Department of Mathematics and Statistics
Metric Geometry
Exercise 5
15.3.2006

Return by **Wednesday, March 15**.

No classes on Monday, March 6, and on Wednesday, March 8.

1. Prove that $\langle x, y\rangle_{n,1} \leq -1$ for all $x, y \in \mathbb{H}^n$ and that $\langle x, y\rangle_{n,1} = -1 \iff x = y$.

2. Let $x \in \mathbb{H}^n$, let $u \in x^\perp$ be a unit vector (w.r.t. $\langle \cdot, \cdot\rangle_{n,1}$) and let $\gamma \colon \mathbb{R} \to \mathbb{H}^n$,
$$\gamma(t) = (\cosh t)x + (\sinh t)u,$$
be as in (2.10) in the lecture notes. Find $\gamma'(t) \in \mathbb{R}^{n+1}$ and show that $\gamma'(t) \in \gamma(t)^\perp$. Compute
$$\|\gamma'(t)\| := \langle \gamma'(t), \gamma'(t)\rangle_{n,1}^{1/2}.$$

3. Let $Z = \{0, 1, 1/2, 1/4, \ldots, 2^{-n}, \ldots\}$. Glue isometrically together two copies of $\mathbb{R}$ along Z and let X be the resulting metric space (cf. Theorem 1.87). Let $\alpha \colon [0, \infty) \to X$ and $\beta \colon [0, \infty) \to X$ be two geodesics emanating from 0 (more precisely, from [0]) such that
$$\alpha(t) = \beta(t) \iff t \in Z.$$
Find $\angle_0(\alpha, \beta)$ and show that the angle does not exists in strong sense.

4. Let $\gamma_n \colon [0, 1/n] \to (\mathbb{R}^2, d_\infty)$,
$$\gamma_n(t) = \big(t, t^n(1-t)^n\big),\ n \in \mathbb{N},\ n \geq 2,$$
be geodesics emanating from the origin. Prove that $\angle_0(\gamma_n, \gamma_m) = 0$ for all $n, m \geq 2$.