28.11.2003
CERN

Rolf
“the temperature”
Hagedorn

1919-2003

Scientific Biography

K. Kajantie, Helsinki

Hagedorn: in Proceedings of Quark Matter 1984
Letessier, Gutbrod, Rafelski, Hot Hadronic Matter, 1995

Early Years

- Born in 1919 in Wuppertal-Barmen, graduation there from high school (Abiturium) in 1937
- Entered military service (Luftwaffe) in 1937 to complete it before university studies
- When he was to be released, WWII started and he had to stay with Luftwaffe
- Served in North Africa, prisoner in May 1943, taken to prisoner of war-camp in Tennessee
- Started studying math and physics, taught physics in a high school
- Returned to Germany in 1946

Almost ten years were lost

University studies in Göttingen

- M.Sc. thesis on Lamb shift in QED in 1950
- Thesis on a statistical model of Barium Titanate ($BaTiO_3$) in 1952. The topic is solid state physics, but statistical concepts, Bessel functions, sums over n, start appearing
- Vol 133 of Zeitschrift für Physik, where this thesis was published, also contains an article by Heisenberg on meson production ("high occupation numbers → classical fields", "renormalised theories contain only weak interactions and do not give rise to multiproduction of mesons")
- On 26 Jan 1954 Heisenberg writes a - rather laconic - letter of recommendation for RH to the CERN PS Group (then at Institut de Physique, Genève!). Heisenberg was a key player in the creation of CERN.

The stage: CERN is born

Imagine a time when the present CERN site was just agricultural fields, pions, “mu-mesons” and nucleons were elementary particles....

- May 52: 1st CERN Provisional Council (theory in Copenhagen - until Oct 57)
- Jul 52: Geneva selected as the site
- Summer 52: Idea of Alternating Gradient (Strong) focusing born in Brookhaven; CERN plans changed, PS became an innovative project, much work needed
- Oct 53: PS group in Geneva, grows to 50 people by Oct 54; one of them is Rolf Hagedorn
- May 54: First earth moved at the present CERN site
- Jun 55 - End 59: Construction of the PS

Clearly Rolf Hagedorn's career was shaped by:

- Postwar reconstruction of Europe in politics and science
- The position of Heisenberg both in the reconstruction and as a professor in Göttingen, where Hagedorn studied
- On a more grass roots level: G. "CPT" Lüders from Göttingen

also worked on beam optics, Hagedorn's first work assignment

Work in the PS group, March 1954-57

Essential task for CERN: convert the idea of strong focussing using alternating gradients to a practical accelerator.

Beam optics in an AGS (basically solving $d\mathbf{p}/dt = e(\mathbf{v} \times \mathbf{B})$ for the appropriate AG $\mathbf{B}$-configuration) leads to two-dimensional non-linear differential equations (in Hagedorn's notation):

$$\ddot{x} + n_{11}(\Theta) + n_{12}(\Theta) = -\frac{\partial K(x,y,\Theta)}{\partial x} + f_x(\Theta)$$

$$\ddot{y} + \underbrace{n_{21}(\Theta)}_{\text{periodic}} + n_{22}(\Theta) = \underbrace{-\frac{\partial K(x,y,\Theta)}{\partial y}}_{\text{nonlin.terms}} + \underbrace{f_y(\Theta)}_{\text{misalignment}}$$

Hagedorn developed the theory of the stability of nonlinear oscillations in a two-dimensional system with a periodic Hamiltonian.

Applications were presented in a large number of PS reports - and Hagedorn also started his career as a prolific yellow report writer.

To the Theory Division

PS was getting ready in 1957 but what would be coming out of the p+Target collisions in it?

This question is still unsolved today!
What is the average number of (charged) particles from an A+A collision at ALICE/LHC?

$$\frac{dN_{\mathrm{ch}}^{\mathrm{AA}}}{dy} = 2200 \text{ or what?}$$

Hagedorn was recruited to estimate particle (π,N) spectra. Teamed up with F. Cerulus.

Nothing was known of strong interaction dynamics. What else can you do than start counting states with energy and momentum constraints?

