ExampleTrendDFT: Introduction

ExerciseTrendDFT **trend plus signal** model is

$$g(t) = \beta_1 + \beta_2 T + \beta_3 T^2 + \beta_4 \sin\left[2\pi(t-\beta_5)/\beta_6\right] \quad (1)$$

where $\Delta T = t_n - t_1$, $t_{\mathrm{mid}} = (t_n - t_1)/2$, and $T = 2(t - t_{\mathrm{mid}})/\Delta T$. The free parameter values are fixed to $\beta_1 = -5$, $\beta_2 = -10$, $\beta_3 = 50$, $\beta_4 = 2$, $\beta_5 = 3$ and $\beta_6 = 2.4$. The quadratic trend (parabola)

$$p(t) = \beta_1 \ + \ \beta_2 T + \beta_3 T^2 \quad (2)$$

order is $K_3 = 2$. There is only $K_1 = 1$ one period $P = \beta_6 = 2.4$ signal

$$h(t) = \beta_4 \sin\left[2\pi(t-\beta_5)/\beta_6\right]. \quad (3)$$

This model gives simulated $n = 100$ observations during (Figure 1).

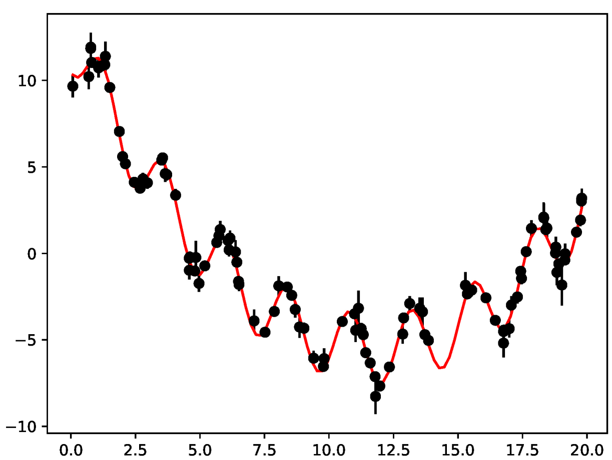

Figure 1: Simulation model (red line) and simulated data (black dots).

DFT for original data is shown in Figure 2. The correct period $\beta_6 = P = 2.4$ is not detected!

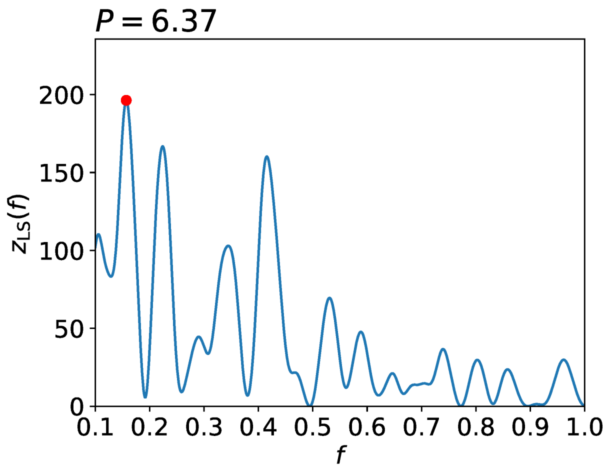

Figure 2: $K_2 = 2$ case DFT for original data.

ExerciseTrendDFT.py solution proceeds through two stages

1. Removing trend from the simulated data gives the detrended data.

2. DFT searches for periodicity from the detrended data.

Model solution **python** program **ExerciseTrendDTF.py** uses an input file **dft.dat**. The format is

```
1 = Tag    = ExerciseTrendDFT
2 = file1  = TrendDFTData.dat
3 = K1     = 1
4 = K3     = 2
5 = PMIN   = 1.
6 = PMAX   = 10.
7 = OFAC   = 40.
```

We obtain the correct result, because we know that the $K_3 =$ **K3=2** order is used in the data simulation. The polynomial fit to original data, and the detrended data are shown in Figure 3 (Upper and lower panel).

