Confining, quasiconformal and conformal gauge theories in gauge/gravity duality

or:

Dynamics of flavour in gauge/gravity duality

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Alanen-Alho-Kajantie-Tuominen 1107.3362 PRD
Järvinen-Kiritsis – Alho-Kajantie-Tuominen, in preparation

Gauge/gravity duality

$$\langle \exp\left[i \int \overset{-4}{d^4x}\, \overset{\Delta_-}{\phi_0(x)}\, \overset{4-\Delta_-}{\mathcal{O}(x)}\right] \rangle$$

$$\exp\left[i \int d^4x \int dz \sqrt{-g}\, \mathcal{L}_{\mathrm{grav}}[g_{\mu\nu}, \ldots, \phi(x,z)]\right]$$

$$\phi(x,z) = \phi_0(x) z^{\Delta_-} + \langle \mathcal{O} \rangle z^{4-\Delta_-} + \ldots$$

Dofs of gravity ~ area, not volume!

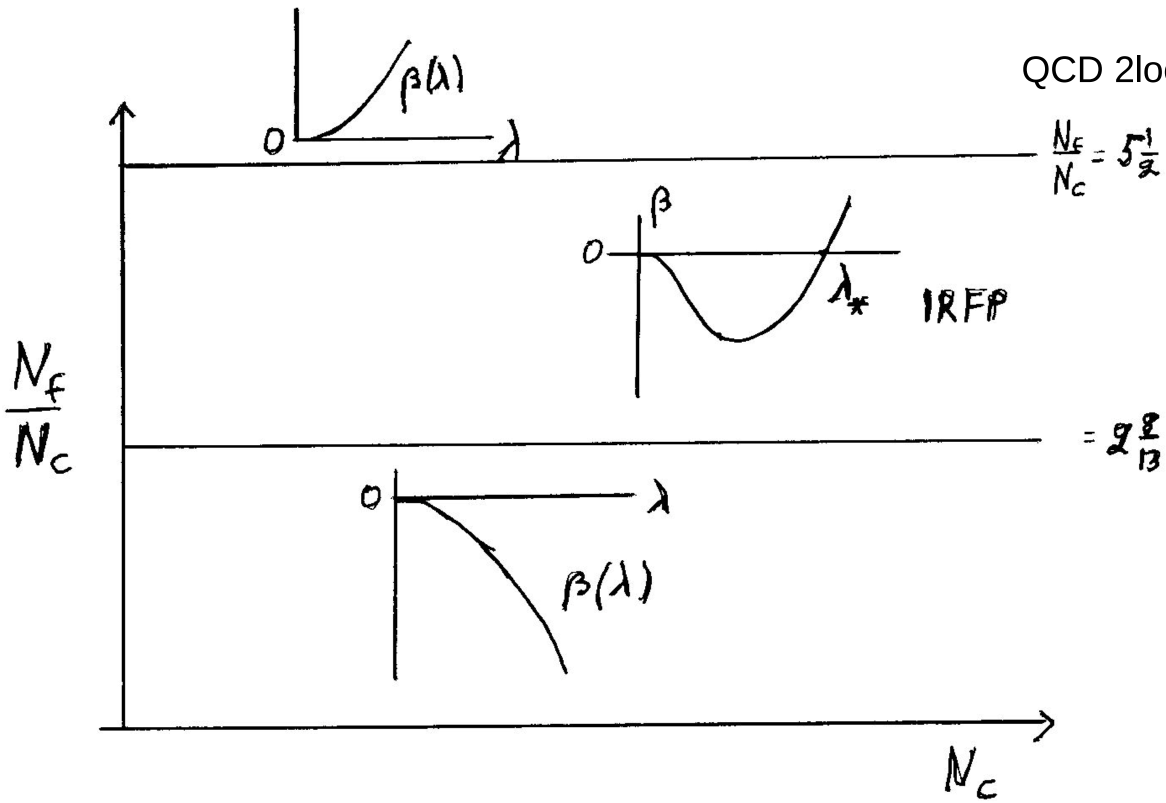
AdS_5 has boundary at z=0 and scale L

N_c, $g^2 N_c$ large

4dim physics one wants to dualize:

SU(N_c) Y-M (confinement, as.freedom) + N_f fermions (chiral symmetry)

QCD 2loop:



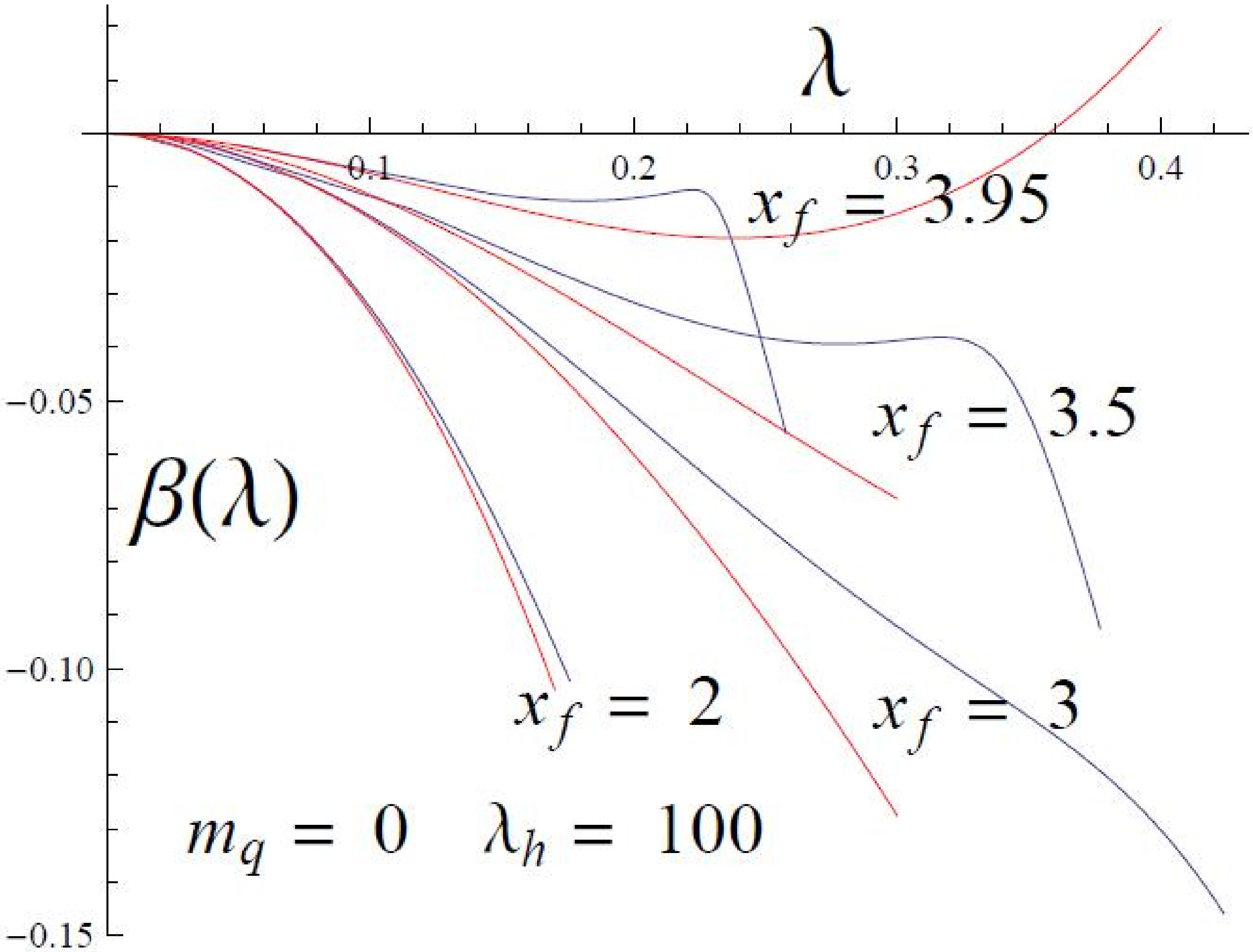
IRFP, conformal

walking, quasiconformal

QCD-like:
-confining
-chiral symm broken

How are the boundaries really in various theories? Conformal window?

Appearance of ”walking” with increasing N_f/N_c:



Below conformal window: quasiconformal, walking technicolor

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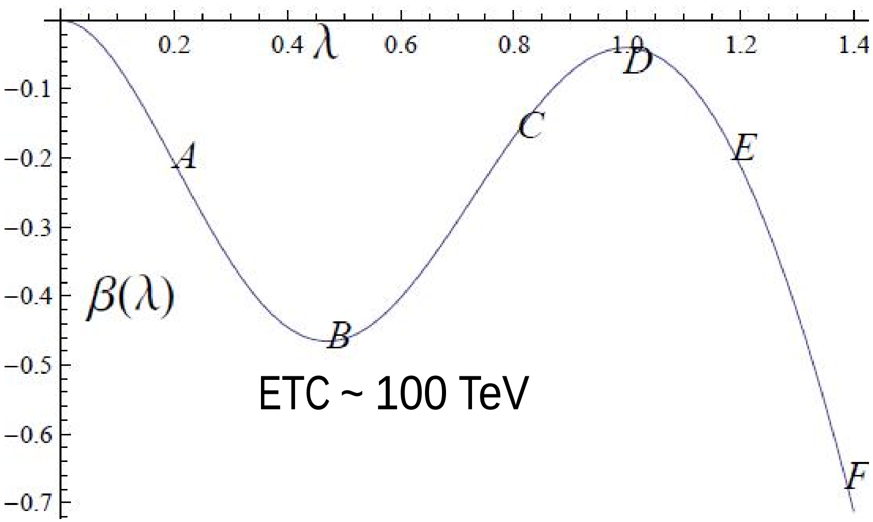
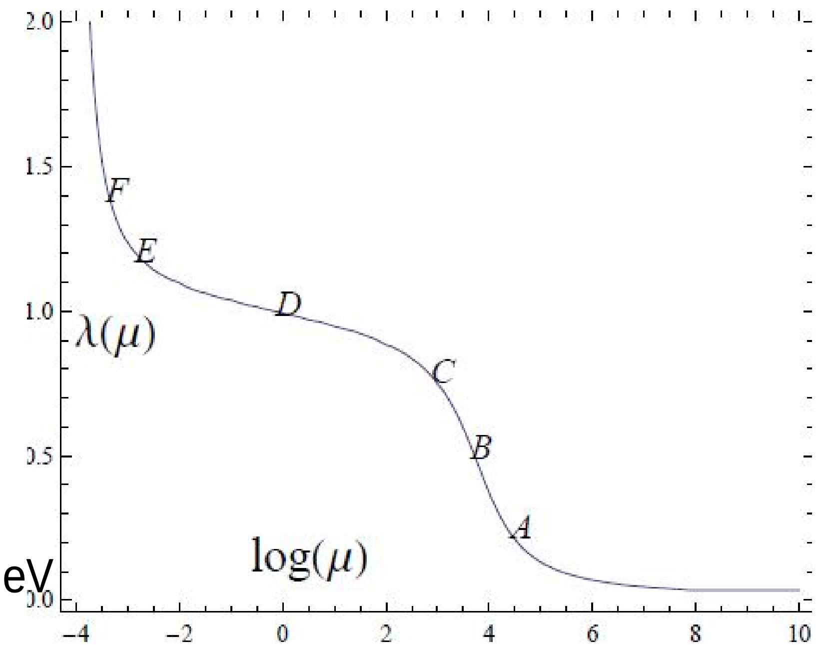
$$\beta(\lambda) = -c\lambda^2 \frac{(1-\lambda)^2 + e(N_f)}{1 + a\lambda^3} \qquad c = 8, \ \ a = 1, \ \ e = 0.01$$