Fermi 1950, “High Energy Nuclear Events”:

$$\mathrm{rate}(\mathrm{N}+\mathrm{N}\to 1+2+\ldots+\mathrm{n}) = \int \prod_1^n \frac{V}{(2\pi)^3} d^3p_i\, \delta(E - \sum_{i=1}^n \sqrt{p_i^2+m_i^2})$$

Fermi has Lorentz contraction, “The only parameter is $V = V_0/\gamma,\ V_0 \sim R_\pi^3$ ” and thermal equilibrium.

From 1957 onwards Hagedorn applies and develops further the Fermi model:

- Nuovo Cimento 1958. In the application of Fermi's model a technical problem appeared: How to compute the phase space integrals? Fermi had used analytic formulas for $m = 0$ or $m \to \infty$.

  ⇒ Monte Carlo methods, which required computers !

  Ferranti-Mercury at CERN

- Nuovo Cimento 1960.

  "Particle production in 6.2 GeV pp collisions treated by the statistical model",

  "... theory of particle production with numerical results for pp collisions at 25 GeV". NN → NN+π, NN+2π,...,NN+5π, NN*+nπ, Λ+N+K, Λ+N+K+nπ

  Parameters: V_π, V_K, V_Y.

TABLE I. – *Reactions considered.*

Reactions	Prob. in %	
	(α)	(β)
$2\mathcal{N}$ (elastic, incoherent)	1.32, − 1	1.41, − 1
$2\mathcal{N}+\pi$	4.59, + 0	4.68, + 0
$2\mathcal{N}+2\pi$	1.09, + 1	1.16, + 1
$2\mathcal{N}+3\pi$	6.51, + 0	5.80, + 0
$2\mathcal{N}+4\pi$	8.23, − 1	8.74, − 1
$2\mathcal{N}+5\pi$	3.70, − 2	3.93, − 2
$\mathcal{N}^*+\mathcal{N} \to 2\mathcal{N}+\pi$	5.01, − 1	6.32, − 1
$\mathcal{N}^*+\mathcal{N}+\pi \to 2\mathcal{N}+2\pi$	1.09, + 1	1.15, + 1
$\mathcal{N}^*+\mathcal{N}+2\pi \to 2\mathcal{N}+3\pi$	2.43, + 1	2.58, + 1
$\mathcal{N}^*+\mathcal{N}+3\pi \to 2\mathcal{N}+4\pi$	8.70, + 0	9.24, + 0
$\mathcal{N}^*+\mathcal{N}+4\pi \to 2\mathcal{N}+5\pi$	6.92, − 1	7.35, − 1
$\mathcal{N}^*+\mathcal{N}+5\pi \to 2\mathcal{N}+6\pi$	1.91, − 2	2.03, − 2
$2\mathcal{N}^* \to 2\mathcal{N}+2\pi$	4.70, − 1	5.00, − 1
$2\mathcal{N}^*+\pi \to 2\mathcal{N}+3\pi$	1.06, + 1	1.13, + 1
$2\mathcal{N}^*+2\pi \to 2\mathcal{N}+4\pi$	1.21, + 1	1.29, + 1
$2\mathcal{N}^*+3\pi \to 2\mathcal{N}+5\pi$	2.55, + 0	2.71, + 0
$2\mathcal{N}^*+4\pi \to 2\mathcal{N}+6\pi$	1.05, − 1	1.12, − 1
$2\mathcal{N}^*+5\pi \to 2\mathcal{N}+7\pi$	1.13, − 3	1.20, − 3
$3\mathcal{N}+\bar{\mathcal{N}}$ (without final state annihil.)	9.40, − 3	1.00, − 2
$\Lambda+\mathcal{N}+K$	3.43, − 1	4.35, − 2
$\Lambda+\mathcal{N}+K+\pi$	8.21, − 1	1.08, − 1
$\Lambda+\mathcal{N}+K+2\pi$	2.61, − 1	3.50, − 2
$\Lambda+\mathcal{N}+K+3\pi$	1.77, − 2	2.21, − 3
$\Lambda+\mathcal{N}^*+K \to \Lambda+\mathcal{N}+K+\pi$	4.23, − 1	6.26, − 2
$\Lambda+\mathcal{N}^*+K+\pi \to \Lambda+\mathcal{N}+K+2\pi$	5.68, − 1	7.08, − 2
$\Lambda+\mathcal{N}^*+K+2\pi \to \Lambda+\mathcal{N}+K+3\pi$	1.04, − 1	1.20, − 2
$\Lambda+\mathcal{N}^*+K+3\pi \to \Lambda+\mathcal{N}+K+4\pi$	2.35, − 3	2.96, − 4

- Nuovo Cimento 1963.