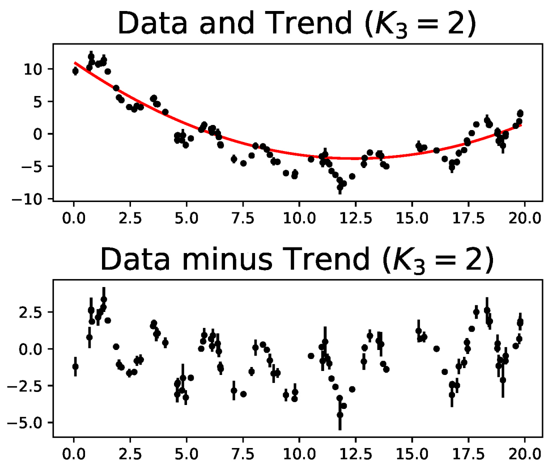

Figure 3: Upper panel: Original data and $K = 2$ order polynomial fit. Lower panel: Original data minus polynomial fit are the detrended data.

DTF for detrended data is shown in Figure 4. The best period $P = 2.41$ is close to the simulated period $\beta_6 = 2.4$.

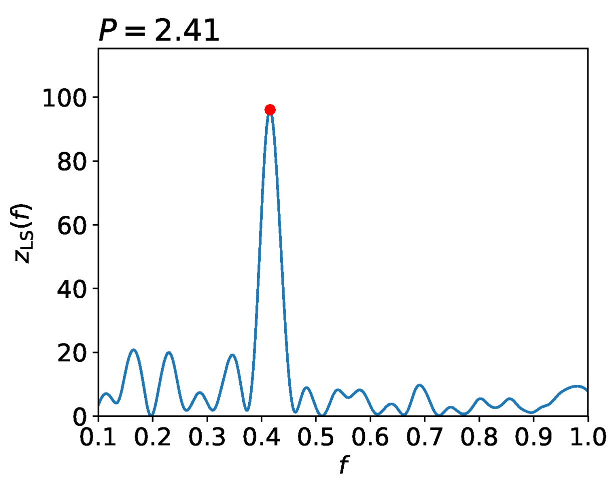

Figure 4: **Detrended data** DFT

Case I: Wrong linear $K_3 = 1$ trend

Next, we test what results would have been obtained for the linear trend $K_3 = 1$. This can be done by simply editing the K_3 value in **dft.dat** to **K3=1**. Note that the **Tag** has also been changed, because this eliminates unnecessary naming confusion in the output file names.

```
1  = Tag    = Linear
2  = file1  = TrendDFTData.dat
3  = K1     = 1
4  = K3     = 1
5  = PMIN   = 1.
6  = PMAX   = 10.
7  = OFAC   = 40.
```

The linear $K_3 = 1$ polynomial fit to the original data, and the detrended data, are shown in Fig. 5

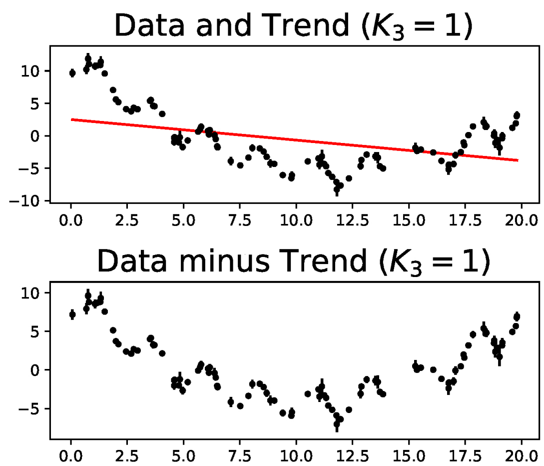

Figure 5: Case I ($K_3 = 1$). Upper panel. Linear fit to original data. Lower panel. Detrended data.

DFT for detrended data gives the best period $P = 6.03$ (Figure 6). This value is not correct, because the simulated period value is $\beta_6 = P = 2.4$.

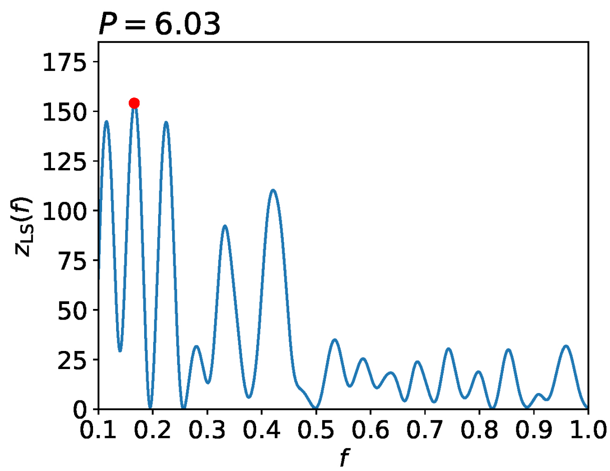

Figure 6: Case I ($K_3 = 1$). Detrended data DFT.

