AdS/QCD and hot QCD matter

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Bielefeld, 10 December 2009

"Hot QCD matter" is finding p(T) from the integral:

$$Z(T,V) = e^{p(T)\frac{V}{T}} = \int \mathcal{D}[A\bar{\psi}\psi] e^{-\int_0^{1/T} d\tau d^3x \, \mathcal{L}_{\rm QCD}}$$

$$\mathcal{L}_{\rm QCD} = \frac{1}{4g^2} \sum_{a=1}^{N_c^2-1} F_{\mu\nu}^a F_{\mu\nu}^a + \sum_{i=1}^{N_f} \bar{\psi}_i [\gamma_\mu D_\mu + m_i] \psi_i$$

1. Lattice: $N_t \cdot N_s^3$ $\qquad U_\mu(x) = e^{igaA_\mu(x)}$

$$\frac{1}{T} = N_t a \ll N_s a = V^{1/3}$$

Derivatives of Z are expectation values;

Trace of en-mom tensor:

$$\frac{-1}{VT^3}\, a\frac{d\log Z}{da} = \frac{\epsilon - 3p}{T^4} = T\frac{\partial}{\partial T}\frac{p(T)}{T^4}$$

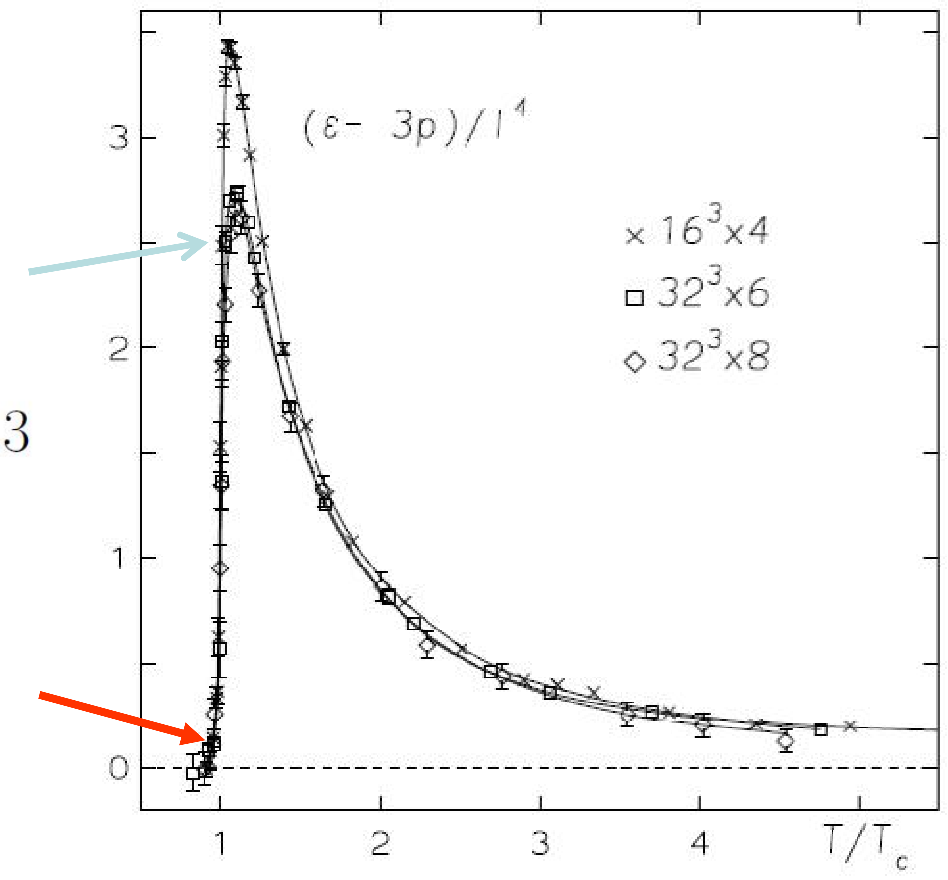
Determine this and by integration $p(T)$

