Department of Mathematics and Statistics
Metric Geometry
Exercise 4
22.2.2006

Return by **Wednesday, February 22**.
In exercises 1, 2, and 3, let $(X_\alpha, d_\alpha)_{\alpha\in\mathcal{A}}$, Z, $i_\alpha\colon Z \to Z_\alpha$, and $(\bar{X}, \bar{d})$ be as in the isometric gluing along Z (cf. Theorem 1.87).

1. Show that $\bar{X}$ is complete if each X_α is complete.

2. Suppose that each X_α is locally compact and that the index set $\mathcal{A}$ is finite. Prove that $\bar{X}$ is locally compact.

3. Construct an example, where X_i is a complete geodesic space, $i \in \mathcal{A} = \{1,2\}$ but $\bar{X}$ is not a geodesic space.

4. Let (X,d) be a metric graph (cf. Example 1.86 (2)). Suppose that for each $v \in V$
$$\inf\{\ell(e)\colon e \in E, v \in \{\partial_0 e, \partial_1 e\}\} > 0.$$
Prove that (X,d) is a length space. Is it complete?

5. Let (X,d) be a metric space, $\mathcal{T}_d$ the topology determined by d, and let $\sim$ be an equivalence relation in X. Suppose that the quotient pseudometric $\bar{d}$ associated to $\sim$ is a metric in $\bar{X}$. Then it determines a topology $\mathcal{T}_{\bar{d}}$. On the other hand, $\bar{X}$ has the so-called quotient topology $\mathcal{T}_\sim$, where $U \in \mathcal{T}_\sim \iff \pi^{-1}U \in \mathcal{T}_d$. Prove that $\mathcal{T}_{\bar{d}} \subset \mathcal{T}_\sim$.