

Department of Mathematics and Statistics
Metric Geometry
Exercise 3
15.2.2006

Return by **Wednesday, February 15.**

1. Let (\mathbb{R}^2, d) be a metric space, where

$$d((x_1, y_1), (x_2, y_2)) = |x_1 - x_2| + \sqrt{|y_1 - y_2|}.$$

Find the generalized inner metric d_s associated to d . What is the topology \mathcal{T}_{d_s} determined by d_s ?

2. Let X be a length space and $x, y \in X$, $x \neq y$. Prove that

$$\text{dist}(x, B(y, r)) = |x - y| - r$$

if $r < |x - y|$.

3. Prove that the completion of a length space is a length space.

4. Construct a complete length space which is not a geodesic space.

5. Construct a locally compact geodesic space whose completion is neither geodesic nor locally compact.

[Hints overleaf.]

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4. Join the points $(0, 0)$ and $(0, 1)$ in the plane by paths of lengths $1 + 1/i$ intersecting only at endpoints $(0, 0)$ and $(0, 1)$. Equip the set $(\subset \mathbb{R}^2)$ thus obtained with the induced length metric.

5. Equip the set

$$((0, 1] \times \{0\}) \cup ((0, 1] \times \{1\}) \cup \bigcup_{i=1}^{\infty} \{1/n\} \times [0, 1]$$

with the induced length metric.