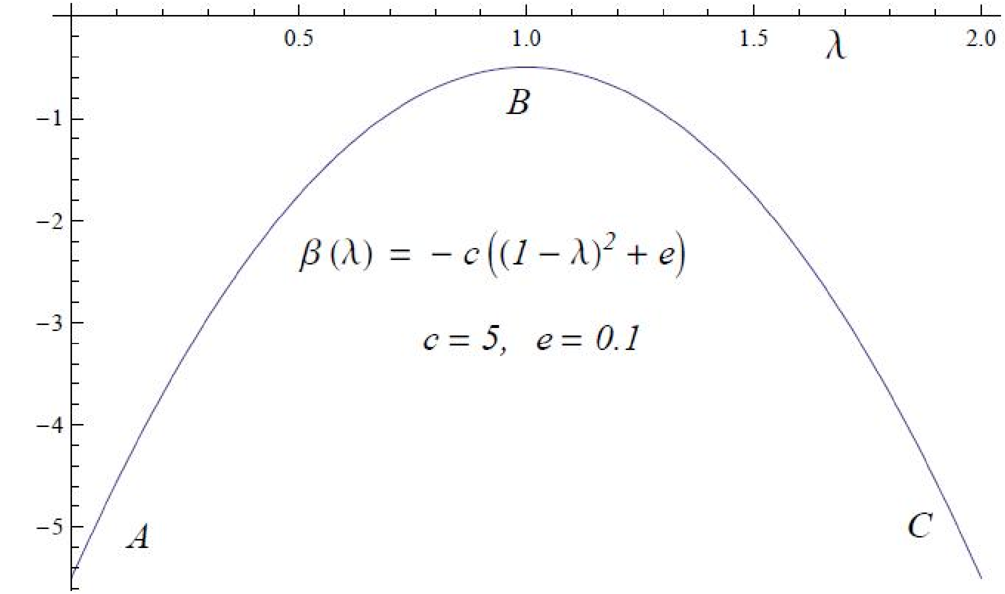


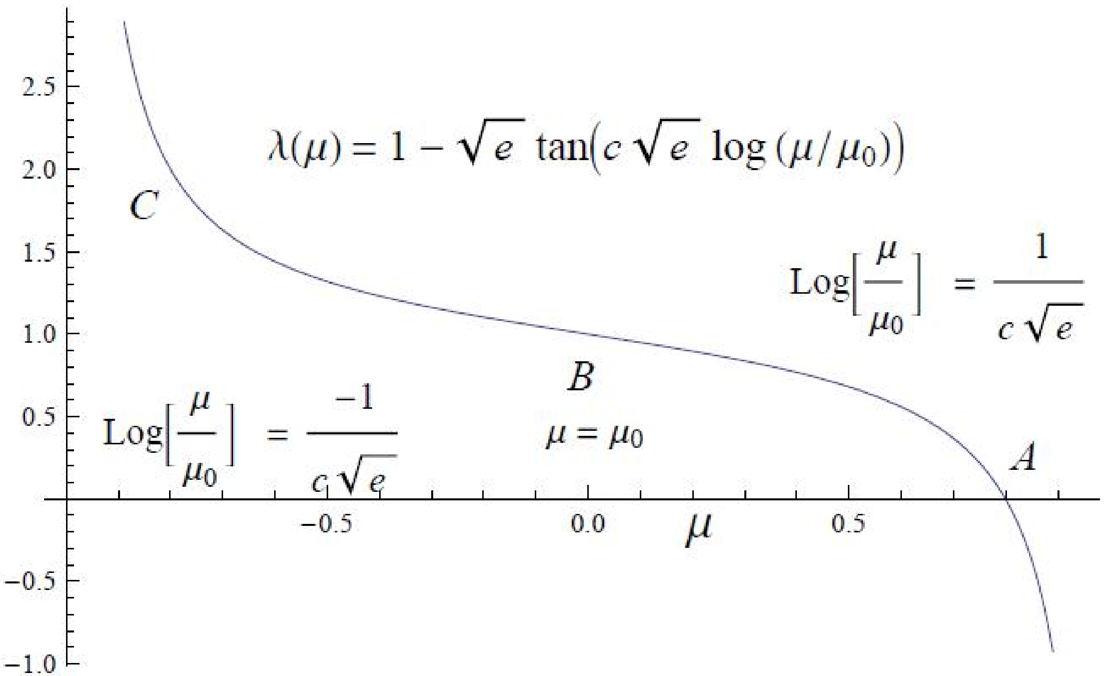
TC ~ 246 GeV



Coupling runs -> condensate walks

Coupling walks -> condensate runs (want this)





Approaching conformality:
KT-Miransky scaling:

$$\frac{d\lambda}{d\log\mu} = -c[(1-\lambda)^2 + e]$$

$$\lambda(\mu) = 1 - \sqrt{e}\tan(c\sqrt{e}\log\frac{\mu}{\mu_0})$$

$$\frac{\mu_{\mathrm{IR}}}{\mu_{\mathrm{UV}}} = C\,e^{-\frac{\pi}{\sqrt{e}}}$$

QCD:

$$Z = \int \mathcal{D}A\mathcal{D}\psi \; e^{-\int d^4x \left[\overset{0}{\frac{1}{g^2(\mu)}}\overset{4}{F^2} + \overset{1}{m(\mu)}\overset{3}{\bar{\psi}\psi}\right]}$$

$$\lambda(\mu) \equiv \frac{N_c g^2(\mu)}{8\pi^2} = e^{\phi(\mu)} \qquad x_f = \frac{N_f}{N_c}$$

$$\mu\frac{d\lambda}{d\mu} = \beta(\mu) = -\underbrace{\frac{11-2x_f}{3}}_{b_0}\lambda^2 - \underbrace{\frac{34-13x_f}{6}}_{b_1}\lambda^3 - ..$$

$$\frac{d\log m}{d\log\mu} = \gamma_m(\mu) = -\frac{3}{2}\lambda - .. = -\gamma_1\lambda - \ldots$$

$$m(\mu) = m_0(\log\mu)^{-\frac{9}{2(11-2x_f)}}$$

1loop

Two operators, F^2 $\bar{\psi}\psi$, need two scalars in bulk

Technical aside:

When string tension grows, strings become points

Closed string theory, containing gravity, becomes supergravity

Open string theory, containing gauge theory, goes to DBI:

Dirac-Born-Infeld

$$\mathcal{L}_{\rm DBI} = -\frac{1}{\ell^4}\sqrt{-\det(g_{\mu\nu} + \ell^2 F_{\mu\nu})}$$

Finite covariant
nonlinear ED

$$= -\frac{1}{\ell^4}\sqrt{1 - \ell^4(E^2 - B^2) - \ell^8(E\cdot B)^2} = \tfrac{1}{2}\,(E^2 - B^2) + \tfrac{1}{2}\,\ell^4(E\cdot B)^2 + \dots$$

$$\ell^2 = 1/T = 2\pi\alpha'$$

Gravity dual

$$S = \frac{1}{16\pi G_5}\int d^5x\,\sqrt{-g}\,\mathcal{L} = S[g_{\mu\nu}, \lambda, \tau]$$

dilaton $\quad \lambda = \lambda(z) = e^{\phi(z)}, \quad \tau = \tau(z), \quad$ tachyon

$$ds^2 = b^2(z)\left[-f(z)dt^2 + d\mathbf{x}^2 + \frac{dz^2}{f(z)}\right]$$

$$\mathcal{L} = R + \left[-\tfrac{4}{3}(\partial_\nu\phi)^2 + V_g(\lambda)\right] - \; x_f V_f(\lambda) e^{-3\tau^2}\sqrt{1 + g^{zz}\dot{\tau}^2}$$

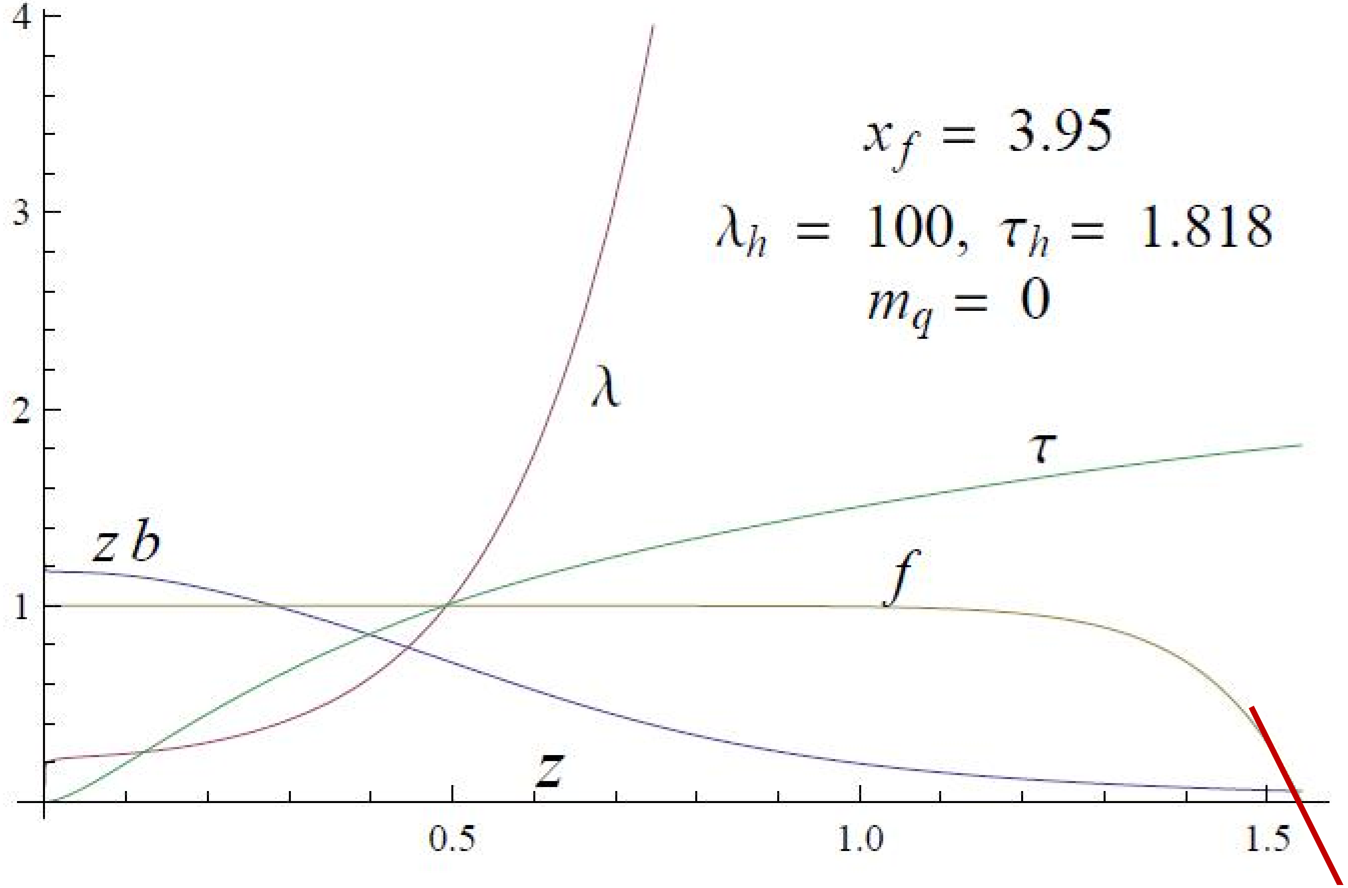
Model 1

Model 2

$$\mathrm{EOM}: \; \frac{\delta S}{\delta g^{\mu\nu}} = 0, \quad \frac{\delta S}{\delta \lambda} = 0, \quad \frac{\delta S}{\delta \tau} = 0$$

