Binary stars in general

and

Two case studies

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Structure of lecture

- All links to web-sites marked with **magenta colour**
- **General part** from text book version
  - **Binary Stars and Stellar Masses (p241-p246)**
  - **Evolution of Close Binary stars (p273, Fig. 12.10)**
  - **Main point: Binary star observations give direct estimates for stellar masses.**
- **Two case studies**: Case I & II (Own research)
  - **Ancient Egyptians** recorded period of eclipsing binary **Algol** 3000 years before modern astronomers.
  - Currently accepted **three members** of Algol system are Algol A, Algol B and Algol C. **I claim** that there are **many more members** in this system.
  - Other **extra** material outside text book = extra
- **Ambitious goal:** 70 p/90 min → Ideal pace → 1 min

Single stars and multiple systems

Optical binary stars

- Two stars close in sky, but far is space: **single stars**
- Two stars orbit each other: **binary stars = binaries**

Single stars

- Like **the Sun**, less than 50% of all stars

Multiple systems

- **Over 50% of all stars** in hierachial structures:
Two or more members
- **Binary**: two stars
- **Triple system**: binary and single star
- **Quadruple system**: two binaries
- **Case study II**: Undetected members? [7, 8]

Binary classification

- **Binary classification: based on discovery method**
  Not on physical properties of binaries

1. **Visual binaries**
   Both members seen $\equiv$ separation ≥ 0.1 arc sec
   Relative position changes can also be seen
2. **Astrometric binary stars**
   One member can only be seen
   Proper motion reveals invisible member
3. **Spectroscopic binary stars**
   Spectral line Doppler shifts follow P_{orb}
   Lines of **both** members seen (SB2)
   Lines of only **one** member seen (SB1)

Binary classification 5 min

4. **Photometric binary stars or Eclipsing variables**

   Members pass in front of each other during $P_{\rm orb}$

   Total or partial **eclipses** change apparent magnitude

- **Alternative classification: mutual separation**

1. **Distant binaries**

   Separation $\geq$ 1 AU

   $P_{\rm orb} =$ tens to thousands years

2. **Close binaries**

   $1{\rm AU} >$ **separation** $>$ stellar radius

   years $> P_{\rm orb} >$ hours

3. **Contact binaries**

   No **separation**: members touch each other

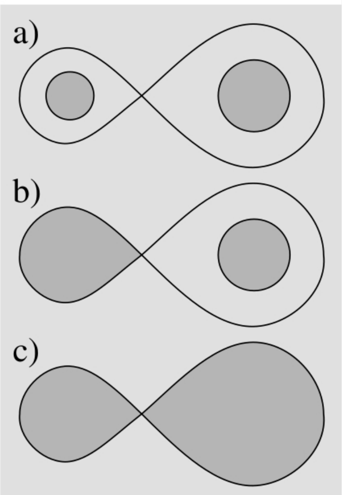
Separation versus Roche lobe

- **Roche lobe** concept: **Region around** binary member where orbiting **material is gravitationally bound** to this member
  → **Material outside** Roche lobe can **escape**
  → **Member** may, or may not, fill its Roche lobe

a) **Detached binary**

b) **Semi-detached binary**

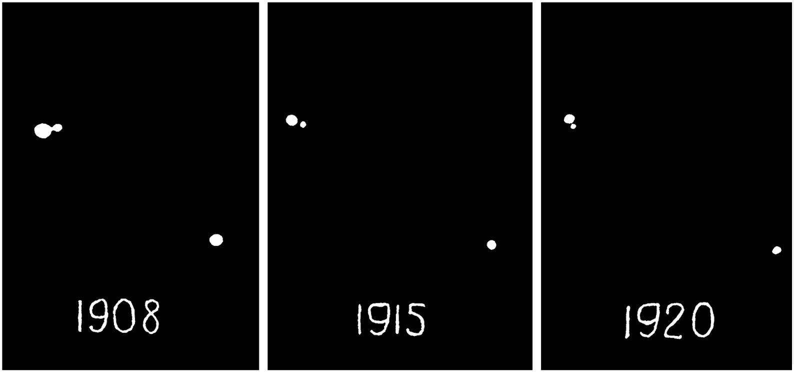
c) **Contact binary**



Visual binaries

Fig. 10.1 Visual binary Krüger 60:

- **Two red dwarfs** orbit each other in 44.6 years
- **Red dwarf** is a faint K or M type main-sequence star
- **Distance** 13.18 ± 0.08 ly $= 4.04 \pm 0.02$ pc

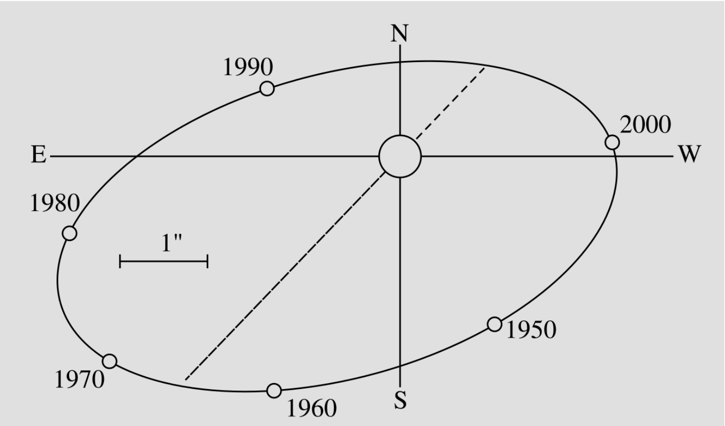


Visual binaries

- **Real motion**
  Both members elliptic orbit around common centre of mass of system (barycentre)
- **Relative motion**
  One member relative elliptic orbit around other member
  $\rightarrow$ Assume brighter **primary stationary**
  $\rightarrow$ **Measure** fainter **secondary** separation and direction with respect to **primary**
  $\rightarrow$ Long-term observations
  $\rightarrow$ **Relative orbit** of **secondary**

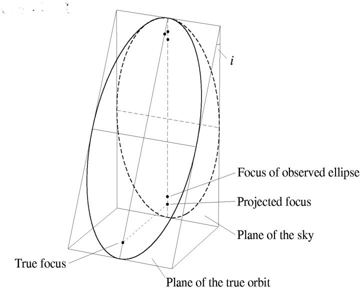
Visual binaries 10 min

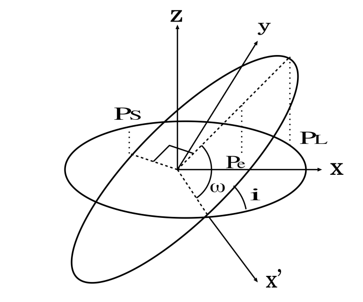
Fig. 10.2 First binary orbit determined: ξ UMa (xi Uma) in 1830 ($m_1 = 1.3M_\odot$, $m_2 = 1.0M_\odot$)



Visual binaries

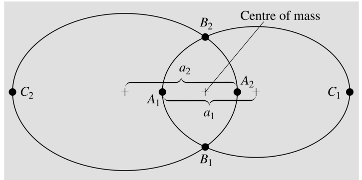
- **Observed ellipse** is a **projection** $\rightarrow$ Primary should be at relative orbit **focal point** $\rightarrow$ **Deviation** gives true orbit **orientation**
- extra Solutions for $i =$ **inclination** $\omega =$ **orientation** are not trivial **(Asada et al. 2004 Fig. 1, Eqs. 38 and 41)**