  "Large angle elastic scattering..."

  Idea: small angle, few pions ⇒ not statistical

  large angle, central, lots of pions ⇒ statistical

$$\frac{d\sigma}{d\Omega} \sim e^{-3.17(E-1.4)}$$

Illuminating statements from this heroic era:

- "Dr. Cerulus took over the tedious job of preparing some of the data tapes"
- "..this work had been impossible without constant collaboration of the engineers and the tape punching group of the CERN-computer"

Yellow reports (density matrix, isospin, field theory, kinematics, polarisation); Book on Relativistic Kinematics (1964)

Parenthesis: Landau

Landau 1953 takes the Fermi model with Lorentz contraction and thermal equilibrium, but observes that

- Fermi's calculation appears unconvincing to us and incorrect at several points
- Fermi's reasoning appears to us to be completely unconvincing

and corrects it by putting in dynamics via

- initial conditions à la Fermi
- relativistic hydrodynamics

$$\partial_\mu((\epsilon + p)u^\mu u^\nu - p g^{\mu\nu}) = 0$$

Remarkably modern outcome:

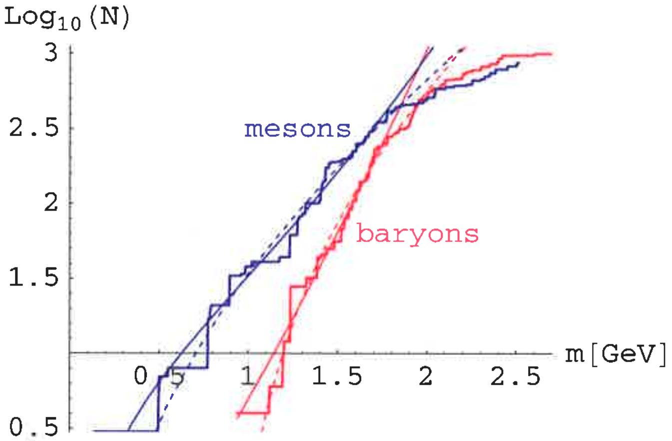
$$N_{\rm tot}^{AA} = K A^{3/4} \left(\frac{E}{2M_p}\right)^{1/4}, \quad K = \mathcal{O}(1) = 2$$

$$\frac{dN}{d\eta} \sim \exp[-\frac{\eta^2}{2L}], \quad L = \frac{1}{2}\log\frac{E}{2M_p}, \quad \eta = -\log\tan(\theta/2)$$

No reference to Landau in Hagedorn!? Even though the idea was also published in Nuovo Cimento Suppl in 1956 (Belenki-Landau).

The highlight: exponential mass spectrum

Take today's 3182 light-flavour (u,d,s) states (Hagedorn had 609 in Oct 64, 971 in Apr 66, 1432 in Jan 67) and plot separately the number of mesonic and baryonic states with mass $< m$ (Broniowski-Florkowski)



Exponential spectrum is seen over a range of m, but slopes may be different for various classes of particles!

Any motivation for the exponential spectrum?

Counting states:

1. One $m = 0$ particle in a 3d box of volume V:

$$dN = \frac{V}{(2\pi)^3} \int d^3p \to \frac{dN}{dE} = \frac{VE^3}{2\pi^2 E}$$

2. N $m = 0$ particles (Fermi had this formula!):

$$\frac{dN}{dE} \equiv \rho(E,N) = \frac{1}{N!} \int \delta(E - \sum_{i=1}^{N} \sqrt{p_i^2}) \prod_{i=1}^{N} \frac{V}{(2\pi)^3} d^3p_i$$
$$= \frac{1}{N!(3N-1)!} \frac{1}{E} \left(\frac{VE^3}{\pi^2}\right)^N$$

3. Summing over N with weight z^N ("grand canonical"):

$$\rho(E,z) = \sum_{N=1}^{\infty} z^N \rho(E,N) \approx cE^{-5/8} \exp(bE^{3/4})$$

($b = 4/3(3zV/\pi^2)^{1/4}$, etc.).