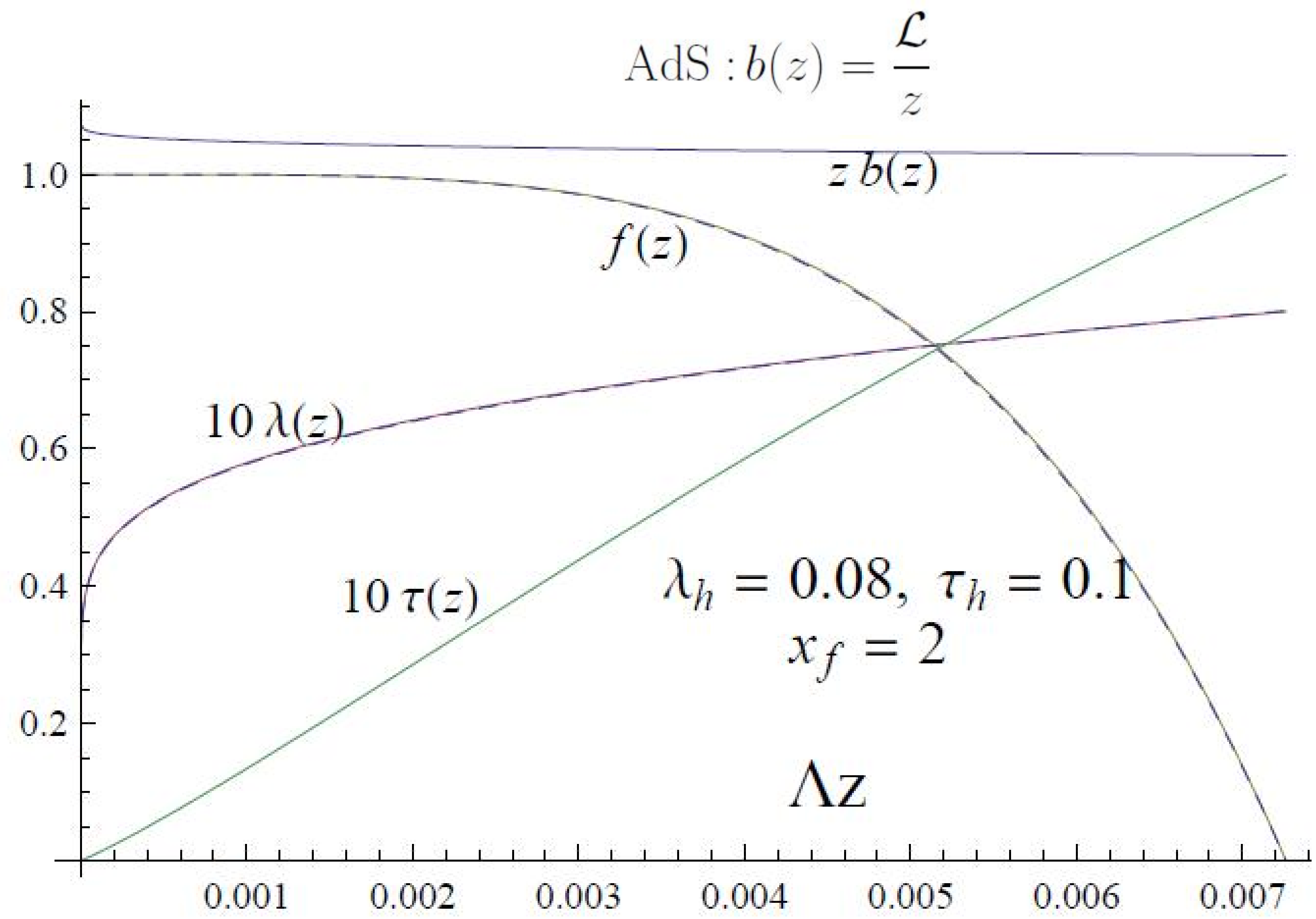
Thermo: $\quad sV_3 = \dfrac{A}{4G_5} = \dfrac{b^3(z_h)}{4G_5} \qquad 4\pi T = -f'(z_h)$

Typical bulk field configuration:



$$4\pi T = -f'(z_h)$$

Another bulk config, large T, nearly conformal $\pi T = \frac{1}{z_h}$



Where is physics hidden?

Field theory scale μ = 1/z, $z \longrightarrow 0: \mathrm{UV} \qquad z \longrightarrow \infty : \mathrm{IR}$

Beta function: $\beta(\lambda) = -z\frac{d\lambda}{dz} \rightarrow b\frac{d\lambda}{db}$ scheme dependent!!

Thermodynamics: $p(T) = \int^T d\bar{T}\, s(\bar{T})$

Green's functions, mass (f=1) or quasinormal mode ($f(z_h)=0$) spectra:

$$-\psi''(z) + \underbrace{\left(\frac{3\ddot{b}}{2b} + \frac{3\dot{b}^2}{4b^2}\right)}_{\frac{15}{4z^2}+2+z^2} \psi(z) = m^2\psi(z)$$