$$s(T) = p'(T), \quad \epsilon(T) = Ts - p$$

1996, pure SU(3): Boyd-Engels-Karsch-Laermann…

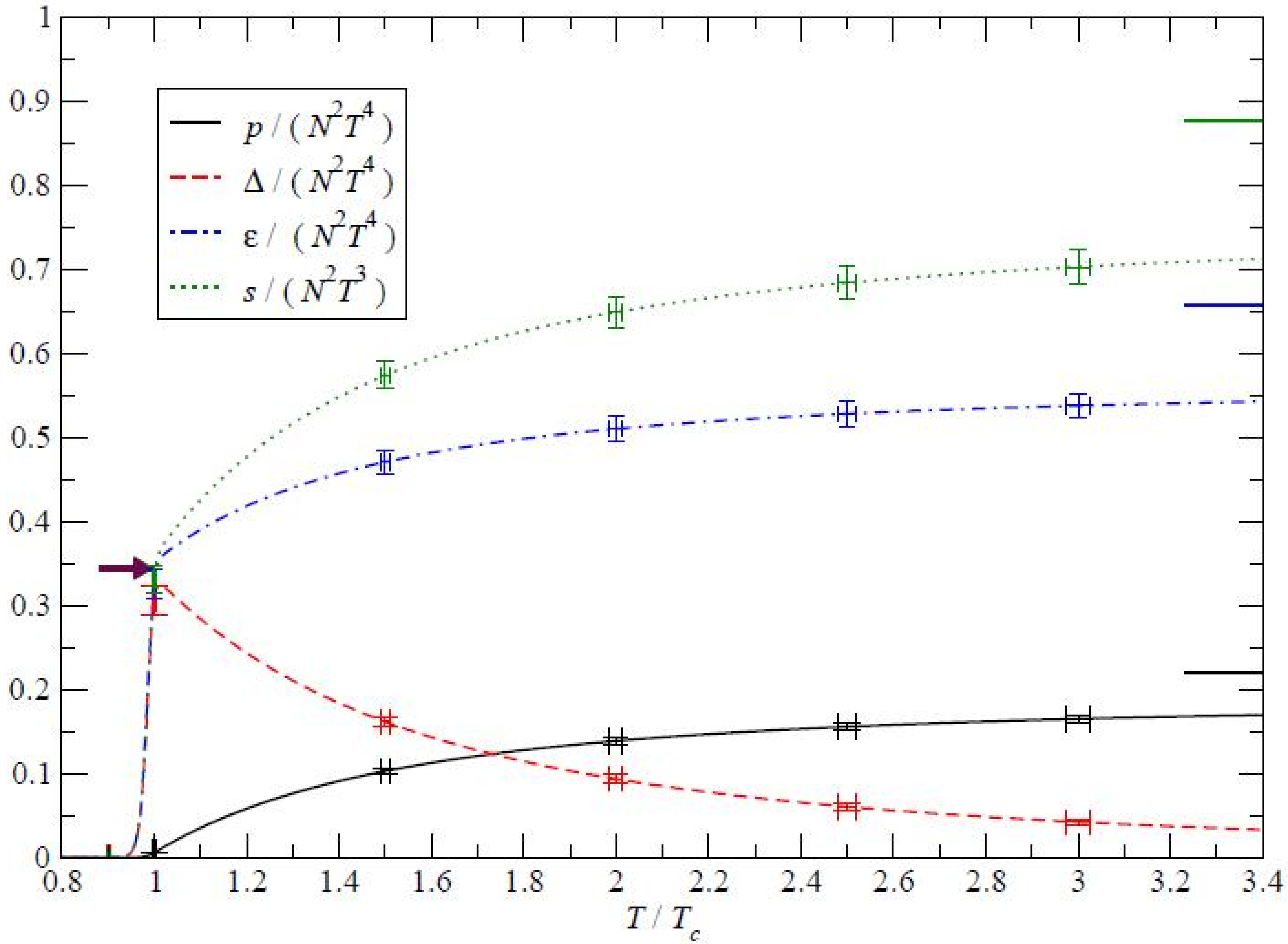
1st order transition

$$\frac{L}{T_c^4 N_c^2} \approx 0.3$$



Extrapolation to the large-N limit

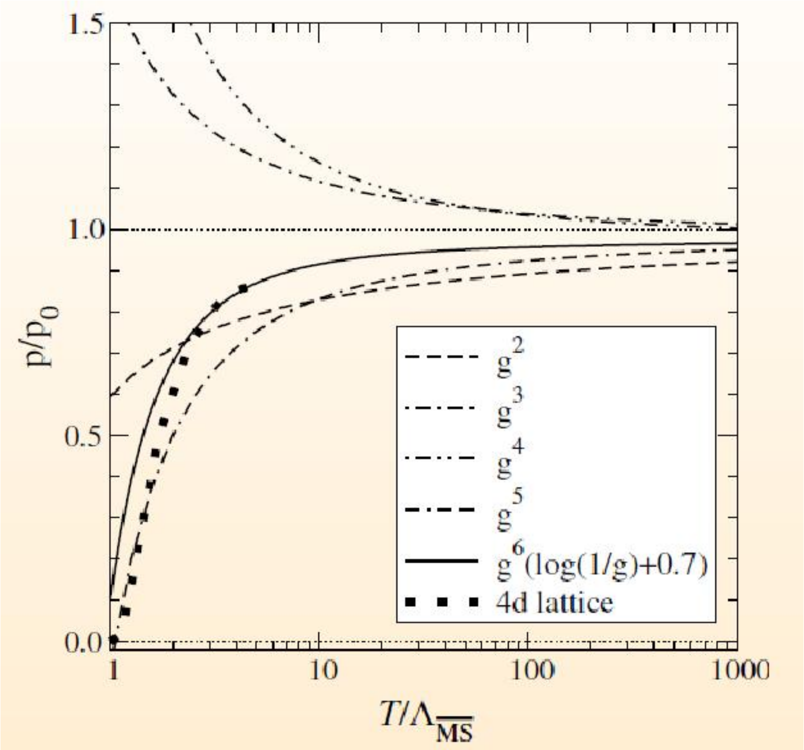
Panero 0907.3719



2. Perturbation theory, large T

$$c_{\rm SB}+c_2g^2+c_3g^3+(c_4'\log g+c_4)g^4+c_5g^5+(c_6'\log g+c_6)g^6+c_7g^7+..$$

c_2 Shuryak 78, c_3 Kapusta 79, c_4' Toimela 83, c_4 Arnold-Zhai 94,
c_5 Zhai-Kastening, Braaten-Nieto 95, c_6' Kajantie-Laine-Rummukainen-Schröder 03



-there is the confining magnetic sector

-pert theory converges slowly

-experiments!

↓

a strongly coupled system

↓

AdS/QCD

3. Operational presentation of computing p(T) from AdS/QCD

Gürsoy-Kiritsis-Mazzanti-Nitti 0903.2859 Alanen-Kajantie-SuurUski 0911.2114

- add 5th dimension z > 0, z=0 is boundary

- write down Einstein gravity for a metric+scalar ansatz:

$$S = \frac{1}{16\pi G_5}\int d^5x\,\sqrt{-g}\left[R - \tfrac{4}{3}(\partial_\mu\phi)^2 + V(\phi)\right] \quad V(0) = \frac{12}{\mathcal{L}^2}$$

$$ds^2 = b^2(z)\left[-f(z)dt^2 + d\mathbf{x}^2 + \frac{dz^2}{f(z)}\right] \qquad \lambda(z) = e^{\phi(z)} \sim N_c g^2$$

flat BH

- find solutions which are "asymptotically (z-> 0) AdS"

$$ds^2 = \frac{\mathcal{L}^2}{z^2}\left[-dt^2 + d\mathbf{x}^2 + dz^2\right] \qquad \lambda(z) = 0$$

and which have a black hole: horizon, Hawking T, entropy:

$$f(z_h) = 0 \qquad 4\pi T = -f'(z_h) \qquad S = \frac{A}{4G_5} = \frac{1}{4G_5} b^3(z_h) V_3$$

- compute p(T) from

$$p(T) = \int^T dT\, s(T)$$

- there are two phases, one with f = 1, s = p = 0 and one with f(z) nontrivial. The latter one is stable when p > 0, phase transition at

$$p(T_c) = 0$$

Need three eqs for b(z), ϕ(z), f(z)