Not yet $\exp(bE)$, which would produce a singularity at $b = 1/T$ in Thermodynamics:

$$Z = \int dE \rho(E) \exp(-E/T).$$

Bootstrap principle Hagedorn 1965, Frautschi 1970

Hagedorn's idea: replace each particle of fixed mass by a mass distribution, $1 \rightarrow \int dm_i \rho_{\rm in}(m_i)$. The output density of states becomes ($E \rightarrow$m):

$$\rho_{\rm out}(m) = \sum_{N=1}^{\infty} \frac{1}{N!} \int \delta(m - \sum_{i=1}^{N} \sqrt{p_i^2 + m_i^2}) \prod_{i=1}^{N} \frac{V}{(2\pi)^3} d^3 p_i [\int dm_i \rho_{\rm in}(m_i)]$$

Now demand that

$$\rho_{\rm out}(m) \underbrace{\rightarrow}_{m \rightarrow \infty} \rho_{\rm in}(m)$$

There, indeed, is a bootstrap solution

$$\rho(m) = \frac{c}{m^{\geq 2.5}} e^{bm}$$

In the bootstrap $\delta(m - \Sigma_i(m_i + E_{\rm kin,i}))$ takes care of e^{bm}, getting the power right is more subtle and the value is only bounded by 2.5 by convergence.

Numerically, from $\rho(m)$ or p_T-distributions $\sim \exp(-6p_T)$

$$b \approx 1/T_H \approx {\rm fm} \approx 1/m_\pi \approx 1/\Lambda_{\rm QCD}$$

Thermodynamics

Laplace of $\rho(E)$:

$$Z = e^{-F(T)/T} = \int dE\ \rho(E) e^{-E/T} \equiv \int dE\ e^{S(E)-E/T}$$

Exponentially increasing $\rho(E) \sim e^{E/T_H}$ necessarily gives a

maximum = Hagedorn temperature T_H;

feeding energy to the system excites more dofs, not a limited number of dofs to an ever increasing energy.

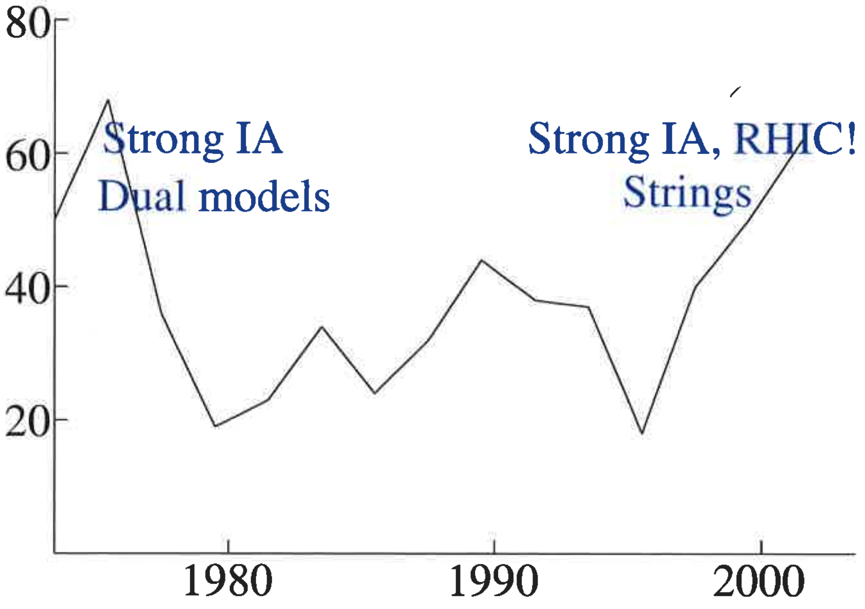
$$T_H \approx 1/\mathrm{fm} \approx m_\pi \approx \Lambda_{\mathrm{QCD}}$$

Cosmology

How had the universe expanded if there were a maximum T (Huang-Weinberg 1970)?