Case II: Wrong cubic $K_3 = 3$ trend

Then, we test what results would have been obtained for the cubic trend $K_3 = 3$. We edit the K_3 value in **dft.dat** to **K3=3**. We also use **Tag=Cubic** to avoid unnecessary output file naming confusion.

```
1  = Tag      = Cubic
2  = file1    = TrendDFTData.dat
3  = K1       = 1
4  = K3       = 3
5  = PMIN     = 1.
6  = PMAX     = 10.
7  = OFAC     = 40.
```

The detrending results are shown in Figure 7. DTF for the detrended data gives the best period $P = 2.41$ (Figure 8). This result is the same as for the quadratic trend $K_3 = 2$. This cubic $K_3 = 3$ trend result also agrees with the simulated period value $\beta_6 = P = 2.40$.

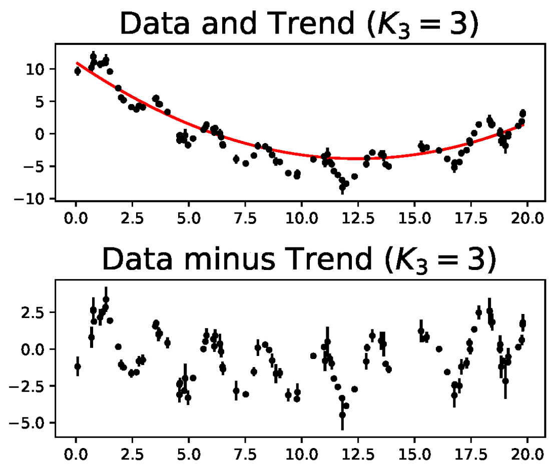

Figure 7: Case II ($K_3 = 3$). Upper panel. Cubic fit to original data. Lower panel. Detrended data.

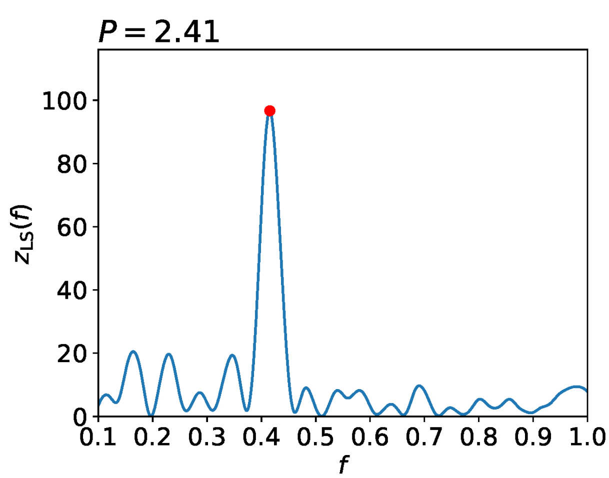

Figure 8: Case II ($K_3 = 3$). Detrended data DFT.

Discussion

If these would have been real data of a variable star, the correct K_3 trend order would have been unkown. In the first example, the mass transfer between two eclipsing binary members of U-Cep causes a **quadratic** $K_3 = 2$ trend in the observed (O) minus computed (C) eclipse epochs (Figure 9)

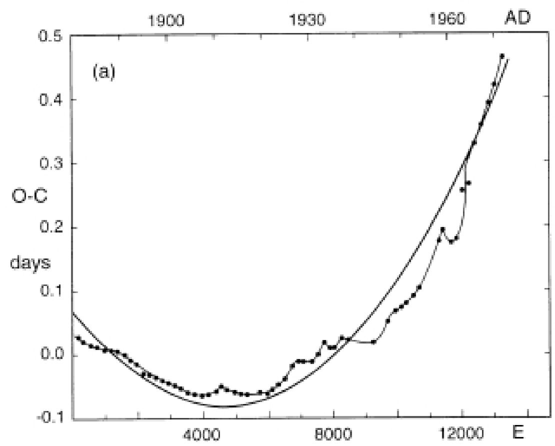

Figure 9: [2] Kiseleva et al. (1998). **Quadratic** O-C changes of eclipsing binary U Cep (continuous black line).