AKS

$$6\frac{\dot{b}^2}{b^2}+3\frac{\ddot{b}}{b}+3\frac{\dot{b}}{b}\frac{\dot{f}}{f}=\frac{b^2}{f}V(\phi)$$

$$6\frac{\dot{b}^2}{b^2}-3\frac{\ddot{b}}{b}=\frac{4}{3}\dot{\phi}^2,$$

$$\frac{\ddot{f}}{\dot{f}}+3\frac{\dot{b}}{b}=0,$$

$$\beta(\lambda)=b\frac{d\lambda}{db}$$

GKMN

Start from $V(\phi)=$
$$\frac{12}{\mathcal{L}^2}\left\{1+V_0\lambda+V_1\lambda^{4/3}[\log(1+V_3\lambda^2)]^{1/2}\right\}$$

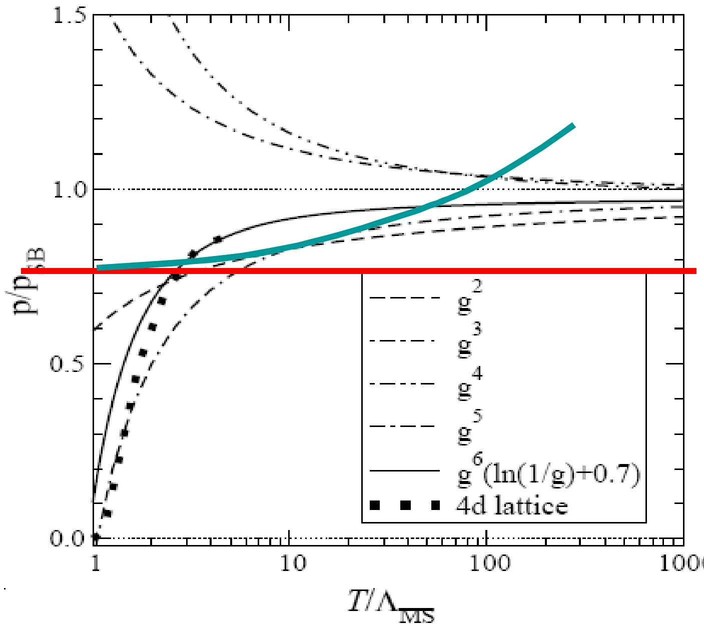
Start from the beta fn of bdry field theory;
λ runs with $b(z)\sim\mathcal{L}/z$ as energy scale

Conformally invariant solution: $p = aT^4$

$$b(z) = \frac{\mathcal{L}}{z} \qquad f(z) = 1 - \frac{z^4}{z_h^4} \qquad \lambda = 0 \qquad \beta(\lambda) = 0$$

$$\pi T = \frac{1}{z_h} \qquad S = \frac{A}{4G_5} = \frac{1}{4G_5}\frac{\mathcal{L}^3}{z_h^3} \cdot V_3 = \frac{\pi^2 N_c^2}{2} T^3 \cdot V_3$$

The famous ¾:



Beta functions:

IR fixed pt

$$\beta(\lambda) = -\beta_0 \lambda^q \qquad \beta(\lambda) = \frac{-\beta_0 \lambda^q}{1 + \frac{2}{3}\,\beta_0 \lambda^{q-1}} \qquad \beta(\lambda) = -\beta_0 \lambda^2 (1 - \lambda)$$

$$\beta(\lambda) \;=\; \frac{-\beta_0 \lambda^q}{1 + \frac{2}{3}\,\beta_0 \lambda^{q-1}} \left[1 + \alpha (q-1) \frac{\log(1 + \frac{2}{3}\,\beta_0 \lambda^{q-1})}{\log^2(1 + \frac{2}{3}\,\beta_0 \lambda^{q-1}) + 1} \right]$$

Logic of this monster: GKMN have shown that in the IR

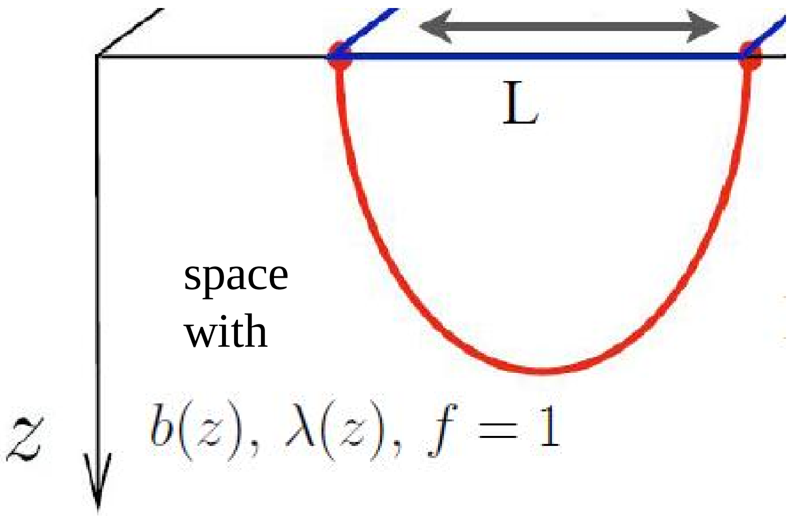
$$\beta \to -\frac{3}{2}\,\lambda \left(1 + \frac{\alpha}{\log \lambda} \right) \qquad \alpha > 0$$

in confining theories. We find q=10/3, α = ¼ gives good thermo

$\alpha = 0$: continuous transition

How does confinement enter?