Citations to Hagedorn's Nuovo Cimento/1965 per two years

The Hagedorn temperature has become an evergreen:

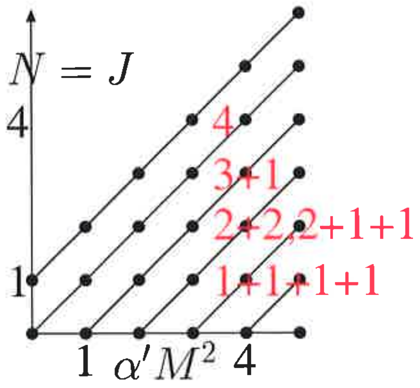


Citations from two sources:

- Strong interaction phenomenology (2/3 recently)
- String physics (1/3 recently)

Dual model, strings

In the 60s, before QCD, strong interaction theory developed via Regge poles and duality to dual models (which later developed into string theories).
Remarkably, in 1969 a count (Fubini-Veneziano) of the states of the dual model gave also an exponential density of states:



The exponential density of states follows from the large degeneracy: the same $M^2 = (N-1)/\alpha'$ is obtained by exciting the string in $\sim \exp(2\pi\sqrt{N/6})$ different modes (number of partitions of an integer as a sum of integers, D types)

$$P_{ND} \to \frac{1}{\sqrt{2}}\left(\frac{D}{24}\right)^{(D+1)/4} \frac{1}{N^{(D+3)/4}} \exp\left[2\pi\sqrt{\frac{DN}{6}}\right]$$

Applies even for a harmonic oscillator, $E \sim N$, density $\sim e^{b\sqrt{E}}$
But now $M^2 \sim N$, $\Rightarrow$ density $\sim e^{bE}$.

String theories have an exponential density of states

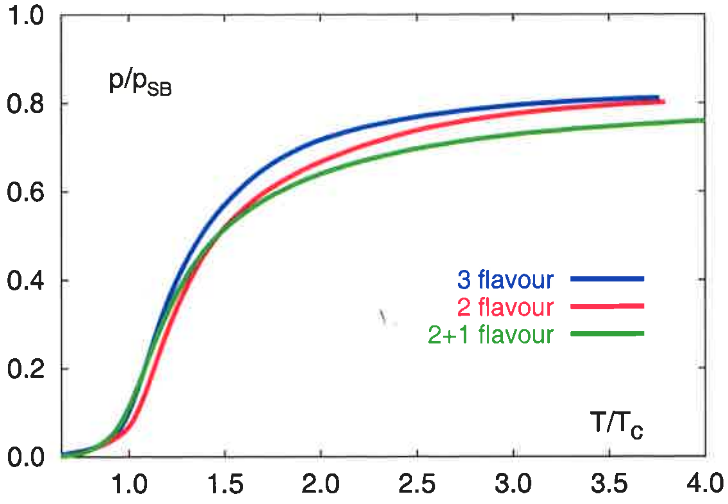
$\Rightarrow$ they have a Hagedorn temperature and Hagedorn transition but to what? What is the true nature of the transition?

Open question

Note: String theories contain gravity but gravity and thermodynamic limit (volume, time $\rightarrow \infty$) are not obviously compatible!

The true nature of the Hagedorn transition in QCD

In QCD we know what happens at T_c; we have a perfect theory, we can solve it numerically and, most importantly, there are experiments (have not the SPS-RHIC experiments convinced everybody?): there is a phase transition(Collins-Perry 1975; lattice data from Bielefeld):



In Hagedorn's 60's we first had data on hadrons, then the true dofs of the theory $(A_\mu^a(x), \psi_{if}(x))$ were discovered.

Two phases, hadron gas and quark-gluon plasma with a phase transition in between - quantitative and many qualitative details still unknown today.

But what happens in string theories? There is an exponential spectrum of string excitations so there for sure is a thermodynamic singularity.
But the true nonperturbative dofs of the low T phase ("hadrons") even less those of the high T one ("q-g") are unknown.

Theoretical candidate dofs of the $T = 0$ theory:

$$\int d^2\sigma\sqrt{-h}h^{ab}g^{\mu\nu}(X)\partial_a X_\mu \partial_b X_\nu + ..$$

or for weak zero-mass fields

$$\int d^dx\sqrt{-G}e^{\phi}[R + (\partial\phi)^2 - \frac{1}{12}H^2 + ..]$$

But what is the true ground state at $T = 0$? The standard model+ sth else?
What is the high T phase? Topological field theory (Atick-Witten 1985)?

A very actively studied topic!

Rolf Hagedorn

- was late for the discoveries of quantum mechanics
- lost ten years in the war
- was early for the discoveries of the standard model

But he got his name into the history of science and to conclude:

The Hagedorn temperature is and remains a hot topic!