Visual binaries

Fig. 10.3 Motion around centre of mass



- **Simultaneous positions** A_1 and A_2, B_1 and ...
- **Real orbits** around **centre of mass** give true semimajor axes a_1 and a_2
- **Centre of mass relation**

$$\frac{a_1}{a_2} = \frac{m_1}{m_2}$$

$m_1 =$ **primary** mass, $m_2 =$ **secondary** mass

Visual binaries 15 min

- Observations: **Orbital period P**
- Observations: **Real orbit** semimajor axes a_1 and a_2
- **Relative orbit** semimajor axis (**primary** stationary)

$$\mathbf{a} = a_1 + a_2$$

If inclination and **distance** known
$\rightarrow$ **a** known
$\rightarrow$ **Kepler's third law** gives **total mass relation**

$$m_{\mathrm{tot}} = m_1 + m_2 = \frac{\mathbf{a}^3}{\mathbf{P}^2},$$

where $[\mathbf{a}] = \mathrm{AU}$, $[\mathbf{P}] = \mathrm{y}$, $[m_{\mathrm{tot}}] = [m_1] = [m_2] = M_\odot$

Visual binaries

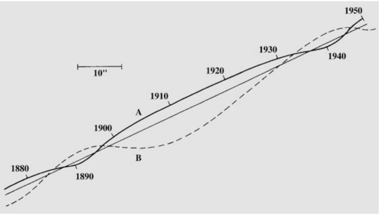
- Masses of both members from pair of equations
  centre mass relation
  and
  total mass relation

$$\begin{cases} m_1 = & (a_2/a_1)m_2 \\ m_1 = & m_{\rm tot} - m_2, \end{cases}$$

where $m_{\rm tot} = \mathbf{a}^3/\mathbf{P}^2$ is total mass of system

Astrometric binary stars

- **Only alternating proper motion** of **brighter** member seen, **fainter invisible**
- **Fig. 10.4 Sirius first astrometric binary discovered in 1830's:** Alternating proper motion

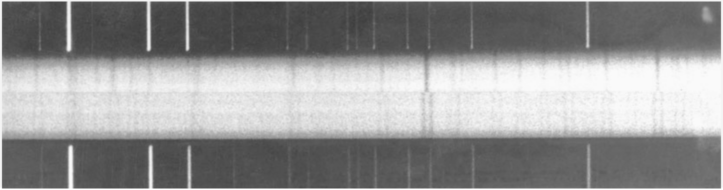


- **Sirius A** (A0 V) brightest star in sky
- **Sirius B** (white dwarf = stellar core remnant)
- **Highly eccentric orbit** ($e = 0.59$, $P_{\mathrm{orb}} = 50.^{\mathrm{y}}1$)
- **First exoplanets** not detected from **proper motion** of nearby stars, but from **radial velocity** observations **(Mayor & Queloz, 1995, Nature, 51 Pegasi b)**

Spectroscopic binaries

- **Appear as single**: members can not be separated
- **Spectra** show **regular** variation
- **First discovered** spectroscopic binary ζ UMa (zeta UMa) discovered in 1880's
- **Doppler shifts** $\rightarrow$ **Lines split** regularly $\rightarrow$ **Largest split** when other **approaching** and other **receding**

Fig. 10.5 Spectrum of κ Arietis.
Upper: single lines, Lower: double lines



Spectroscopic binaries 20 min

SB2 = **double-lined** spectroscopic binary
- **spectral lines** of **both** members observed

SB1 = **single-lined** spectroscopic binary
- **spectral lines** of only one member observed

- **Observed velocity**

$$v = v_0 \sin i, \quad (1)$$

$v_0 =$ **True velocity**
$i =$ **inclination** = **Angle** between **line of sight** and **normal of orbital plane**

- For example, **no radial velocity** changes are observed in spectroscopic binaries having $i = 0^{\circ}$.

Spectroscopic binaries

- **Combining** $m_1 a_1 = m_2 a_2$ and $a = a_1 + a_2$ gives **primary semimajor axis**

$$\mathbf{a_1} = \frac{\mathbf{a} m_2}{m_1 + m_2} \qquad (2)$$

- **Assuming** both orbits **circular** with radii a_1 and a_2
- **True orbital velocity** of **primary**

$$v_{0,1} = \frac{2\pi a_1}{P}$$

where P is **orbital period**
$\rightarrow$ **Observed radial** velocity of **primary**

$$v_1 = \frac{2\pi \mathbf{a_1} \sin i}{P} = \frac{2\pi \mathbf{a}}{P} \frac{m_2 \sin i}{m_1 + m_2} \qquad (3)$$

- This relation gives **primary semimajor axis** a_1, except that **inclination** i remains **unknown**

Spectroscopic binaries

- From this relation, one obtains solution for **semimajor axis of relative orbit**

$$\mathbf{a} = \frac{v_1 P}{2\pi} \frac{m_1 + m_2}{m_2 \sin i}$$

- Inserting this **a** into **Kepler's third law** gives

$$P^2 = \frac{4\pi^2}{G(m_1 + m_2)} \mathbf{a}^3 = \frac{4\pi^2}{G(m_1 + m_2)} \left[\frac{v_1 P}{2\pi} \frac{m_1 + m_2}{m_2 \sin i} \right]^3$$

- Re-arranging this, gives **mass function equation**

$$\frac{m_2^3 \sin^3 i}{(m_1 + m_2)^2} = \frac{v_1^3 P}{2\pi G} \qquad (4)$$

determined **only by observed** v_1 and P

Spectroscopic binaries

- **Case 1. If only primary** lines observed, radial velocities give v_1 and P, and mass function value

$$\frac{m_2^3 \sin^3 i}{(m_1 + m_2)^2} = \frac{v_1^3 P}{2\pi G}$$

$\rightarrow$ Masses m_1, m_2 and $m_{\mathrm{tot}} = m_1 + m_2$ **unknown**

Case 1: One way to proceed

- **Primary mass estimate** m_1 for example from **its spectral class**

$\rightarrow$ **Secondary mass** m_2 can be solved for different **inclinations** i

$\rightarrow$ Numerical value of m_2 usually solved **iteratively**

- **Solution** for inclination $i = 90^{\mathrm{o}}$, gives **lower limit for secondary mass** m_2

Spectroscopic binaries 25 min

- **Case 2. If lines of both** members observed, radial velocity measuments give v_1, v_2 and P.
- One can combine
  $v_1/v_2 = a_1/a_2$ from **circular orbit** relation
  $a_1/a_2 = m_2/m_1$ from **centre of mass** relation
- This gives

$$\mathbf{m_1} = \frac{m_2 v_2}{v_1}$$

- **Substitution of $\mathbf{m_1}$** into mass function of Eq. 4 gives

$$m_2 \sin^3 i = \left[\frac{v_2}{v_1} + 1\right]^2 \frac{v_1^3 P}{2\pi G}$$

- Observed v_1, v_2 and P give **value of** $m_2 \sin^3 i$
- Only inclination i remains unknown
- **Similar substitution of $\mathbf{m_2}$** gives **value of** $m_1 \sin^3 i$

Spectroscopic binaries extra

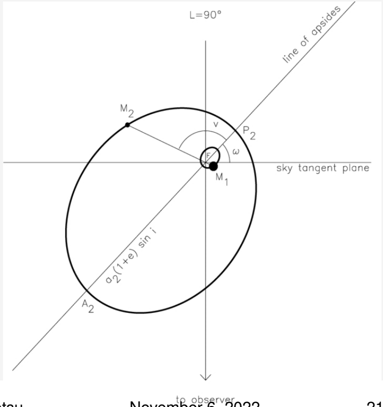
- Former equations assumed **circular orbit** $e = 0$
- All binary orbits **not circular, eccentricity** $e > 0$
- **Observed** radial velocity [12](Karami&Mohebi, 2007)

$$v_r(t) = v_{\odot} + K[\cos(\nu + \omega) + e\cos\omega]$$

- **Amplitude** (Constant!)

