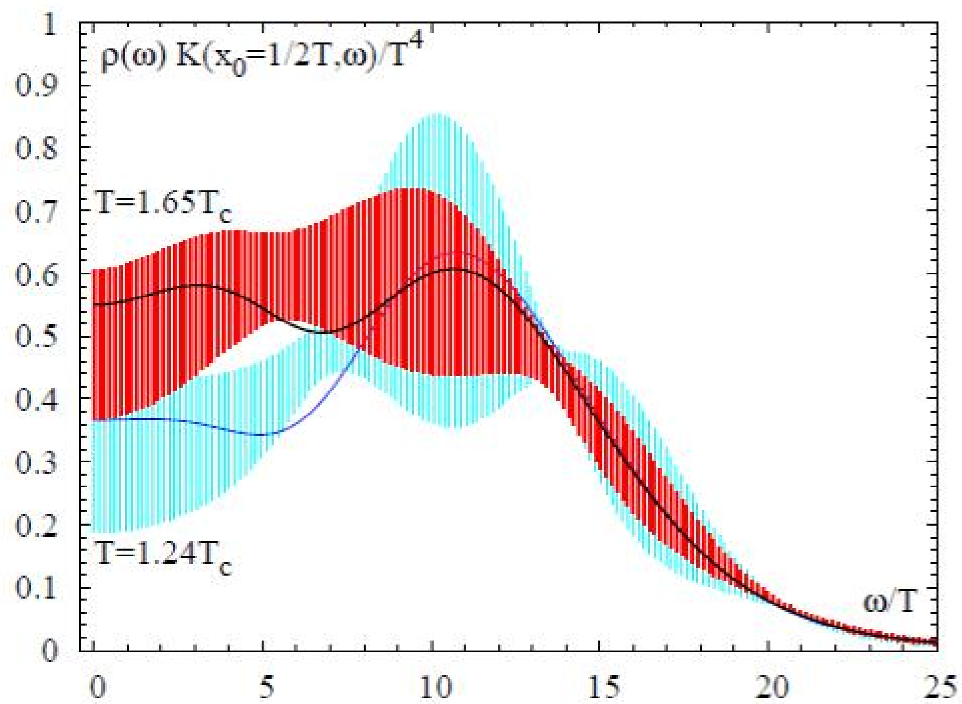
$$C(x_0) = \frac{1}{4}(C_{BB} + C_{EE} + 2C_{EB})$$

$$O(\tau) = \sum_x (P_{10} - P_{20} + P_{13} - P_{23})$$

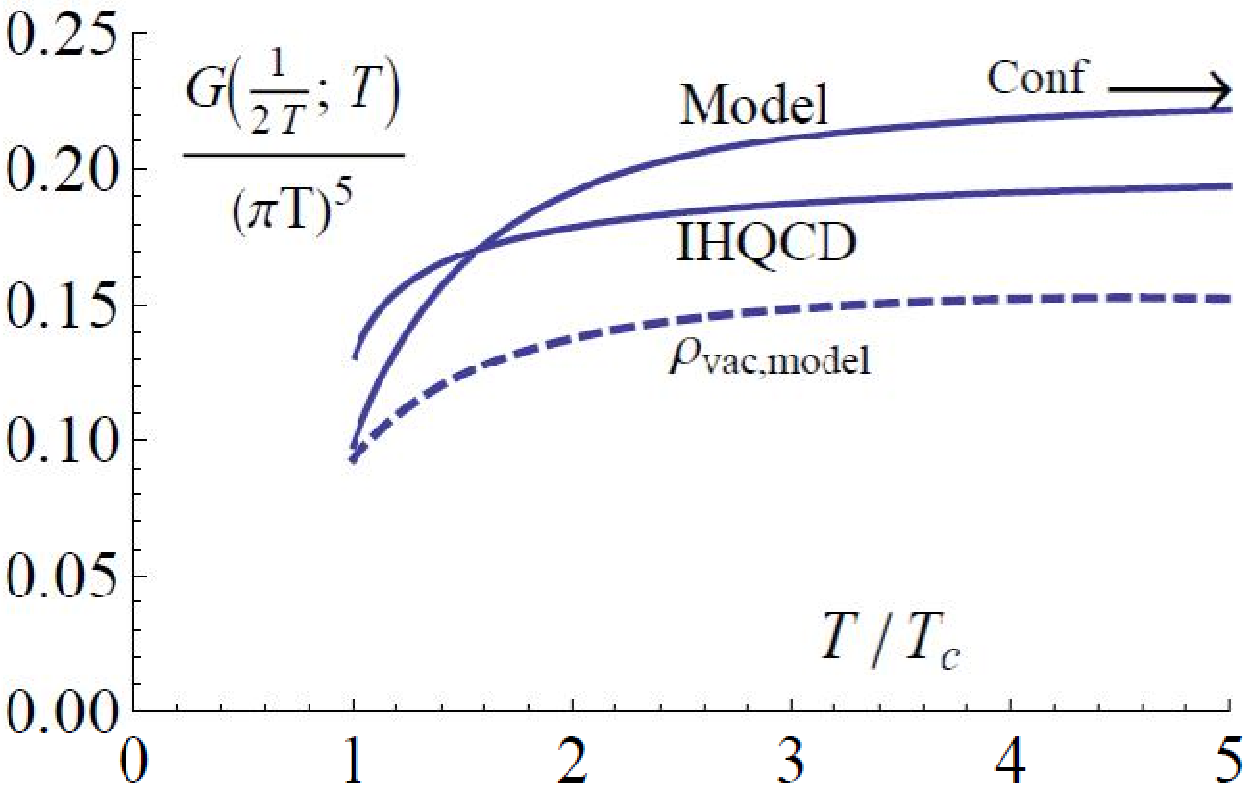
IHQCD




Meyer 0704.1801



Value of imag time correlator at the middle point:




If this really works quantitatively, can we conclude that no transport peak exists, QCD matter really is **s**QGP?

Many more plots of $G(\omega,k;T)$ have been computed, not shown

# Conclusions

We have used AdS/QCD to compute $G(\omega,k)$ for $T_{12}$ using an asymptotically AdS 5d gravity background with first order phase transition and glueballs, all $\omega$,k

Utility: solve QCD matter in the strong coupling domain, where exps are done(?)

Sizable effects of non-conformality are observed; increase when T decreases towards Tc.

We suggest that possible agreement of these computations and lattice MC measurements might shed light on the weak/strong interaction issue

Extension to other correlators possible – fluctuation equation more complicated

Very slow approch to $\frac{\pi}{32}\ \omega^3$ !?

IHQCD