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★**Dependence logic.**

A new approach to independence friendly logic.

London Mathematical Society Student Texts, 70.
Cambridge University Press, *Cambridge*, 2007. *x*+225 *pp*. $45.00.
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In the introduction of the book we read: "Dependence is a common phenomenon, wherever one looks: ecological systems, astronomy, human history, stock markets. . . . But what is the logic of dependence? In this book we set out to do a systematic logical study of this important concept."

In fact, this book is a beautiful monograph devoted to the so-called dependence logic. The dependence logic, $\mathcal{D}$, is an extension of first-order logic in which one is allowed to use expressions of the form $=(t_1,\ldots,t_n)$ as atomic formulas, where $t_1,\ldots,t_n$ are terms. The intuitive meaning of these expressions is as follows: the value of the term t_n depends only on values of the terms $t_1,\ldots,t_{n-1}$. The rules of construction of formulas are those that we find in first-order logic. The semantics of $\mathcal{D}$ are defined in analogy to the compositional semantics for independence friendly logic given by Hodges. Independently, the game theoretic semantics for $\mathcal{D}$ are also formulated and studied. $\mathcal{D}$ does not satisfy the Law of Excluded Middle and therefore, in that sense, is a nonclassical logic. Several examples of concepts defined with the use of dependence logic are given (e.g., infinity, even cardinality, well-foundedness, graph connectedness). It is proved that dependence logic is mutually interpretable with Σ^1_1-logic. This allows one to deduce some model theoretic properties of $\mathcal{D}$, for example, compactness, the Skolem–Löwenheim property and the interpolation property. It is also observed that there is a strict connection between $\mathcal{D}$ and other earlier introduced logics containing dependence or independence concepts, for example, the dependence and independence friendly logics together with logic with a Henkin quantifier. One chapter of the book is devoted to the complexity of validity and satisfiability problems for $\mathcal{D}$. It is proved that the satisfiability problem for $\mathcal{D}$ is Π^0_1-complete and the validity problem is Π_2-complete in set theory. In the last part of the book the so-called team logic is introduced. Roughly speaking, team logic is an extension of dependence logic by adding a classical negation. It is proved that team logic and second-order logic are mutually interpretable.

On one hand the book is a long research paper containing several nonpublished earlier observations and results about dependence logic.

On the other hand it is a textbook suitable for a special course in logic in mathematics, philosophy, or computer science. The material is written in a very friendly way. Definitions of the majority of the important notions are preceded by explanations of the ideas. Moreover, the book contains more than 200 exercises, many of which have a solution at the end of the book. This makes it easier to read the book and understand its material.

The book could definitely be interesting to a wide spectrum of mathematicians, philosophers and computer scientists.

Reviewed by *Michał Krynicki*