$$K = [(2\pi) \ / \ P] \ \ [(a \sin i) \ / \ (\sqrt{1 - e^2})]$$

- **Time** t
- **Barycentre velocity** $v_{\odot}$
- **Semimajor axis** a
- **Inclination** i
- **Eccentricity** e
- **Periastron longitude** ω
- **True anomaly** ν



Spectroscopic binaries extra

- Eccentric $e > 0$ radial velocity curves **not sinusoids**
- Computation of time dependence $v_r(t)$

1. **Mean anomaly** computed from

$$M = \frac{2\pi(t - t_p)}{P_{\mathrm{orb}}},$$

   where t_p is pericentre epoch

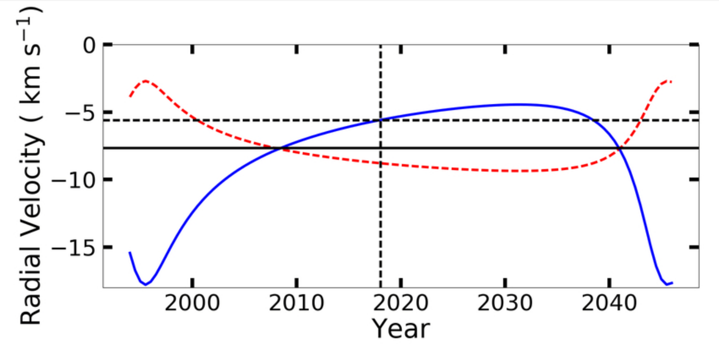
2. **True anomaly** computed from Fourier expansion

$$\begin{aligned}\nu &= M + (2e - \frac{1}{4}e^3)\sin M + \\ &\frac{5}{4}e^2 \sin 2M + \frac{13}{12}e^3 \sin 3M + O(e^4),\end{aligned}$$

   where $O(e^4)$ refers to omitted fourth order terms

Spectroscopic binaries

- **High eccentricity**: Radial velocity curves of **Sirius A** (red) and **Sirius B** (blue) having $e = 0.59$ [11].



- Definitely **not pure sinusoids**
- **Velocity curve shape** can be used to estimate **eccentricity** e and **periastron longitude** ω,
- Only **inclination** $\sin^3 i$ remains unknown

Photometric binary stars 30 min

Variability caused by **motions** of members

Eclipsing binaries

- **Members pass** each other $\rightarrow$ **Variable** brightness
- **Inclination** close to 90^{o}
- **Inclination** of these **spectroscopic binaries** known $\rightarrow$ **masses** known, because **inclination** $\sin^3 i$ known
- **Lightcurve shape** defines type

Algol

β **Lyrae**

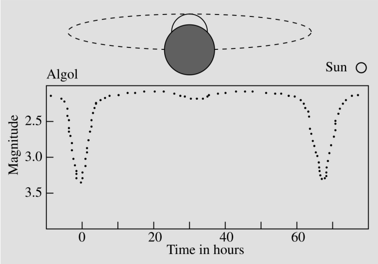
W Ursae Majoris

Algol stars

Note: Size of the Sun

Prototype: Algol (β Per)

- Constant brightness outside **eclipses**
- **Primary** minimum
- **Secondary** minimum



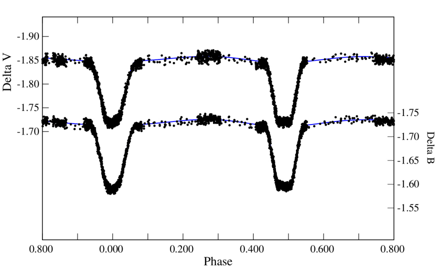
- **Member A** = smaller, hotter, brighter, main sequence
- **Member B** = larger, cooler, dimmer, giant/subgiant
- **Primary eclipse** = B covers A **totally or partially**
- **Secondary eclipse** = A covers B **totally or partially**

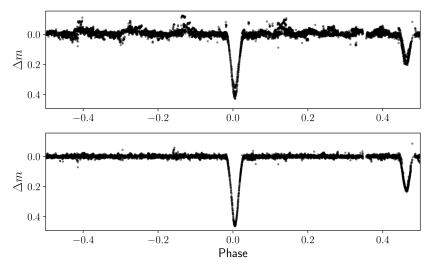
Algol stars

- **Shape** of light curve minima

Top: Total eclipse
$\rightarrow$ Other member **totally invisible**
$\rightarrow$ **flat** bottom minimum
([4] Fekel et al. 2013: VV Crv = HR 4821)

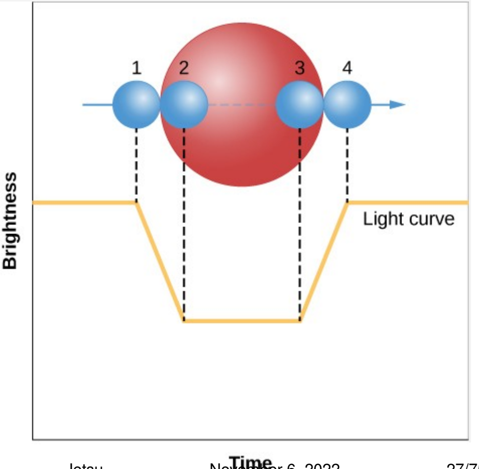
Bottom: Partial eclipse
$\rightarrow$ Other member **partly visible**
$\rightarrow$ **smooth** minimum
([17] Tallens et al. 2018: KW Hya)





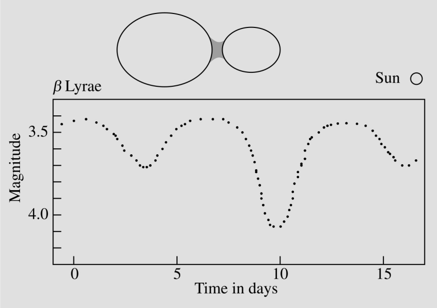
Algol stars extra

- **Duration** of binary eclipse
  $\rightarrow$ **Lightcurve** shape
  $\rightarrow$ **Total** eclipse
  $\rightarrow$ **Inclination** $i \approx 90^0$
  $\rightarrow$ **Radial velocities**
  $\rightarrow$ **Diameter/orbit** ratio
  $\rightarrow$ **Masses, Orbits and Radii** determined (regardless of distance)
  Figure from **lumenlearning.com (@CC BY)**



β Lyrae Stars 35 min

Note: Size of the Sun



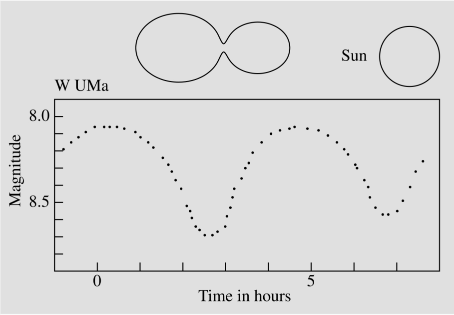
Prototype: β Lyrae
- Roche lobe of **one member** filled
$\rightarrow$ **Mass transfer** to other member

- **Close** in space
  $\rightarrow$ **Gravitational** pull
  $\rightarrow$ **Ellipsoidal** shape
  $\rightarrow$ **Projected** area changes **continuously**
  $\rightarrow$ Magnitude varies **continuously**
  $\rightarrow$ **Smooth** lightcurve

