ExerciseSinesDFT: Introduction

The simulated data **TestData.dat** (**Paper I**, Table 3) contains three signals having periods $P_1 = 1.1$, $P_2 = 1.4$ and $P_3 = 1.9$. Discrete Chi-square Method (DCM) can re-detect these periods (**Paper I**, Sect. 4.1).

Discrete Fourier Transform (DFT) formulated by [2]Scargle (1982, his Eqs. 10 and 11) is frequently used to search for many periods in the data. This technique is called **pre-whitening**. A typical example can be found in [1] (Reihold & Reiners, 2013), where it was applied to light curves of chromosperically active variable stars. In this particular case, the multiple periods were interpreted as periods of individual starspots that trace surface differential rotation.

In the following example, we show how this **pre-whitening** technique can be applied to **TestData.dat**. DFT input parameters are the minimum tested period $P_{\rm min}$ =**PMIN=0.5** and the maximum tested period $P_{\rm max}$ =**PMAX=20.0**. The overfilling factor is OFAC = **OFAC=50**.

1. Apply DFT to **original data**.
   - For **TestData.dat**, the best period is $P = 7.14$ (Figure 1).
2. Fix the best frequency to the constant numerical value $f = 1/P$. The sinusoidal model for **original data** is

$$g(t) = \beta_0 + \beta_1 \cos(2\pi f t) + \beta_2 \sin(2\pi f t) \quad (1)$$

   where the free parameters are $\bar{\beta} = [\beta_1, \beta_2, \beta_3]$. This model gives the **residuals**

$$\epsilon_i = y(t_i) - g(t_i) = y_i - g_i \quad (2)$$

   of **original data**.

- For **TestData.dat**, this sinusoidal model is shown in Figure 2 (black dots = **original data**, red line = model, and blue dots = **residuals**). Note that the residuals are offset to level $\epsilon_i + C$ (dotted blue line), where C is the smallest value of all y_i and g_i values.

3. Apply DFT to **residuals** of original data.

   - The best period is $P = 1.17$ (Figure 3).

4. Fix the best frequency to the constant numerical value $f = 1/P$. Fit the sinusoidal model (Eq. 1) having this frequency to the **residuals**. This model gives the **next residuals**.

   - This sinusoidal model for **residuals** is shown in Figure 4 (black dots = **residuals**, red line = model, and blue dots = **next residuals**).

5. Apply DFT to **next residuals**.

   - The best period is $P = 2.98$ (Figure 5).

6. Fix the best frequency to the constant numerical value $f = 1/P$. Fit the sinusoidal model (Eq. 1) having this frequency to the **next residuals**. This model gives the **second next residuals**.

   - This sinusoidal model for **next residuals** is shown in Figure 6 (black dots = **next residuals**, red line = model, and blue dots = **second next residuals**).

Some notes about this DFT pre-whitening example

- Correct periods $P_1 = 1.1$, $P_2 = 1.4$ and $P_3 = 1.9$ are not detected.
- First period $P_1 = 7.14$ is longer than $\Delta T = 4$.
- Data is not detrended before **pre-whitening**. Therefore, this $P_1 = 7.14$ period signal tries to remove the quadratic $p(t)$ trend.
- DFT tested period interval should not be longer than $\Delta T = 4$. The period interval between $P_{\text{min}} = 0.5$ and $P_{\text{max}} = 20.0$ was chosen, because Fig. 1

shows no high periodogram peaks between $P_{\rm min} = 1.0$ and $P_{\rm max} = 2.0$.
- Residuals (blue dots) show systematic trends in Figures 2, 4 and 6, which confirms that this **DFT pre-whitening** analysis of **TestData.dat** fails.

ExerciseSinesDFT: Problem

Copy file **TestData.dat** from home-page. Edit your program **ExerciseSinesDFT.py**, which can repeat the analysis of **stages 1-4. Note that you do not have to repeat stages 5-6.** In other words, you should produce Figures 1 – 4. Send your files
ExerciseSinesDFT.py
periodogram0.eps
model0.eps
periodogram1.eps
model1.eps
to the assistant. The content of your figures should agree with the results of the introduction, but the form can be different (e.g. size, symbols,...)

ExerciseSinesDFT: Tips

Copy all required subroutines "as such" from model programs **ExerciseScargle.py** and **ExerciseSineFit.py**.
After some editing, you can apply these ready subroutines, and save a lot of unnecessary programming time.

References

[1] T. Reinhold and A. Reiners. Fast and reliable method for measuring stellar differential rotation from photometric data. *A&A*, 557:A11, Sept. 2013.

[2] J. D. Scargle. Studies in astronomical time series analysis. II - Statistical aspects of spectral analysis of unevenly spaced data. *ApJ*, 263:835–853, Dec. 1982.

