Department of Mathematics and Statistics
Metric Geometry
Exercise 8
5.4.2006

Return by **Wednesday, April 5**.

A uniquely geodesic space X is said to be *metrically convex* if, for all constant speed geodesics $\alpha, \beta\colon [0,1] \to X$, we have

$$d\big(\alpha(t), \beta(t)\big) \leq (1-t)d\big(\alpha(0), \beta(0)\big) + td\big(\alpha(1), \beta(1)\big)$$

for all $t \in [0,1]$.

1. Prove that every CAT(0)-space is metrically convex.

2. Prove that a metrically convex (uniquely geodesic) space is contractible.

3. Let X be a CAT(0)-space, $p, q, r \in X$, and let $\alpha\colon [0,a] \to X$ and $\beta\colon [0,b] \to X$ be the unique geodesics from q to p and from r to p, respectively. Show that
$$d\big(\alpha(t), \beta(t)\big) \leq d(q,r)$$
for all $t \leq \min\{a, b\}$.

4. Prove that the product $X_1 \times X_2$ of CAT(0)-spaces X_1 and X_2 is a CAT(0)-space (cf. Exercise 7/6).

5. Let $X = \mathbb{R}^3 \setminus \{(x,y,z) \in \mathbb{R}^3 \colon x > 0, y > 0, z > 0\}$ be equipped with the length metric associated to the induced metric. Prove that X is not a CAT(0)-space.