W UMa stars

Note: Size of the Sun

- **Prototype:**
  W UMa
  F8 V + F8 V
  $P_{\mathrm{orb}} = 0.3336$ days
- **Both members** fill their Roche lobe
  → **Contact binary**
  → **Elllipsoidal** shape
  → **Projected area** changes **continuously**
  → **Brightness** changes **continuously**
  → Lightcurve minima **round** and almost **identical**



Confusing classification

- Roche lobe **filled** → **Drop-like** shape
- **Limb darkening** → **Disk edges** darker
- **Elongated shape**
  → **Gravitational darkening**
- **Reflection effect**
  → **Heating** each other's surfaces
- **Mass transfer**
  → Changes surface **temperature**
- **Starspots** and other actitivy
  → Dark and/or bright **structures**

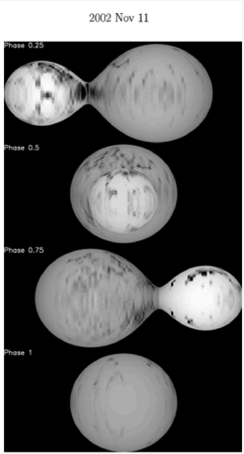


Figure: Contact binary AE Phe ([1] Barnes et al. 2004, Fig. 4)

Algol paradox 40 min

- **Rule of thumb:** *"More massive stars evolve faster"*

  Evolution of binaries can differ from evolution of **single stars**

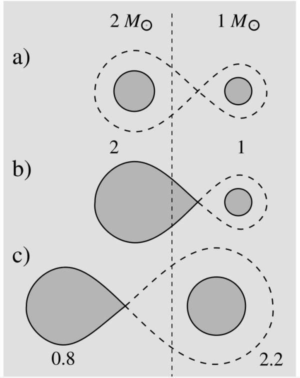
  - Eclipsing binary Algol $P_{\rm orb} = 2.867$ days
  - B8 **main sequence** Algol A has $3.2M_{\odot}$
  - K0 **subgiant** Algol B has $0.7M_{\odot}$

- **Paradox: If** ages same, **why** has less massive Algol B evolved away from main sequence, but not more massive Algol A? $\rightarrow$ **Solution: Mass transfer**

One binary evolution example

Example of one **binary evolution case**

a) Detached binary A+B formed. Initial masses $m_A = 2M_\odot$ and $m_B = 1M_\odot$



b) B **remains** in main sequence but A **evolves** away from main sequence,

$\rightarrow$ Roche lobe **filled**

$\rightarrow$ **Mass transfer** from A to B

$\rightarrow$ **Semidetached binary**

c) B becomes **more massive** than A

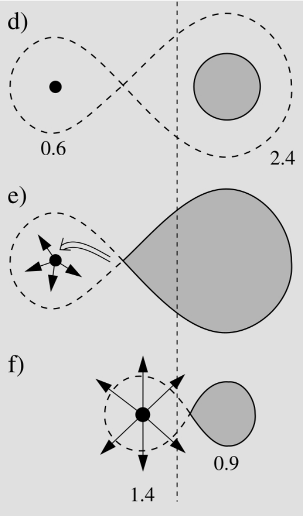
$\rightarrow$ Mass transfer **weakens** $\rightarrow$ **Detached binary**

One binary evolution example ...

d) Mass transfer **ends**
B **contracts** to
$0.6 M_{\odot}$ **white dwarf**

e) $2.4 M_{\odot}$ mass B **evolves**
to a **giant**
→ Roche lobe of B **filled**
→ **Mass transfer** from B to A
→ Nova outburst **explosions**

f) Mass of A **may** exceed
Chandrasekhar mass
limit $1.4 M_{\odot}$
→ A **explodes** as
type I supernova



Own binary research: Case I

Short modern history of variable stars

1. **Mira**
   Fabricius 1596: variability
   Holwarda 1638: 11 months period
   Pulsations: **expands and contracts**
2. **Algol**
   Montanari 1669: variability
   Goodricke 1783: 2.867 days period
   Eclipsing binary
   - 10h **primary eclipse**: observable with **naked eyes**
   - **Secondary eclipse**: observable only with telescope

Our main result: Ancient Eqyptians **detected Algol's variability and periodicity** three millennia earlier.

Own binary research: Case I

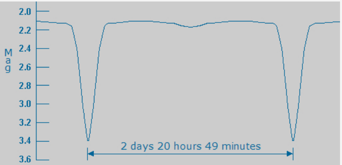
John Goodricke (17 September 1764 - 20 April 1786)

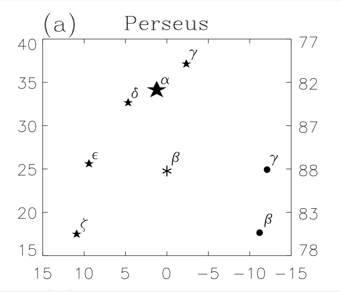
Own binary research: Case I 45 min

- In 3 hours, Algol becomes **dimmer** than all six $\ast$ & $\bullet$
- For 4 hours, Algol the **dimmest**
- In 3 hours, Algol becomes **brighter** than all six $\ast$ & $\bullet$
- **Goodricke's** discovery:
  - **Tabulated** eclipse epochs

  $\rightarrow$ Epochs **multiples** of 2.867^{d}

$\bigstar$	α Per	$1.^{\mathrm{m}}79$
$\ast$	β Per	$2.^{\mathrm{m}}12 \leftrightarrow 3.^{\mathrm{m}}37$
$\star$	ζ Per	$2.^{\mathrm{m}}85$
$\star$	ϵ Per	$2.^{\mathrm{m}}88$
$\star$	γ Per	$2.^{\mathrm{m}}93$
$\star$	δ Per	$3.^{\mathrm{m}}01$
$\bullet$	γ And	$2.^{\mathrm{m}}26$
$\bullet$	β Tri	$3.^{\mathrm{m}}00$





(Upper figure: @nightskyinfo.com)

- **Whole 10h eclipse** observed **only** every 19th night.

Own binary research: Case I

General results presented here. **Details** in papers

- [15] **Porceddu et al. (2008),** “Evidence of Periodicity in Ancient Egyptian Calendars of Lucky and Unlucky Days”, **Cambridge Archaeological Journal**
- [10] **Jetsu et al. (2013),** “Did the Ancient Egyptians Record the Period of the Eclipsing Binary Algol—The Raging One?”, **The Astrophysical Journal**
- [9] **Jetsu & Porceddu (2015),** “Shifting Milestones of Natural Sciences: The Ancient Egyptian Discovery of Algol’s Period Confirmed”, **Plos One**
- [14] **Porceddu et al. (2018),** “Algol as Horus in the Cairo Calendar: The Possible Means and the Motives of the Observations”, **Open Astronomy**

Own binary research: Case I

Analysed data

Ancient Egyptian **"Calendar of Lucky and Unlucky days"** in Papyrus Cairo 86637

- **Written** by Ancient Egyptian **scribes**
- **Dated** to 1271-1163 B.C.
- **Prognoses**: **Lucky** = Good and **Unlucky** = Bad
- **One year**: **Three prognoses** for each day
- Additional prognosis **descriptive texts**
- **"Hour-watchers"** measured **time from stars** for religious purposes
  $\rightarrow$ **Describe** astronomical and mythological events
  $\rightarrow$ **Thousands of years**: 300 clear nights every year
- **Descriptions** of other events: Flood of Nile, weather, seasons, human activity, animals, ...