Model 1: build N_f dependence in the beta function

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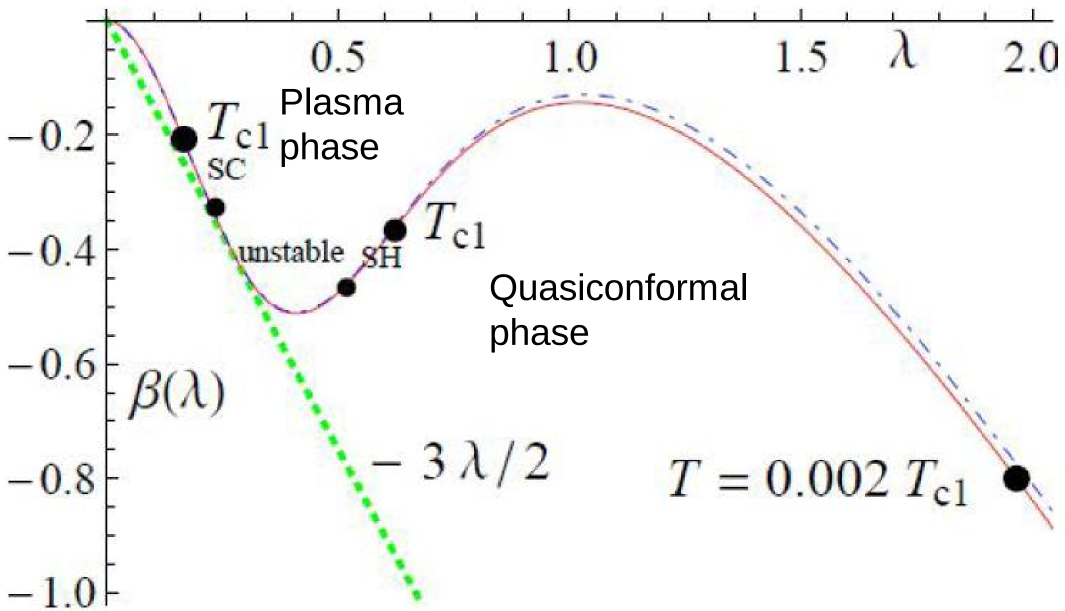
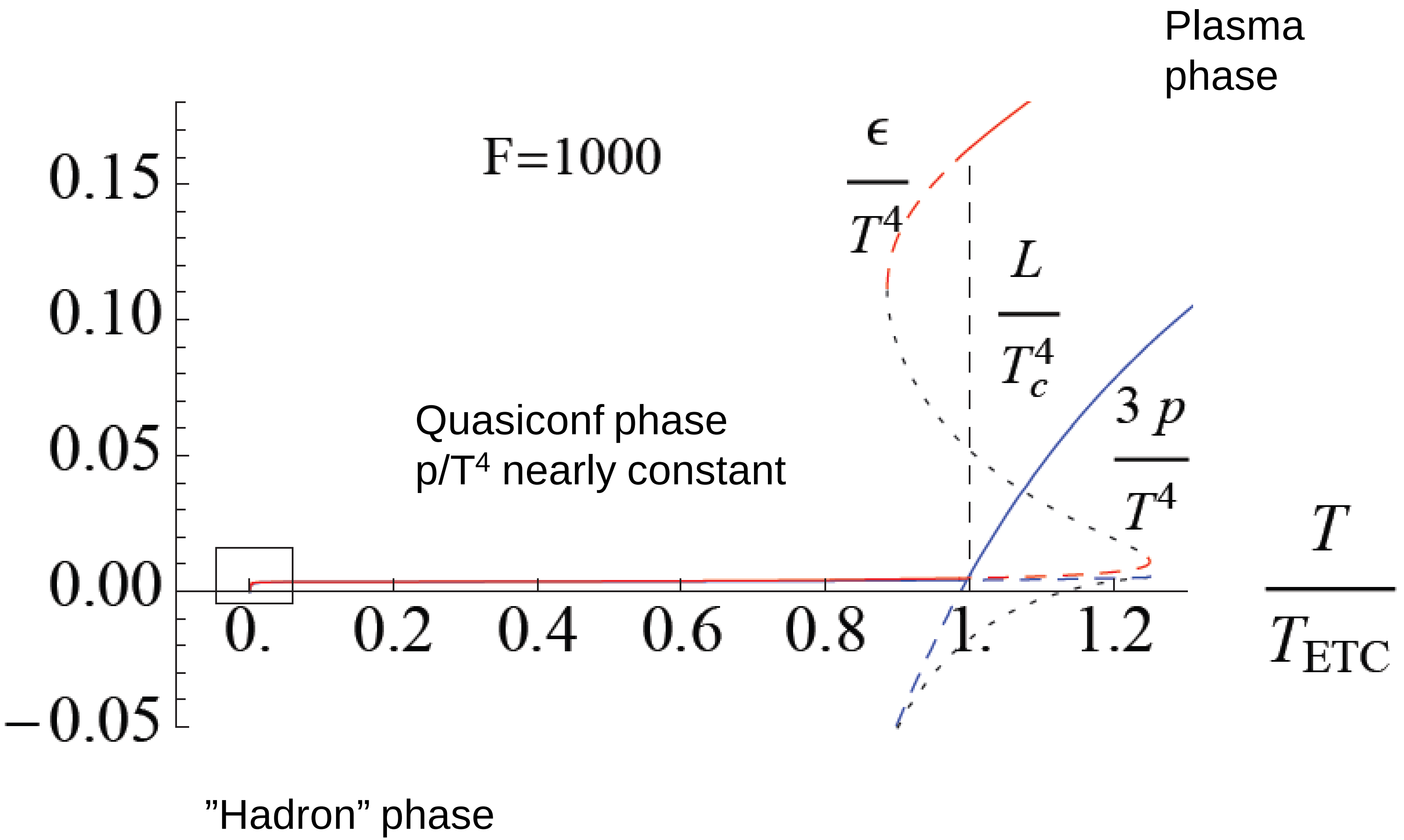
$$\beta(\lambda) = -c\lambda^2 \frac{(1-\lambda)^2 + e}{1 + \frac{2}{3}c\,\lambda^3}$$

Fix two scales, $\Lambda_{\rm ETC} \sim 10^3 \Lambda_{\rm TC}$ 2 transitions, 3 phases