$$V(L) = \sigma L$$



Condition for $L \to \infty$ is

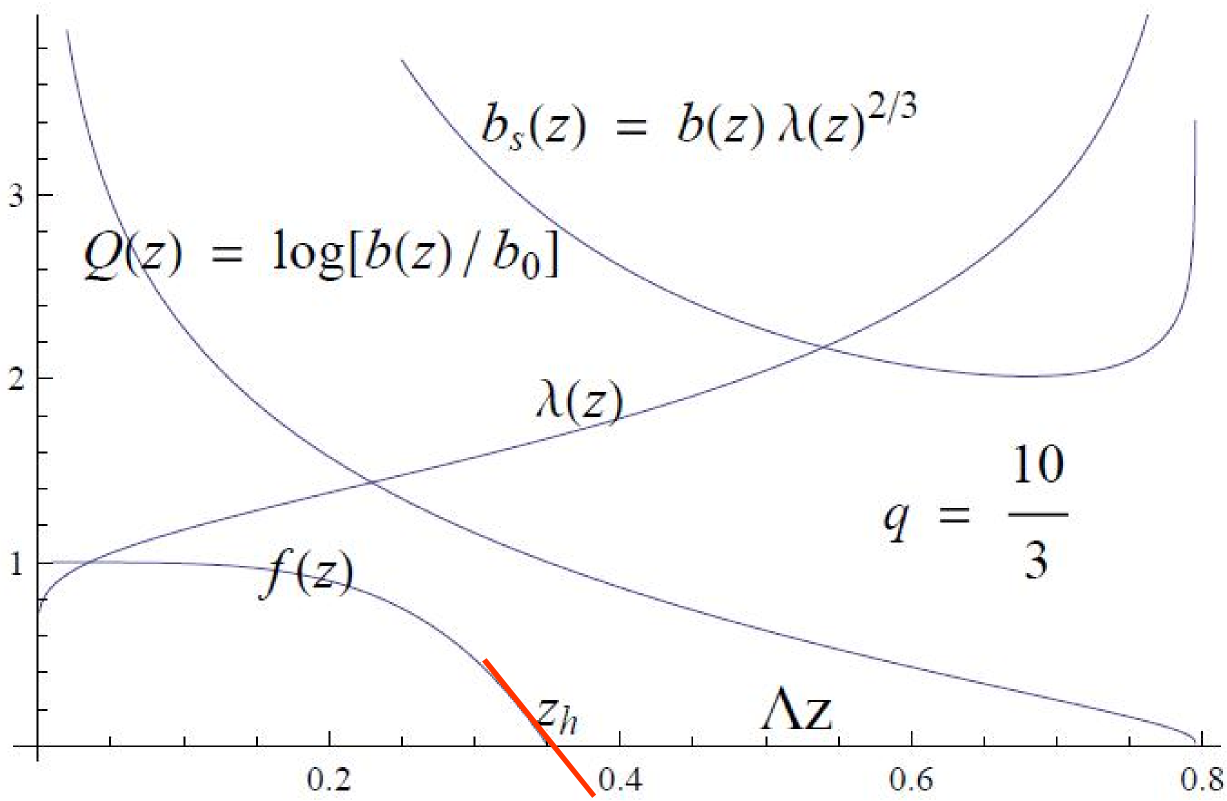
$b(z)\lambda^{2/3}(z)$ have a minimum at some z_{min}

$$\frac{db}{b} + \frac{2}{3}\frac{d\lambda}{\lambda} = 0$$

$$\beta(\lambda_{min}) = -\frac{3}{2}\lambda_{min} \qquad !!$$

Typical field configs:

$$Q = \int^{\lambda} \frac{d\lambda}{\beta(\lambda)} = \frac{1}{(q-1)\beta_0 \lambda^{q-1}}$$



$$\pi T = \frac{1}{z_h} = \text{ big}$$

$$4\pi T = -f'(z_h)$$

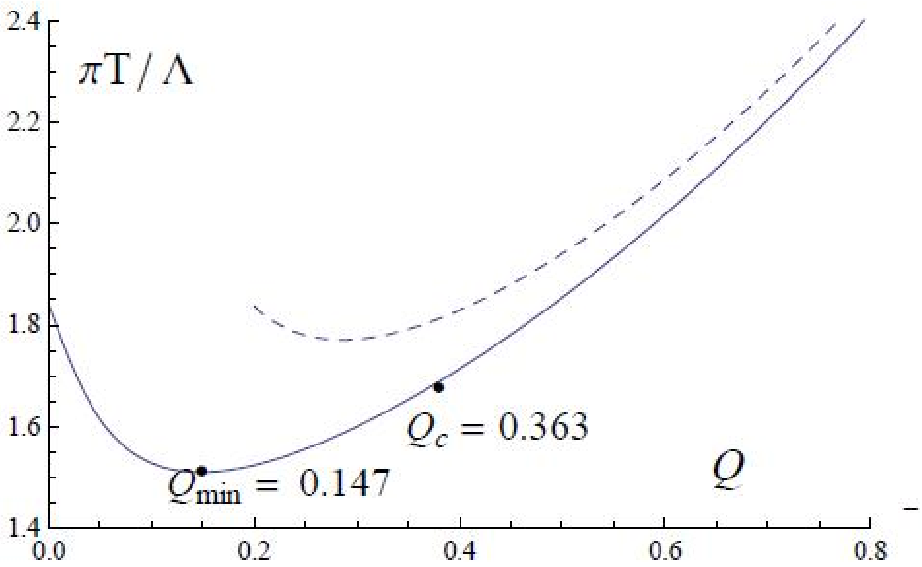
T big again!

T minimum!

UV

spherical BH $\pi T = \frac{1}{2r_h} + \frac{r_h}{\mathcal{L}^2}$

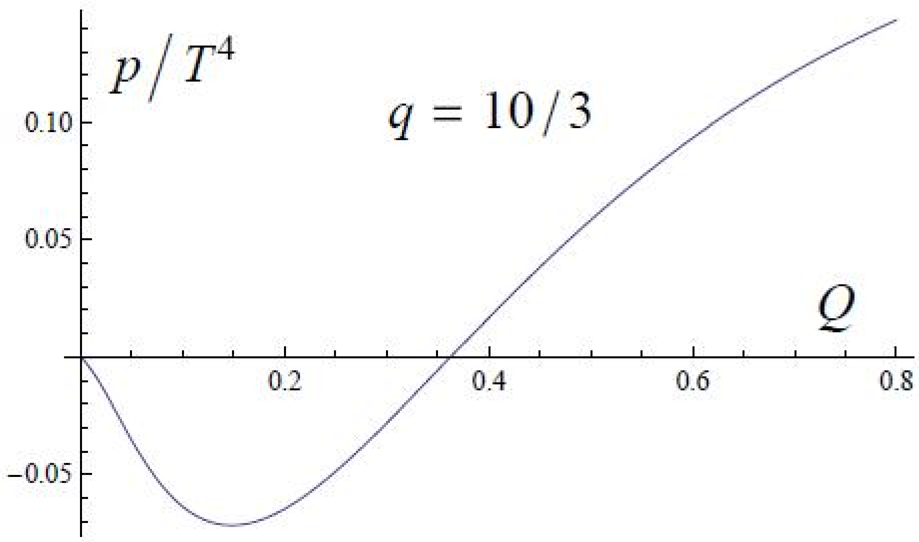
IR



$$p(T) = \int^T dT\, s(T)$$

$$\sim \int_0^Q dQ \frac{dT}{dQ} b^3(Q(T))$$

starts negative!!