Own binary research: Case I

50 min

Own binary research: Case I

- 1 year = 3 Seasons = 12 months (M) = **360** days (D)
- Every year: 5 epagomenal days $\rightarrow$ **365** days
- "G"=Gut=Good, "S"=Schlecht=Bad and "–" = Lost
- Each month **19 days** regularity:
- **Always** $D = 1 \equiv$ GGG
- **Always** $D = 20 \equiv$ SSS

Day D	Akhet I $M=1$	Akhet II $M=2$	Akhet III $M=3$	Akhet IV $M=4$	Peret I $M=5$	Peret II $M=6$	Peret III $M=7$	Peret IV $M=8$	Shemu I $M=9$	Shemu II $M=10$	Shemu III $M=11$	Shemu IV $M=12$
1	GGG	GGG	GGG	GGG	GGG	GGG	GGG	GGG	GGG	GGG	GGG	GGG
2	GGG	GGG	–	GGG	GGG	GGG	GGG	GGG	–	–	GGG	GGG
3	GGS	GGG	GGG	SSS	GGG	–	–	SSS	GGG	GGG	SSS	SSS
4	GGS	SGS	–	GGG	GGG	GGG	GSS	GGG	SSS	SSS	GGG	SSG
5	GGG	SSS	–	GGG	GSS	GGG	GGG	SSS	–	GGG	SSS	GGG
6	SSG	GGG	GGG	SSS	GGG	–	GGG	SSS	GGG	–	–	SSS
7	GGG	SSS	GGG	SSS	SSS	GGG	SSS	GGG	GGG	SSS	SSS	–
8	GGS	GGG	–	GGG	GGG	GGG	GGG	GGG	–	GGG	SSS	GGG
9	GGG	GGG	SSS	GGG	GGG	GGG	GGG	–	GGG	GGG	GGG	GGG
10	GGG	GGG	GGG	GGG	SSS	SSS	SSS	–	–	GGG	SSS	GGG
11	SSS	GGG	GGG	GGG	SSS	GGG	GGG	SSS	–	SSS	SSS	SSS
12	SSS	SSS	–	SSS	–	GGG	GGG	SSS	–	GGG	–	GGG
13	GSS	GGG	SSS	GGG	GGG	SSS	GGG	SSS	–	GGG	–	GGG
14	–	GGG	SSS	GGG	SSS	SGG	–	–	–	GGG	SSS	GGG
15	GSS	GSS	SSS	–	GGG	–	SSS	GGG	–	SSS	GGG	SSS
16	SSS	GGG	GGG	GGG	GGG	–	SSS	GGG	GGG	GGG	SSS	GGG
17	SSS	GGG	–	–	SSS	GGG	SSS	SSS	GGG	SSS	–	GGG
18	GGG	SSS	SSS	SSS	GGG	SSS	GGG	–	GGG	SSS	SSS	SSG
19	GGG	GGG	SSS	SSS	SSS	GSS	–	GGG	GGG	SSS	SSS	GGG
20	SSS	SSS	SSS	SSS	SSS	SSS	SSS	–	SSS	SSS	SSS	–
21	GGG	SSG	GGG	SSG	GGG	–	–	–	SSS	SSG	GGG	GGG
22	SSS	–	–	GGG	GGG	GGG	SSS	SSS	GGG	SSS	SSS	GGG
23	SSS	–	SSS	GGS	GGG	GGG	GGG	–	GGG	GGG	SSS	SSS
24	GGG	SSS	GGG	–	GGG	SSS	SSS	SSS	–	GGG	GGG	GGG
25	GGS	SSS	GGG	–	GGG	GGG	–	SSS	GGG	GGG	GSG	GGG
26	SSS	SSS	GGG	GGG	SSS	–	SSS	–	GGG	SSS	GGG	GSG
27	GGG	SSS	GGG	GGS	GGG	–	SSS	SSS	–	SSS	SSS	SSS
28	GGG	GGG	GGG	SSS	GGG	GGG	GGG	GGG	–	GGG	SSS	GGG
29	SGG	GGG	GGG	SSS	GGG	SSS	GGG	GGG	GGG	GGG	GGG	GGG
30	GGG	GGG	GGG	GGG	GGG	SSS	GGG	GGG	GGG	GGG	GGG	GGG

Own binary research: Case I

Computing series of time points t_i

- t_i **integer** part: $N_E = 30(M-1) + D$
- t_i **decimal** part:
  Texts suggest: morning, day, evening/night
  Alternative 1: Three daytime
  Alternative 2: Two daytime and one night-time
- **Integer** part + **Decimal** part = **Final** time points t_i
- Exact computation [10] (Jetsu et al. 2013: Eqs. 1-3)
  24 alternative samples $3 \times 2 \times 4$**:**
  - **3 alternatives**: Transformation to Gregorian days
  $\rightarrow$ Daytime and night-time length
  - **2 alternatives**: Analyse G and S separately
  - **4 alternatives**: Analyse also $D = 1$ and $D = 20$, or reject them

Own binary research: Case I

- **Time points** $t_i = t_1, t_2, ..., t_n$
- **Phase angles** $\Theta_i = 2\pi(t_i/P)$
- This gives **one round during** P **If time, draw!**
- **Unit vectors** $\bar{r}_i = [\cos\Theta_i, \sin\Theta_i]$
- **Sum of unit vectors** $\bar{R} = \sum_{i=1}^{n} \bar{r}_i$
- **Rayleigh test statistic**

$$z = \frac{|R|^2}{n} = \frac{|\sum_{i=1}^{n} \bar{r}_i|^2}{n} = \frac{1}{n}\left[\left(\sum_{i=1}^{n} \cos\Theta_i\right)^2 + \left(\sum_{i=1}^{n} \sin\Theta_i\right)^2\right]$$

- Vectors $\bar{r}_i$ point to **same** direction $\rightarrow |R|$ large $\rightarrow |z|$ large $\rightarrow t_i$ spacing **regular** with $P \equiv$ **Periodicity**
- Vectors $\bar{r}_i$ point to **different** direction $\rightarrow |R|$ small $\rightarrow |z|$ small $\rightarrow t_i$ spacing **irregular** with $P \equiv$ **No periodicity**

Own binary research: Case I

55 min

Finding the best period with Rayleigh test

Since **best period unkown**, we **test many possible periods.**

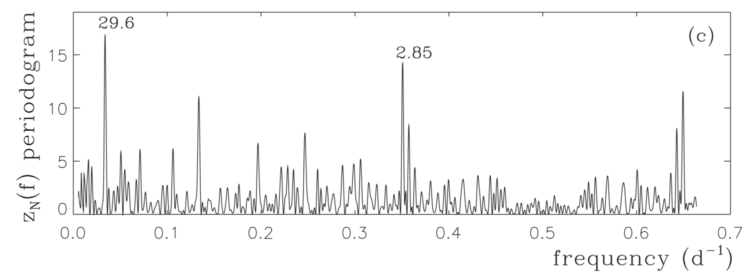
- **Select minimum** tested period $P_{\rm min}$.
- **Select maximum** tested period $P_{\rm max}$.
- Create a **dense grid of tested periods** P_j between $P_{\rm min}$ and $P_{\rm max}$.
- Compute **periodogram** $z = z(P_j)$ for every P_j.
- **Best period** $P_{\rm best}$ gives **highest peak** of periodogram, $z_{\rm max} = z(P_{\rm best})$ **maximum**.

- **Probability (significance=critical level)** for this $z_{\rm max}$ **peak** can be computed [10] Jetsu et al. (2013, Eq. 4)
- We tested periods from **1.5 days** (exceeds t_i spacing) to **90 days** (at least four rounds during 360 days).

