Department of Mathematics and Statistics
Metric Geometry
Exercise 2
8.2.2006

Return by **Wednesday, February 8**.

1. Let (X, d) be a metric space and $0 < \alpha < 1$. Find all rectifiable paths in the metric space (X, d^α).

2. Let $f\colon [0,1] \to [0,1]$ be the Cantor 1/3-function and let $\gamma\colon [0,1] \to \mathbb{R}^2$ be the path
$$\gamma(t) = (t, f(t)).$$
Compute $V_\gamma(0,t)$ for $t \in [0,1]$. Study the existence and values of the metric derivative $|\dot{\gamma}|(t)$. Draw conclusions.

3. Construct a rectifiably connected metric space (X, d) such that $\mathcal{T}_{d_s} \not\subset \mathcal{T}_d$. In other words, that there exist open sets in the topology given by the inner metric d_s that are not open in the original topology given by d.

4. Prove that metric spaces $(\mathbb{R}^2, d_1)$ and $(\mathbb{R}^2, d_\infty)$ are not uniquely geodesic spaces by giving examples of points x and y that can be joined by more than one geodesic. Here d_1 and d_∞ are the metrics given by norms $\|\cdot\|_1$ and $\|\cdot\|_\infty$, respectively.

5. Prove Theorem 1.64 (b) in the lecture notes.