$$p(Q_c) = p(Q(T_c)) = 0$$

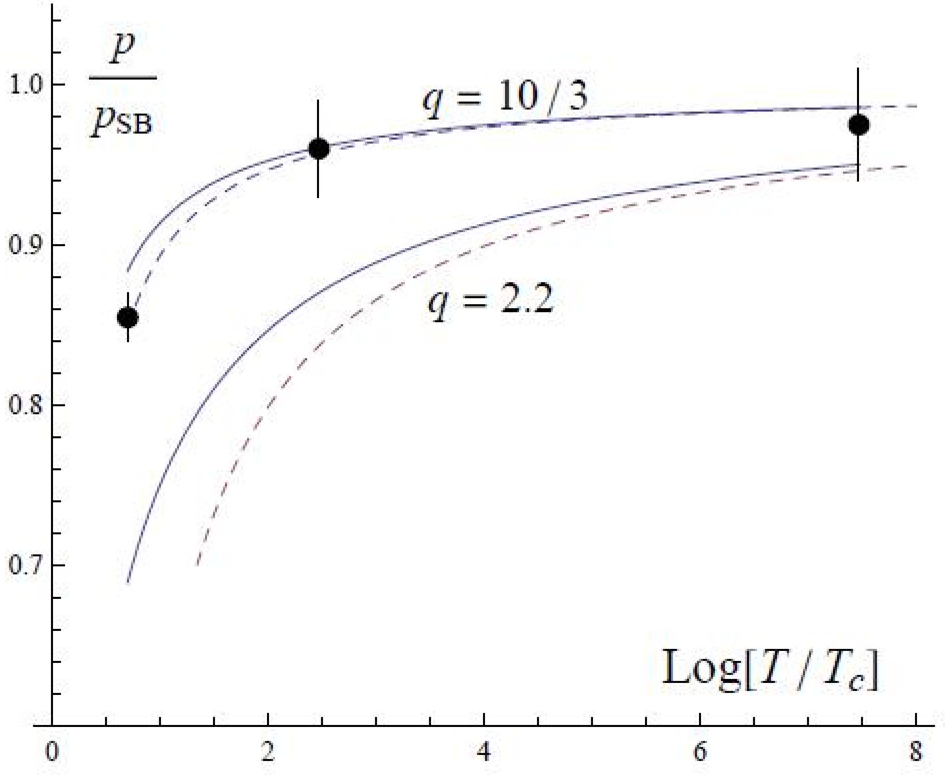
Now you have T_c !
but in units of Λ!

p/T^4 in units of $\mathcal{L}^3/G_5$

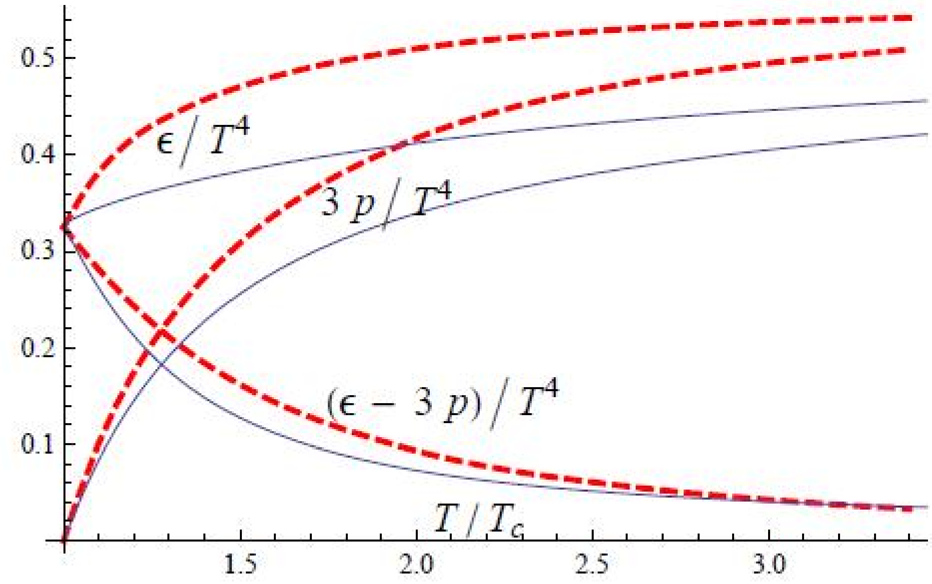
Try the very simple $\beta(\lambda) = -\beta_0 \lambda^q$

value of β_0 never enters, only Q!

Large T:
q=10/3 looks fine but L too big!