Own binary research: Case I

All good $n = 564$ prognoses G

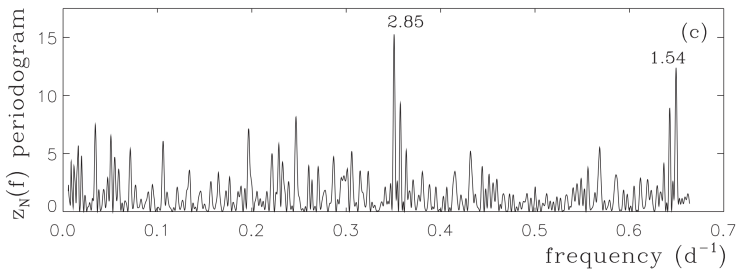
- **Best period $P_1 = 29.6$ days** reaches simulated critical level $Q^\star = 0.000012$
- **Second best period** $P_1 = 2.850$ days reaches simulated critical level $Q^\star = 0.00014$
- Exact computation of these **simulated critical levels** explained in [10](Jetsu et al. 2013: Eqs. 7 and 8)



Own binary research: Case I

Good $n = 528$ prognoses G, where D=1 removed

- Period $P_1 = 29.6$ days **vahishes**
- **Best period $P_1 = 2.850$ days** reaches simulated critical level $Q^{\star} = 0.000094 \rightarrow$ **significance increased!**
- **Second best period $P_1 = 1.540$ days** reaches simulated critical level $Q^{\star} = 0.00059 \rightarrow$ **spurious (unreal) periodicity** caused by data spacing



Own binary research: Case I

Bad $n = 351$ **prognoses S**

- **No significant periodicity**
- $D = 20$ removed $\rightarrow$ **No significant periodicity**

All 24 tested samples

- **Only significant periods** are 29.6 and 2.850 days in G prognoses
- This result **does not depend on**
  Chosen **decimal part** (3 daytime, or 2 daytime and 1 night-time)
  Transformation to **Gregorian days** (daytime and night-time length)

Main question: Could these be periods of the Moon (29.53 days) and Algol (2.867 days)?

Own binary research: Case I 60 min

Problem: Why is Algol's current $2.^{\rm d}867$ period longer than Algol's $2.^{\rm d}850$ period in Cairo Calendar?

Answer: Algol paradox

[16] Sarna 1993 (Algol's best evolutionary model)

- Zero-age main sequence:
  $m_B = 2.81 m_\odot$ and $m_A = 2.50 m_\odot$
- After 450 million years:
  Algol B evolves away from main sequence
  $\rightarrow$ Algol B fills its Roche-lobe
  $\rightarrow$ Mass transfer from Algol B to Algol A
  $\rightarrow$ Algol A becomes more massive than Algol B
  $\rightarrow$ **Orbital period** of Algol A and B system **increases**

Own binary research: Case I

- Equation of **mass transfer** [13] (Kwee 1958)

$$\frac{\dot{P}_{\mathrm{orb}}}{P_{\mathrm{orb}}} = -\frac{3\dot{m}_B (m_A - m_B)}{m_A m_B}$$

  $\dot{P}_{\mathrm{orb}} =$ period change
  $\dot{m}_B =$ mass transfer from Algol B to Algol A

- **Period increase** from 2.850 days to 2.867 during three thousand years gives period change $\dot{P}_{\mathrm{orb}}$ $\rightarrow$ **mass transfer** $\dot{m}_B = -2.2 \times 10^{-7} M_{\odot}\mathrm{yr}^{-1}$
- Best evolutionary model by [16] Sarna 1993 **predicted!** $\dot{m}_B = -2.9 \times 10^{-7} M_{\odot}\mathrm{yr}^{-1}$
- **Main result: Mass transfer** can explain 0.017 days increase of Algol's period during past three millennia.

Own binary research: Case I

Problem: Algol is **triple system**. [3] Curtiss (1908) discovered Algol C having $P_{\rm orb} = 1.9$ years.
$\rightarrow$ Algol C **perturbs** Algol A-B orbital plane
$\rightarrow$ **No Algol A-B eclipses** three millennia ago?

- This Algol A-B and Algol AB-C interaction relation is quite **complicated** [10] (Jetsu et al. 2013: Eq. 10)
- **Essential meaning** of this relation is **simple**: Algol A-B system orbital plane is **stable**, if **angle** between Algol A-B and Algol AB-C **orbital planes** is $\Psi = 0^{\rm o}$ **or** $90^{\rm o}$

  [18] Zavala (2010): $\Psi = 86^{\rm o} \pm 5^{\rm o}$ $\rightarrow$ Yes/No eclipses?

  [2] Baron et al. (2012): $\Psi = 90.^{\rm o}2 \pm 0.^{\rm o}32$ $\rightarrow$ Yes eclipses! **Note:** Our analysis predicted this result!

Own binary research: Case I

Problem: Why only periods of the Moon (29.6 days) and Algol (2.850 days) present in Cairo Calendar?

Solution: What shorter than 90 days periodic variability detectable in the sky with naked eyes?

- The Sun and planets $\rightarrow$ No
- The Moon and variable stars $\rightarrow$ Yes
- **Problem:** This would explain the Moon, but **why only** Algol out of all 40 000 variable stars?
- **Eight elimination criteria** C_1 ... C_8 applied to all 40 000 known variable stars
- **Naked eyes** limitations $m = 6^{\mathrm{m}}$ and $\Delta m = 0.^{\mathrm{m}}1$
- Variable star parameters: maximum brightness ($m_{\max}$), amplitude (Δm) and period (P)

Own binary research: Case I 65 min

- **Elimination** from 40 000 variable star candidates

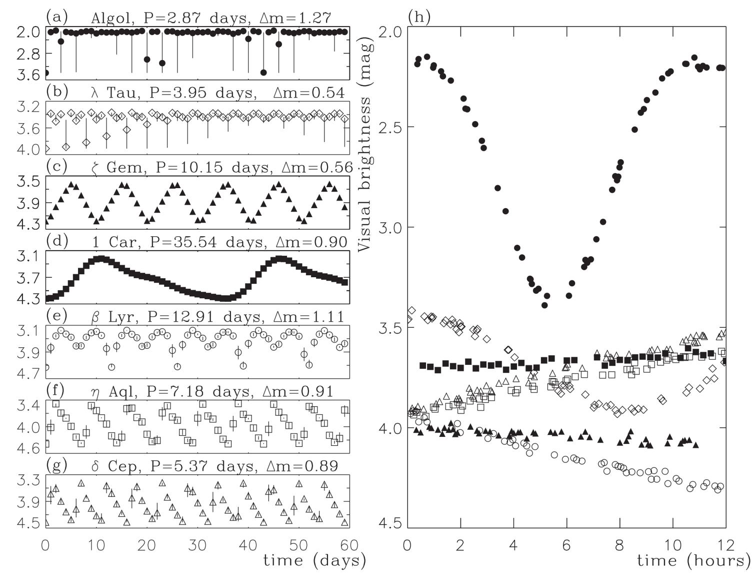
C_1 **Variability** fulfills $m_{\max} \leq 4$ and ($\Delta m > 0.4$)
$\rightarrow$ 109 candidates

C_2 **Period** known and fulfils $1.^{\mathrm{d}}5 \leq P \leq 90^{\mathrm{d}}$
$\rightarrow$ 13 candidates

C_3 Variable was not below, or too close to, **horizon**
$\rightarrow$ 10 candidates