In the second example, the chosen constant orbital period value used in computing O-C changes of Algol causes a **linear** $K_3 = 1$ trend (Figure 10.)

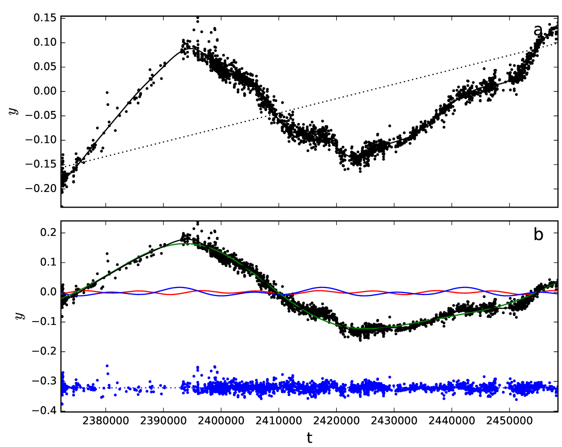

Figure 10: [1] Jetsu (2021). **Linear** O-C changes of eclipsing binary Algol (Upper panel: dotted black line)

What is the correct trend in real data? $\rightarrow$ Far from trivial problem!

Use wrong trend(-s) $\rightarrow$ Get wrong period(-s)?

Use wrong period(-s) $\rightarrow$ Get wrong trend(-s)?

Polynomial trend argument alternatives

- DCM polynomial K_3 order trend alternatives

$$p(t) \ = \ p(t, K_3) = \sum_{k=0}^{K_3} p_k(t)$$

Paper I (Eq. 5)

$$p_k(t) \ = \ M_k \left[\frac{2t}{\Delta T} \right]^k$$

- Even and odd $k \rightarrow p_k(t)$ can only increase or decrease during ΔT $\rightarrow$ Worse predictability outside ΔT?

Paper II (Eq. 6)

$$p_k(t) = p_k(t) = M_k \left[\frac{2(t - t_{\mathrm{mid}})}{\Delta T} \right]^k$$

- Even $k \rightarrow p_k(t)$ can increase and decrease during ΔT $\rightarrow$ Better predictability outside ΔT?

- **Paper II** alternative used in **ExerciseTrendDFT.py** solution

Bigger picture

- We aim for a general DFT **python** program **dft.py**
  - Control file **dft.dat** **only** edited
  - Removes **trends**
  - Detects many sinusoidal **signals** $\equiv$ **"Pre-whitens"**

- Subroutine **dftInput(Val)** in **dft.py**
  - **Any** control file **dcm.dat** variable can be used inside **any** subroutine or main program part of **dft.py**

```
# ______________________________________________________________________________
# - Read input file 'dft.dat' variables.
# ______________________________________________________________________________
def dftInput(Val):
    file0='dft.dat'
    file = open(file0, 'r')
    rivi = file.readline()
    while (len(rivi) > 0):
        osat=rivi.split("=")
        luku=np.float(osat[0])
        loppu=osat[2].strip()
        if ((Val == 1.) & (luku == 1.)):
            NY=loppu                          # Tag: String!
        if ((Val == 2.) & (luku == 2.)):
            NY=loppu                          # file1: String
        if ((Val == 3.) & (luku == 3.)):
            NY=np.int(np.float(loppu))        # K1: Integer!
        if ((Val == 4.) & (luku == 4.)):
            NY=np.int(np.float(loppu))        # K3: Integer!
        if ((Val == 5.) & (luku == 5.)):
            NY=np.float(loppu)                # PMIN
        if ((Val == 6.) & (luku == 6.)):
            NY=np.float(loppu)                # PMAX
        if ((Val == 7.) & (luku == 7.)):
            NY=np.float(loppu)                # OFAC
        if ((Val == 24.) & (luku == 24.)):
            NY=np.float(loppu)                # PrintScreen
        rivi = file.readline() # Read next line: Important in while!
    file.close()
    return NY
# ______________________________________________________________________________
```

References

[1] L. Jetsu. Say Hello to Algol's New Companion Candidates (**Paper II**). *ApJ*, 920(2):137, Oct. 2021.

[2] L. G. Kiseleva, P. P. Eggleton, and S. Mikkola. Tidal friction in triple stars. *MNRAS*, 300(1):292–302, Oct. 1998.