$$\beta(\lambda) = -\beta_0 \lambda^{2.2}$$

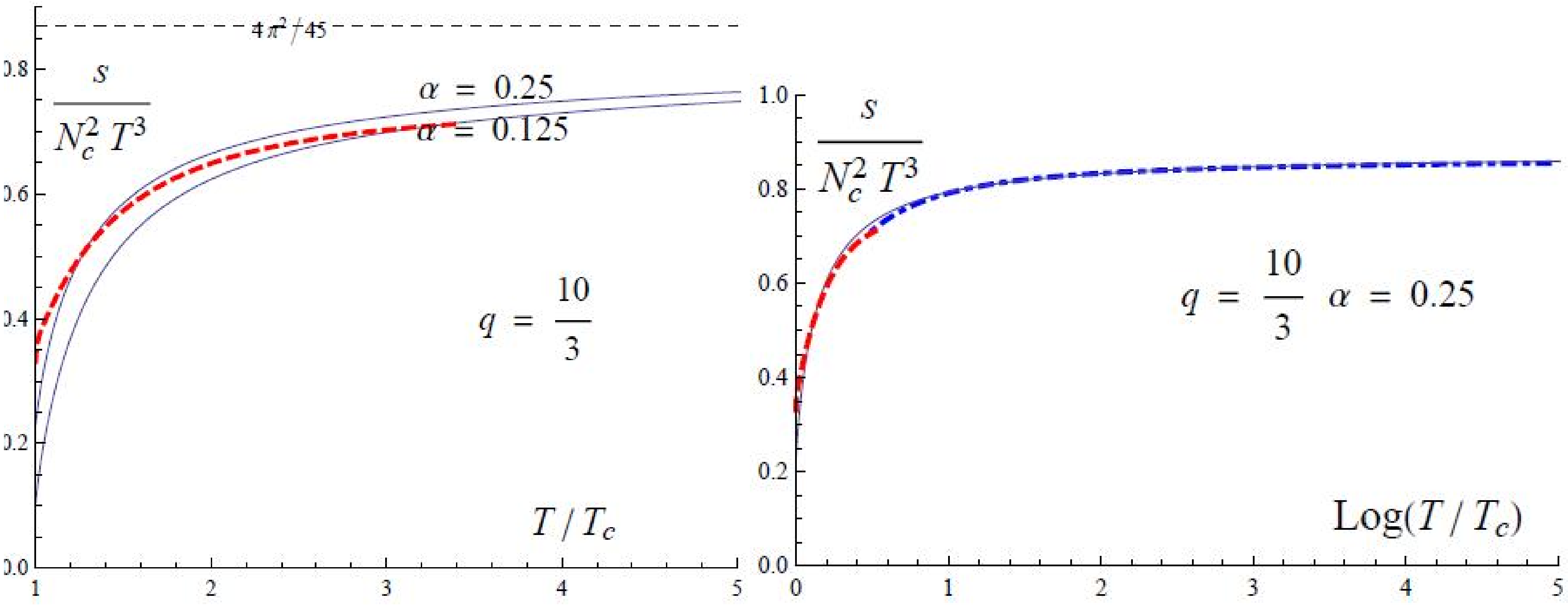


Red = SU(N_c) data/N^2_c

Panero 0907.3719

For detailed fit need 2 parameters!

For a good fit to SU(N) thermo need the monster beta fn (or the monster potential $\frac{12}{\mathcal{L}^2}\left\{1+V_0\lambda+V_1\lambda^{4/3}[\log(1+V_3\lambda^2)]^{1/2}\right\}$



4. Spatial string tension σ(T)

Alanen-Kajantie-SuurUski 0905.2032, PRD

Finite T QCD: 3d space + imaginary time $0 < \tau < 1/T$

Measure string tension in the 3d spatial sector for varying T, get σ(T)

But can also measure σ in the 3d spatial sector without any 4th dim, string tension in 3d SU(3) Yang-Mills

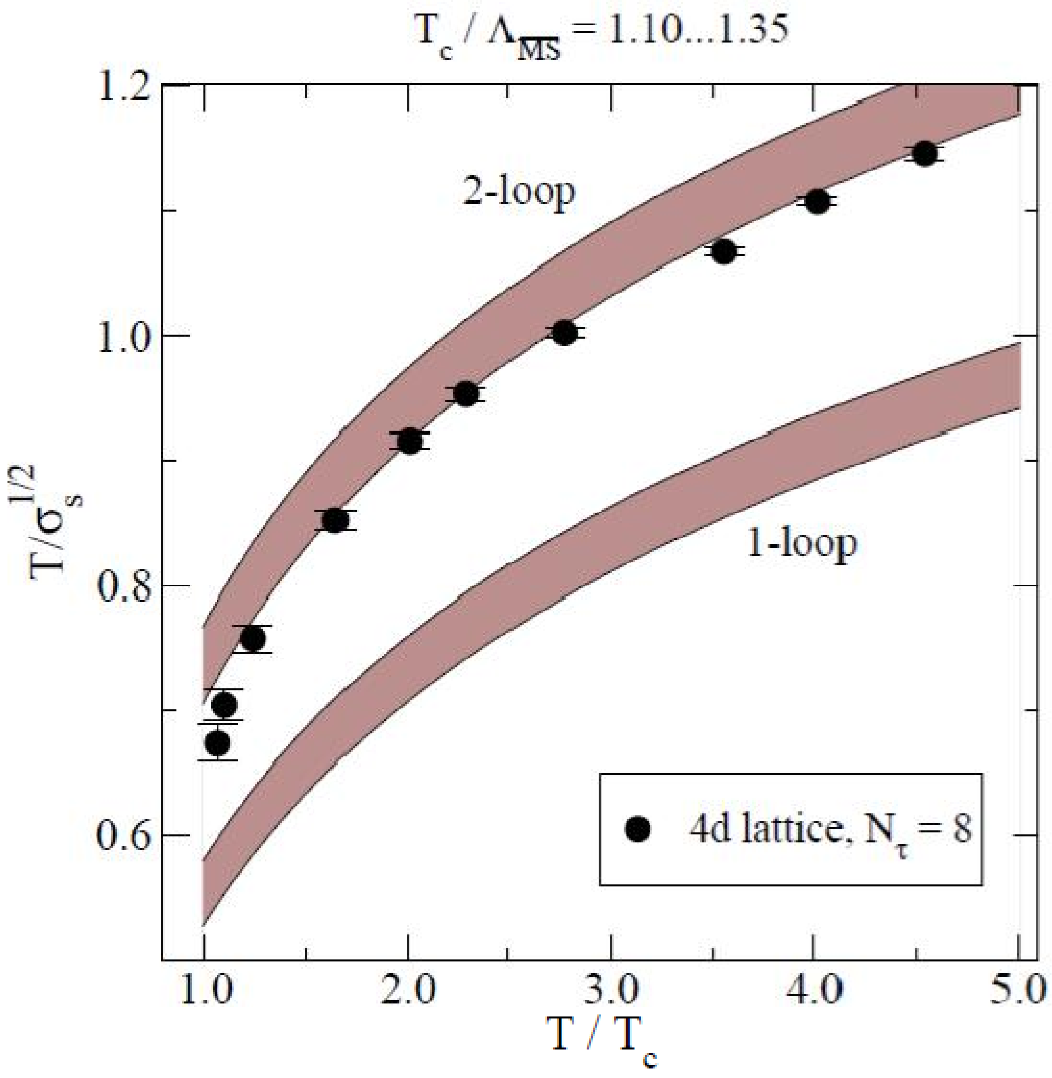
non-pert → $\sqrt{\sigma_s} = 0.553(1) g_M^2$