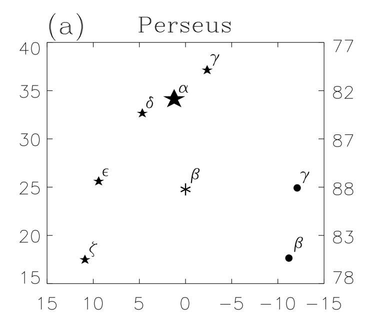
C_4 Variability can be **predicted**
$\rightarrow$ 7 candidates

C_5 Variability can be detected during a **single night**
$\rightarrow$ 2 candidates (Next page figure: Algol and λ Tau)


(a) Algol, P=2.87 days, Δm=1.27
(b) λ Tau, P=3.95 days, Δm=0.54
(c) ζ Gem, P=10.15 days, Δm=0.56
(d) 1 Car, P=35.54 days, Δm=0.90
(e) β Lyr, P=12.91 days, Δm=1.11
(f) η Aql, P=7.18 days, Δm=0.91
(g) δ Cep, P=5.37 days, Δm=0.89
(h)
Visual brightness (mag)
time (days)
time (hours)

Own binary research: Case I

- C_6 Variability changes **constellation pattern**.
  $\rightarrow$ 2 candidates
  (Only Algol and λ Tau)
- C_7 Period of variability **could be detected** by Ancient Egyptians
  (Only Algol and λ Tau)



- C_8 Variability and periodicity **was determined first**, Goodricke (1783: Algol) and Baxendell (1848: λ Tau)
  - **Main result:** Algol is clearly the best candidate of all 40 000 variable stars. This would explain why **only the Moon and Algol** are detected in Cairo Calendar.

Own binary research: Case I 70 min

Vocabulary in Cairo Calendar

- 28 **Selected Words** (SW)
- Periods of the Moon and Algol **known**
  Phase angles **known**

$$\Theta = (t_i - t_0) \times 360^{\mathrm{o}}$$

- SWs having high Rayleigh test statistic z **identified**
- SWs connected to Algol

  Horus, Re, Wedjat, Followers, Sakhmet, Ennead
- SWs connected to the Moon

  Earth, Heaven, Busiris, Rebel, Thoth, Onnophris
- **Good prognoses** connected to **bright phases** of Algol and the Moon

Own binary research: Case I

Good prognoses phase angles Θ for P=2.850 days

- Time runs counter-clockwise on this circle

  $\Theta = +6^{\mathrm{o}}$ *“It is the day of receiving the white crown by the Majesty of Horus”*

- **Horus**

  closed squares

- **Wedjat**

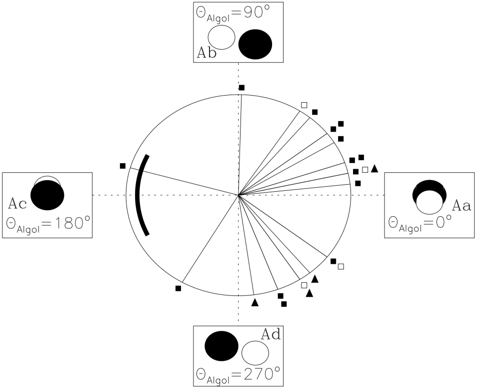
  open squares

- **Sakhmet**

  closed triangles

- 10 h eclipse

  thick curve

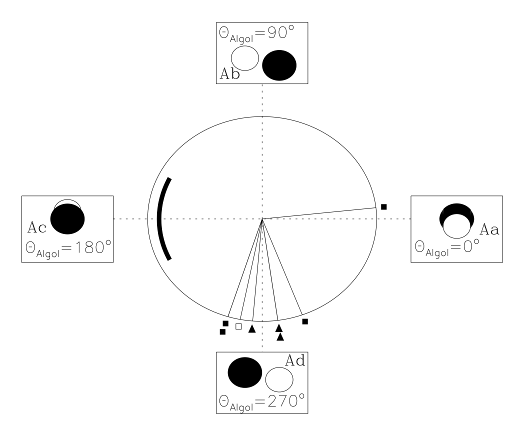


Own binary research: Case I

Bad prognoses phase angles Θ for P=2.850 days

$\Theta = +278^{\rm o}$ *"Do not go out of your house to spend time until the setting of the sun to the horizon. This is the day of the hidden-named "slaughterer-demons" of Sakhmet..."*

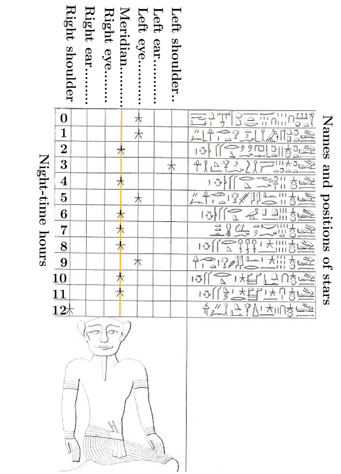
- **Horus** closed squares
- **Wedjat** open squares
- **Sakhmet** closed triangles



Own binary research: Case I

Most difficult to prove **why Algol?** → **Ten arguments**

- Daytime and night-time: **12 hours**
- Scribes called **“hour-watchers”** measured night-time with **hour-stars**
- This required at least **three stars in 24 hour-patterns** (72 stars)
- Algol 51st brightest star in Ancient Egypt
- **Argument 1.** Algol was an hour-star or belonged to an hour-star pattern



Own binary research: Case I **75 min**

- Scribes responsible for **both** Astronomy (hour-watching) and Priesthood (religious rituals)
- Night-time the Sun passed through **underworld**
- **Each hour, prayers and rituals** opened one of twelve underworld gates guarded by terrible gate-keepers.
- **Timing** of night-time religious rituals was **crucial**
- **Mess up** this timing and **the Sun may not rise!**
- **Argument 2.** Proper timing of the nightly religious rituals relied on fixed hour-star patterns.
- Algol's **eclipse** causes a clear **constellation change**
- **Argument 3.** A naked eye can easily discover the significant hour-star pattern change caused by Algol's eclipse.

Own binary research: Case I

$P = 2.^{\mathrm{d}}850 = 57^{\mathrm{d}}/20 \rightarrow$ 20 eclipses in 57^{d}

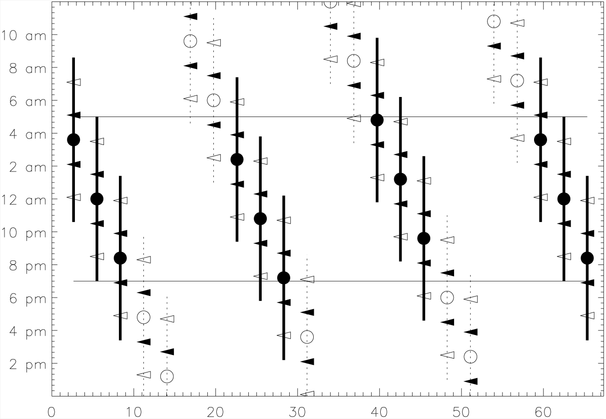
- **Nine night-time eclipses** in 57 days (observed)
- **Eleven daytime eclipses** in 57 days (not observed)
- **Every eclipse exactly** at same time in 57 days
- Scribes: *"This 19 days pattern always repeated"*

  1st night: **morning eclipse**
  2nd and 3rd night: **no eclipses**
  4th night: **mid-night eclipse** 10 hours!
  5th and 6th night: **no eclipses**
  7th night: **evening ecplipse**
  8th-19th night: **no eclipses**
  20th night: **morning eclipse** Again!