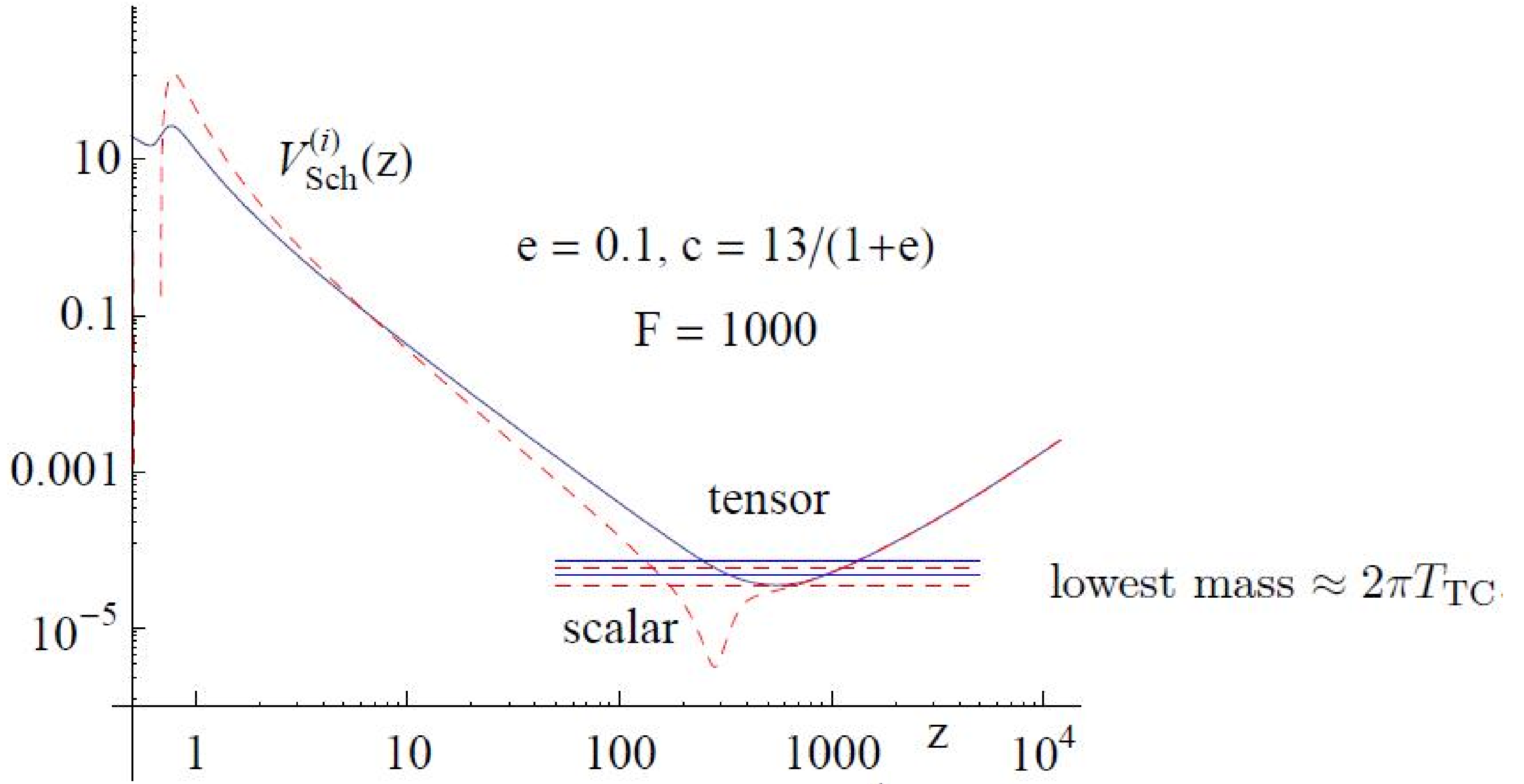


Confining, chiral
symm broken phase



Plasma
phase
F=1000
$\frac{\epsilon}{T^4}$
$\frac{L}{T_c^4}$
$\frac{3p}{T^4}$
Quasiconf phase
p/T⁴ nearly constant
0.15
0.10
0.05
0.00
−0.05
0.
0.2
0.4
0.6
0.8
1.
1.2
$\frac{T}{T_{ETC}}$
"Hadron" phase

Stable states on the TC level:

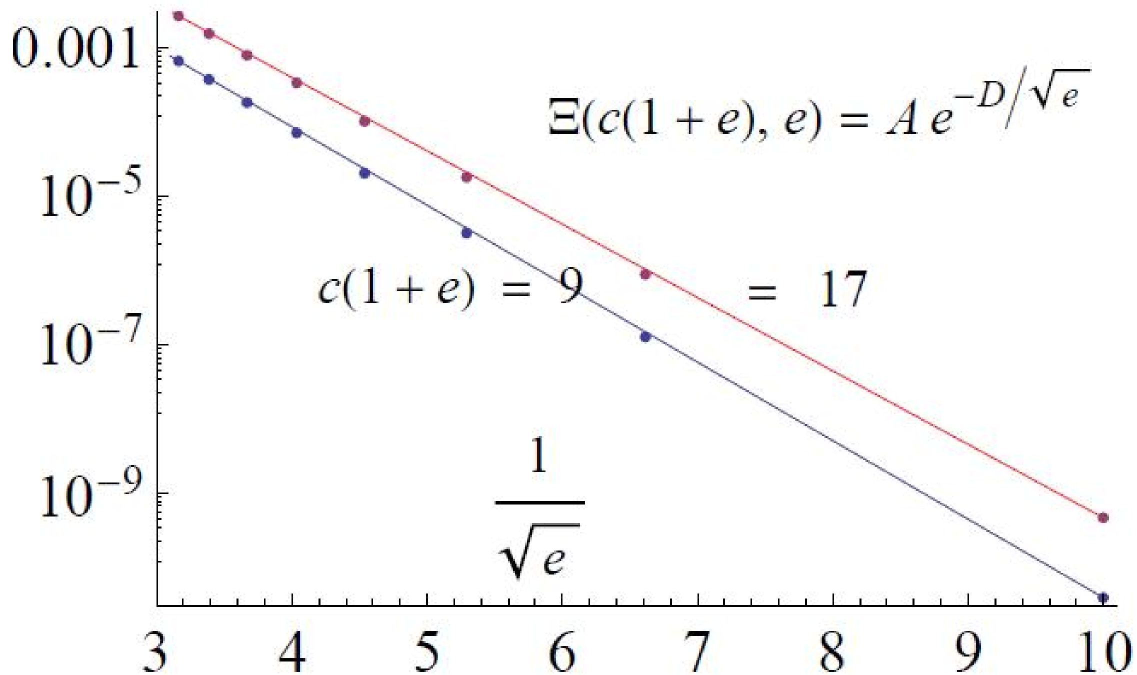


$$\Xi(c,e) = \frac{1}{z_{\rm min}}$$

The lowest scalar is effectively the Higgs: we hoped it would be very light; the potential is deep but too narrow to bind a very light state!