pert → $g_M^2 = g^2(T)T$

$$\frac{T}{\sqrt{\sigma_s}} = 1.81\,\frac{1}{g^2(T)} = 0.25\left[\log\frac{T}{\Lambda_\sigma} + \frac{51}{121}\log\left(2\log\frac{T}{\Lambda_\sigma}\right)\right]$$

$$\Lambda_\sigma = ? = T_c/7.753$$

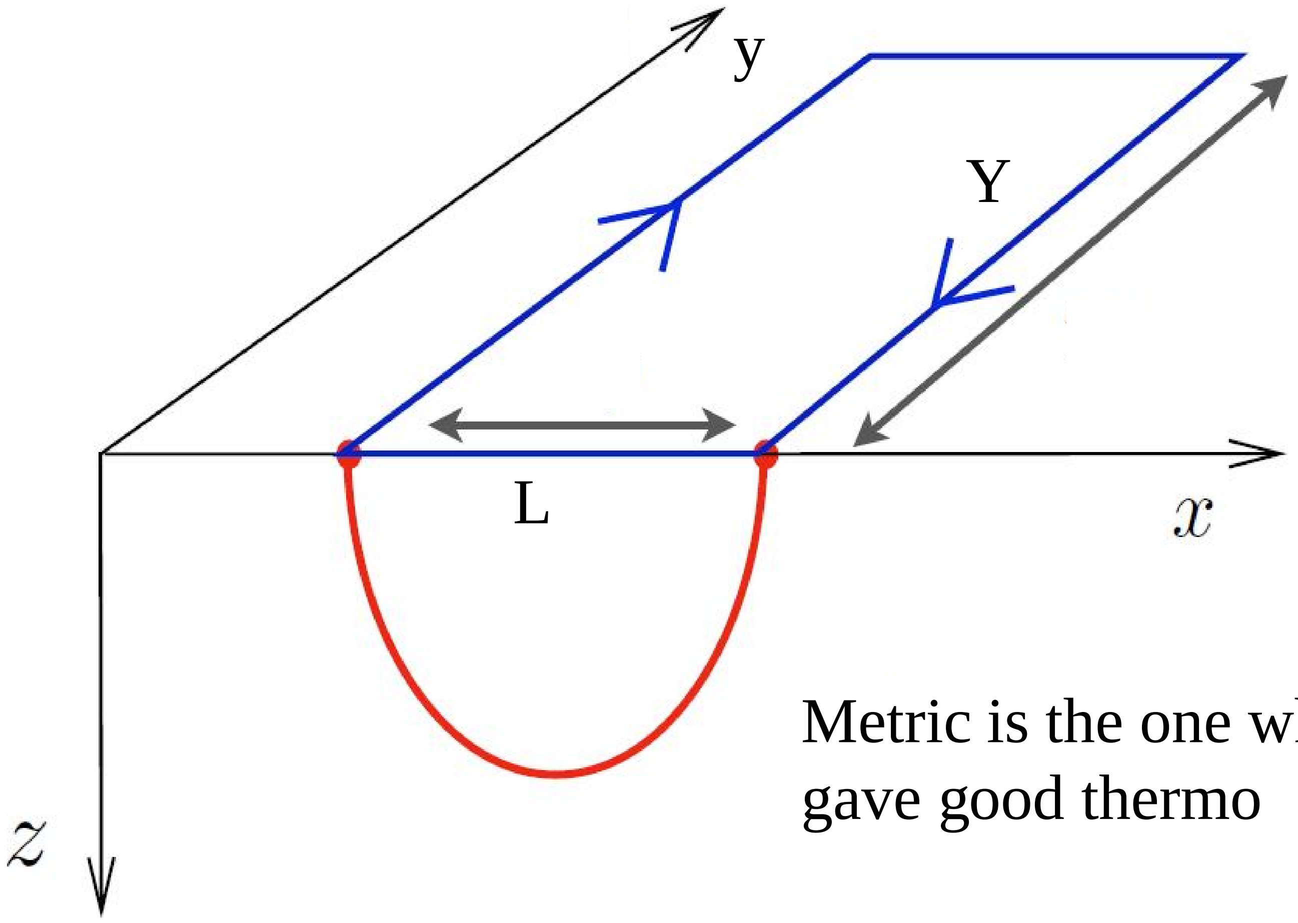
Hot QCD to 2 loops, Laine-Schröder hep-ph/0503061



Reproducible
well defined

3 loop ?
Lattice cont ?

<Wilson loop> : value of extremal action of string sheet hanging from the loop to 5th dim



Metric is the one which gave good thermo

Find spatial string tension:

$$\sigma_s = \frac{1}{2\pi\alpha'} b^2(z_h)\lambda^{4/3}(z_h)$$

But: new parameter α’, normalisation of λ = ?

Try to fit T-dependence with leading log b, λ :

$$\frac{T}{\sqrt{\sigma_s}} = \sqrt{2\pi\alpha'}\frac{1}{\pi\mathcal{L}\left[1-\frac{4}{9(q-1)^2}\log^{-1}\frac{\pi T}{\Lambda}\right]}\left[(q-1)\beta_0\log\frac{\pi T}{\Lambda}\right]^{2/(3q-3)}$$