Figure 1: **periodogram0.eps**

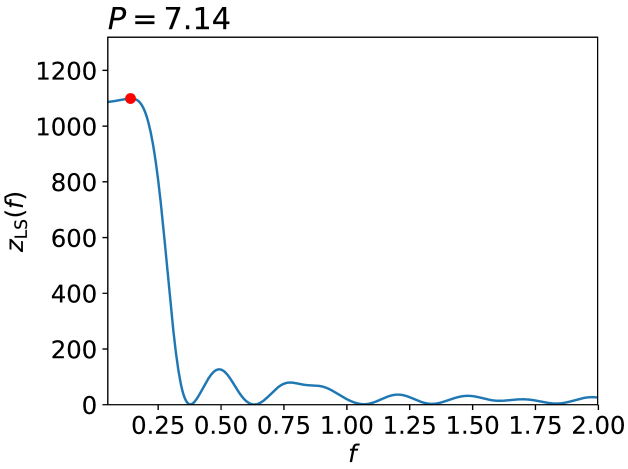


Figure 2: **model0.eps**

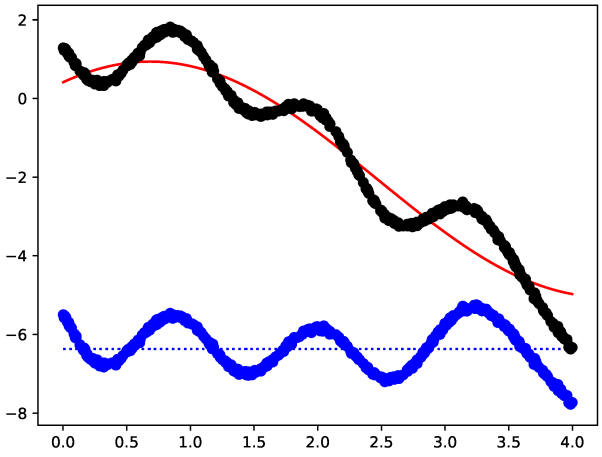


Figure 3: periodogram1.eps

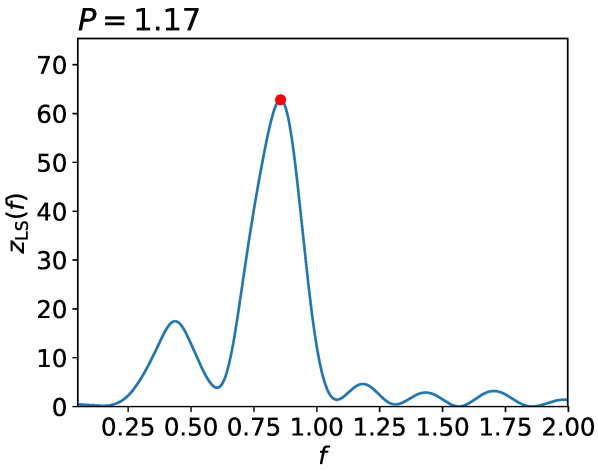


Figure 4: model1.eps

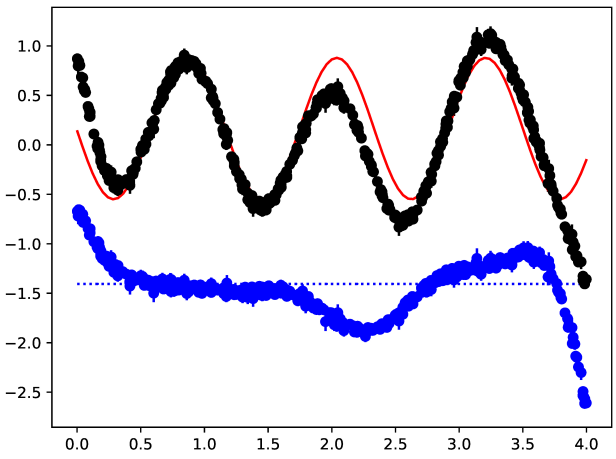


Figure 5: periodogram2.eps

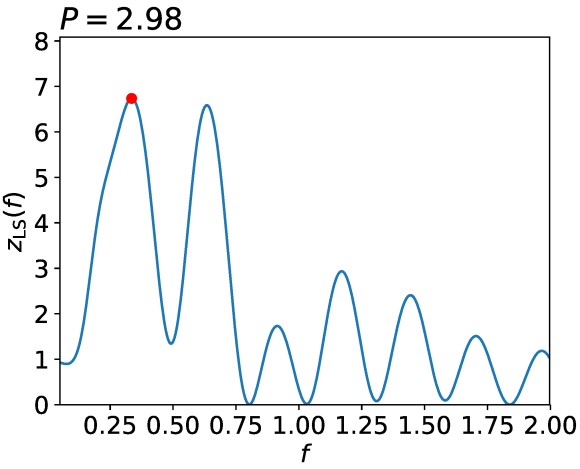


Figure 6: **model2.eps**