- Every month: $D = 1$ good, after 19 days $D = 20$ bad

Own binary research: Case I

- **Two rules**
  3+3+13=19
  19+19+19=57
- Eclipses **begin again** after 13 days
  → **Unlucky number is 13?**



- **Argument 4.** Scribes could have discovered Algol's 2.850=57/20 days period from regular 19 and 57 days eclipse cycles.

Own binary research: Case I

Argument 5. Three alternatives why Cairo Calendar contains Algol's 2.850=57/20 days period

- Scribes **solved** 2.850=57/20 days value.
- Scribes **did not solve** 2.850=57/20 days value, but **only recorded** 19 days cycle.
- Scribes **did not solve** 2.850=57/20 days value, but **only recorded** observed night-time eclipses.

- **Why** are there **no direct** references to star Algol?
- Observed **actions of divine deities (gods)** in the Sky (the Sun, the Moon, planets, stars) $\rightarrow$ Writing allowed **communication with gods** $\rightarrow$ Must avoid raising **anger of gods** $\rightarrow$ **Indirect reference** works

Argument 6. To avoid violating cosmic order, scribes would have referred to Algol's changes only indirectly.

Own binary research: Case I **80 min**

Argument 7. Cairo calender contains numerous extracts from two legends
"Destruction of Mankind"
"Contendings of Horus and Seth"

- Read these extracts in **temporal order**
  $\rightarrow$ Stories **make no sense**
- Read these extracts in **order** of the Moon's and Algol's **phases**
  $\rightarrow$ Stories follow these legends and **make sense!**

Argument 8. The above two legends were used to decribe indirectly the changes of the Moon and Algol

Own binary research: Case I

Problem: Why was Algol called “Horus”?

- Rejuvenation, the **power to disappear and re-appear**, was associated with “Horus”.
- Egyptologists have associated “Horus” to **many celestial objects**, depending on the context.

Argument 9. Algol could have been naturally associated with “ Horus” and called as such, because Algol can disappear and re-appear.

Argument 10. Astrophysical considerations support the idea that the 2.850 days period in Cairo Calendar can be the period of Algol.

Own binary research: Case I

- **If** the significant 2.850 days period
  is not connected to Algol, then
  this question must be answered:

  “What was the origin of the phenomenon that occurred every third day, but always 3 hours and 36 minutes earlier than before, and caught the attention of Ancient Egyptians?” In other words, what happened three times in a row at the night-time? Then it occurred during the daytime? After a gap of 13 days, it occurred again during the night-time?”
- So far, no one has been able to answer this question!
- **Case I completed!**

Own binary research: Case II

Light time effect

- Computed times of eclipsing binary (EB) eclipses
  $C = t_0 + i\, P_{\rm orb}$,
  where t_0 is zero epoch and i is an integer
- **Third body:**
  $\rightarrow$ Eclipses occur **earlier** when EB **approaches**
  $\rightarrow$ Eclipses occur **later** when EB **recedes**
  $\rightarrow$ O = Observed eclipse epochs differ from C = Computed epochs
  $\rightarrow$ O-C data may reveal **third, fourth, ... bodies**

- **Figure: CHARA interferometer image of Algol**
- Algol C was detected from Algol A-B **radial velocity changes**, not from Algol A-B **complex O-C changes**.

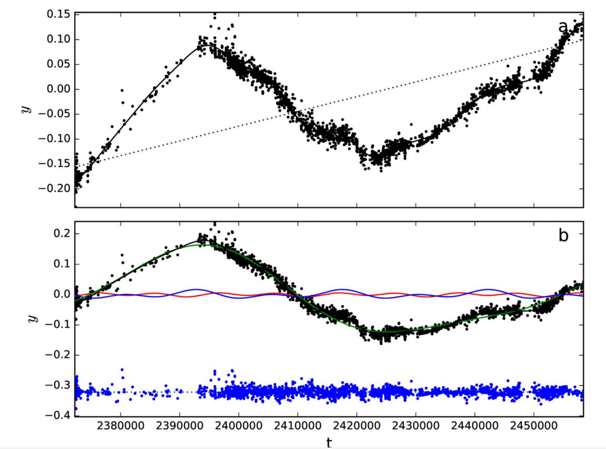
Own binary research: Case II

85 min

- **Algol A** and **Algol B** discovered by Ancient Egyptians
- Period re-detected by [5] Goodricke (1783)
  Data: Photometry
  Period: 2.867 days
- **Algol C** discovered by [3] Curtiss (1908)
  Data: Radial velocities of Algol A-B system
  Period: 1.9 years
- **Algol D, Algol E, Algol F, Algol G and Algol H** candidates discovered by [8] Jetsu (2021)
  Data: Observed minus Computed (O-C) eclipse epochs of Algol during past 237 years
  Periods: 19.96, 27.78, 33.7, 66.4 and 219.0 years

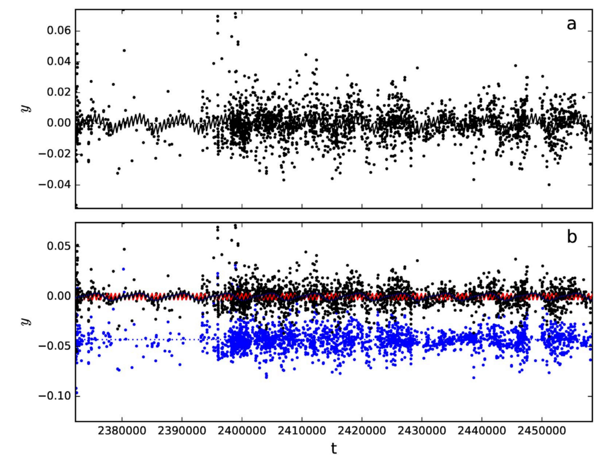
Own binary research: Case II

O-C data of Algol: Linear trend and three strongest signals. For details, see Jetsu (2021: Figure A7)



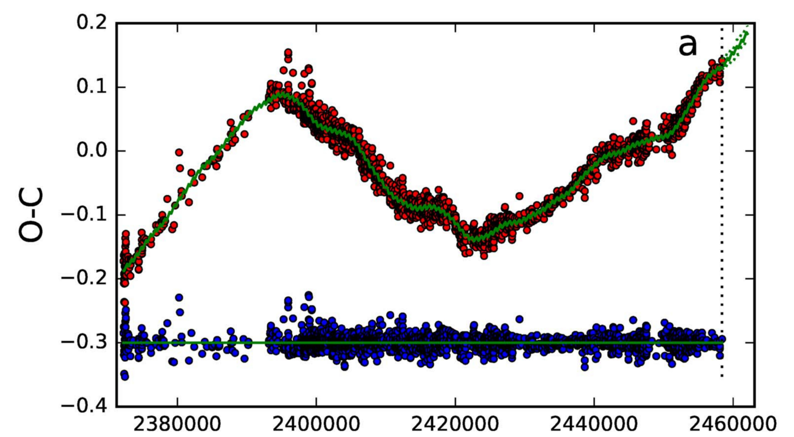
Own binary research: Case II

O-C data of Algol: two weakest signals. The shortest 680 days signal is that of Algol C. For details, see Jetsu (2021: Figure A10)



Own binary research: Case II

- All data (red dots), all signals (continuous black line) and residuals (blue dots). For details, see Jetsu (2021: Figure 1)



Own binary research: Case II

- **"Discrete Chi-square Method (DCM) for Detecting Many Signals"**
  [6](Jetsu, 2020)
  published in
  Open Journal of Astrophysics on April, 2020

→ **O-C data of Algol** analysed with DCM

- **"Say Hello to Algol's new companion candiates"**
  [8](Jetsu, 2021)
  published in
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Thank you!

90 min

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