Pert th had logT

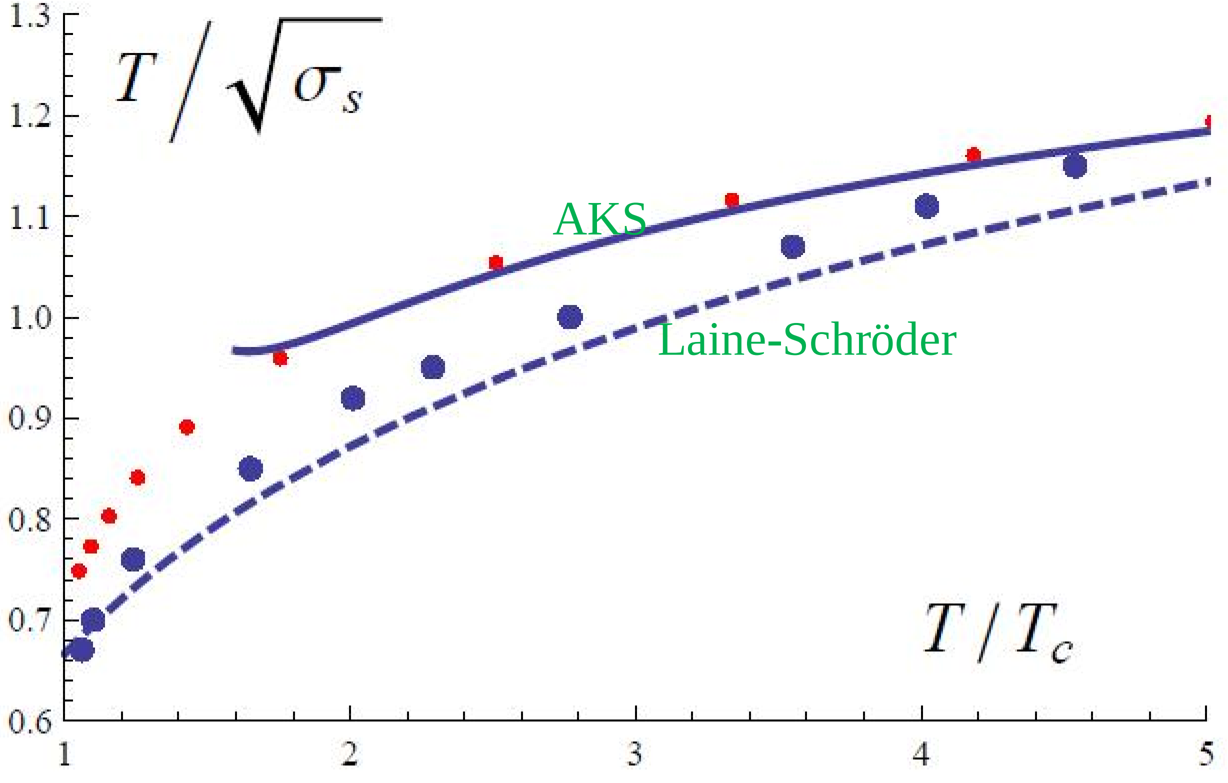
Better: relate this to T=0 (f(z)=1) string tension

$$\sigma = \frac{1}{2\pi\alpha'} b^2(z_{\rm min})\lambda^{4/3}(z_{\rm min})$$

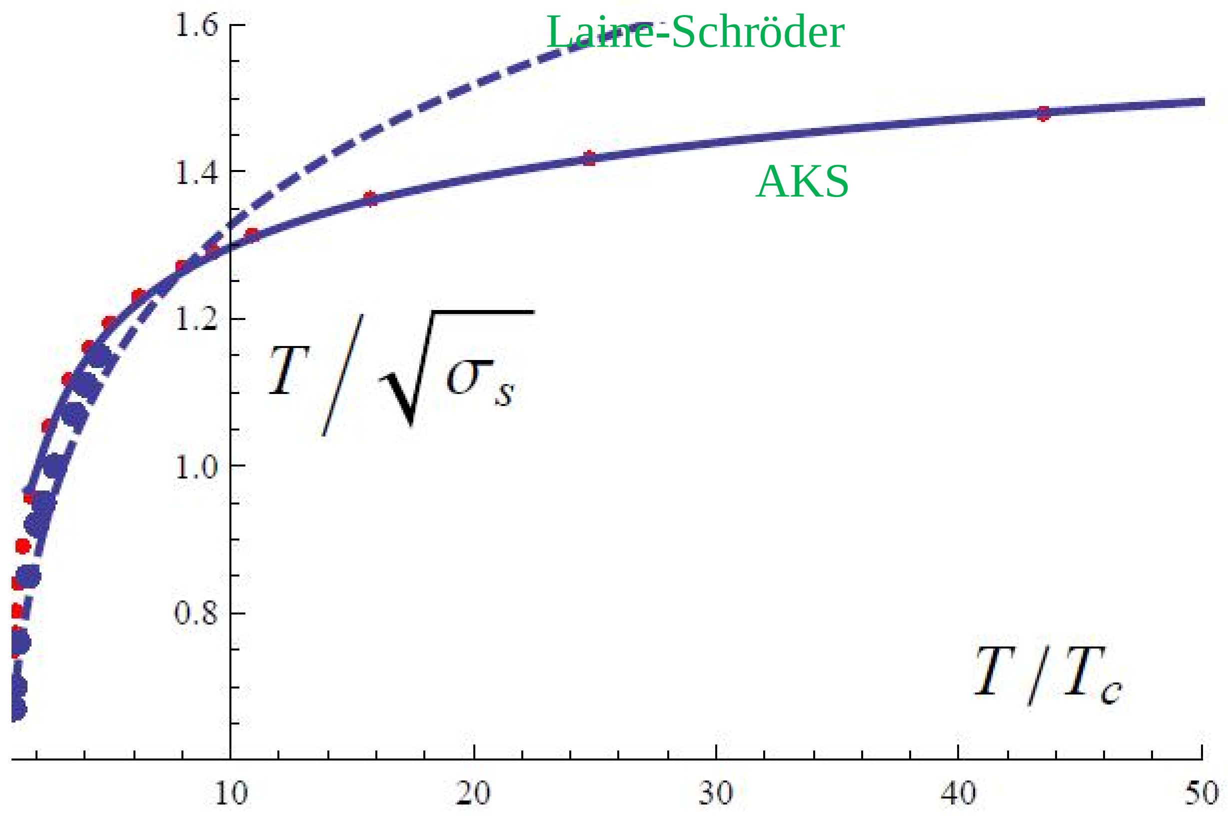
different b,λ

$$\frac{T}{\sqrt{\sigma_s}} = \frac{T_c}{\sqrt{\sigma}} \frac{\Lambda}{\pi T_c} \frac{1}{1 - \frac{4}{9(q-1)^2} \log^{-1} \frac{\pi T}{\Lambda}} \left[\tfrac{3}{2}\, e(q-1) \log \frac{\pi T}{\Lambda}\right]^{2/(3q-3)}$$

Small T:



Clear difference at large T:



Conclusions:

AdS/CFT(theory)…AdS/QCD(model) is popular

This application to thermal QCD seems like a nice way of non-conformizing with a dilaton & beta fn

It still is a model, postdicts. Top-down derivation is missing!