When e approaches 0 all masses should also approach zero, conformality

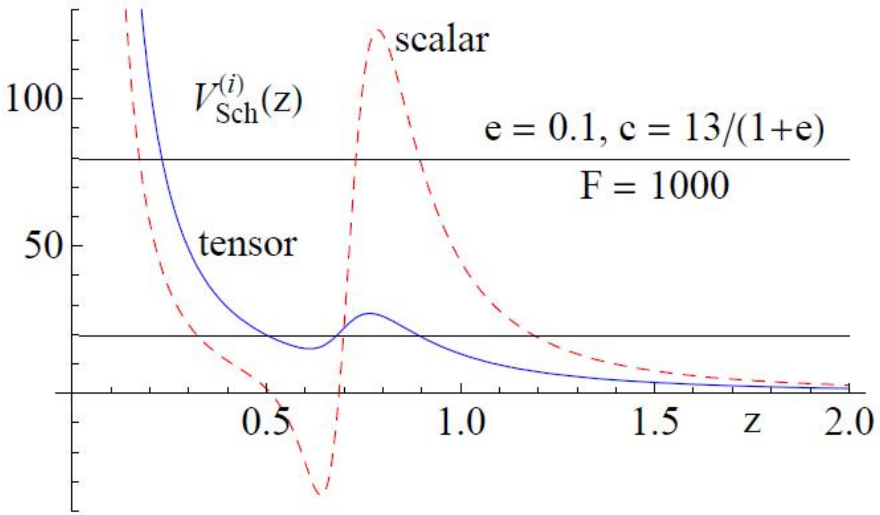
4d prediction: $\exp\left[-\left(\frac{2}{3}+\frac{1}{c}\right)\frac{\pi}{\sqrt{e}}\right]$



5d computation

Path to this prediction: ansatz for 5d bulk metric, solve numerically Einstein’s equations, solve numerically scalar field equations in this background, compute eigenvalues of Schrödinger equation. Striking that the result is as predicted!

States on the ETC level:

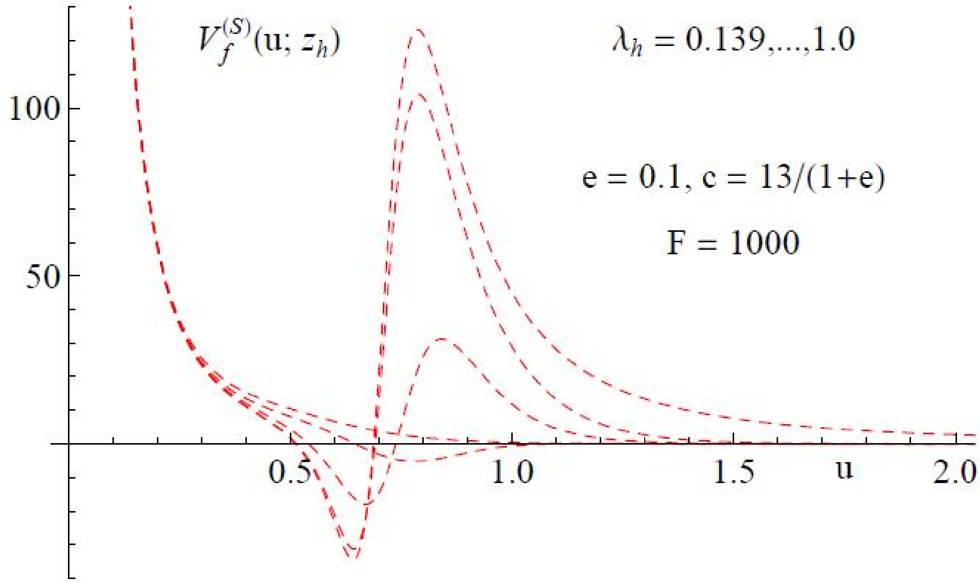


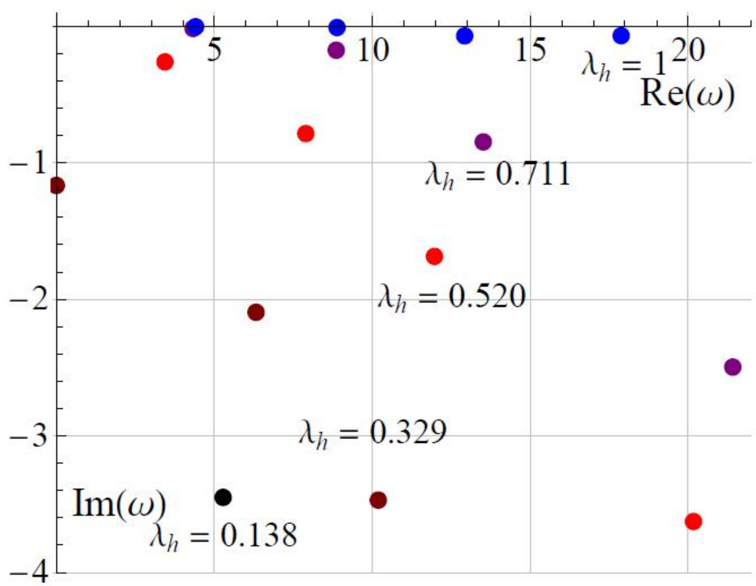
Two candidate scalar states; the other side of the potential is at $z = 10^6$

n=44410

If $f(z_h)=0$ there is no other side of the potential and these states become quasinormal modes, they have imaginary part

Quasinormal modes: hit a black hole, how does it oscillate?





time = R_{Schw}/c
strongly damped!

T grows,
peak
disappears

states get
big Im(energy)

Physics: thermalisation

Model 2: Explicit N_f dependence

Both dilaton and tachyon: confinement and chiral symmetry

Key relation:

$$\tau(z) = m_q(z) z + \langle \bar{q} q \rangle z^3 + ..$$

In progress

Conclusions

Gauge-gravity duality has some monumental predictions ($\eta/s=1/(4\pi)$)

Other good predictions are of type ”find background fitting thermodynamics, calculate correlators in this background”

Works beautifully for special theories

Not a theory but a collection of models

A good method for generating complicated and subtle formulas for curve fitting