Dynamic choice

- In our choices we think also the consequences of them to our future choices
- In particular, how should one model new opportunities or changing preferences
- Are regret, temptations and other emotions undescribable in the decision theory language of decision theory?
- What behavioral, i.e. observational, implications can such dynamic considerations have?
- Normative question: which is better, flexibility or restrictions?

Example

You are choosing a retaurant with different menus

1. {pizza, pasta, salad}
2. {pizza, pasta}
3. {salad}

Not knowing your future preferences may suggest emphasizing flexibility, i.e. restaurant 1. Knowing that one may be tempted to choose unhealthty choice when healthy is present would require taking temptation into account, i.e. restaurant 3. Preferring unhealthy choice and avoiding regret would lead to the choice of restaurant 2?

- Kreps 1979 on preferences for flexibility
- Let there be a set X of alternatives
- The preferences $\succsim$ over $2^X \backslash \emptyset$ describe how desirable it is to choose from $A \subseteq X$ relative to choose from $B \subseteq X$
- If there is an underlying preference relation on $\succsim^*$ on X, then we can construct a preference relation $\succsim$ such that $A \succsim B$ if and only if there is $x \in A$ such that $x \succsim^* y$ for all $y \in B$
- If the converse holds, then $\succsim$ is said to be *strategically rational*
- Observation: Preferences are $\succsim$ strategically rational if and only if $A \succsim B$ implies $A \succsim A \cup B$, for all $A, B \subseteq X$

- But future preferences are not always known when decision is made
- Preferences $\succsim$ are strategically rational under uncertainty if there are k distinct utility functions u_i on X such that

$$A \succsim B \quad \text{if and only if} \quad \sum_{i=1}^{k} \max_{x \in A} u_i(x) \geq \sum_{i=1}^{k} \max_{x \in B} u_i(x)$$

- Independence of the prior distrbution is without loss of generality

Theorem

Preferences $\succsim$ are strategically rational under uncertainty if and only if
(i) $A \succsim B$ if $B \subseteq A$
(ii) if $B \subseteq A$ and $A \sim B$, then $A \cup C \succsim B \cup C$, for any $C \subseteq X$

- The preceding analysis assumed that preferences are independent of the context - the key meta-assumption behind decision theory
- But many aspects, e.g. temptation and regret, of decision making seem to be precisely dependent on the context - leading to the issues of time-inconsistency
- How should one model this in the standard language of preferences?
- The resulting model should still have testable implications, i.e. rely on revealed preferences

- Models of temptation vs. flexibility include Gul and Pesendorfer 2001, and Dekel, Lipman and Rustichini 2009
- Let X be a finite set and 2^X its power set
- The DM is identified with a preference relation $\succsim$ on 2^X

GP1 (weak order) $\succsim$ is complete and transitive

- The preference $\succsim$ represents the DM's ranking of choice problems in period 0 with the understanding that in period 1 one alternative from the set must be chosen for consumption.

- We model a decision-maker who must deal with temptations => adding an alternative x to a choice problem A may make the DM strictly worse off in which case $A \succ A \cup \{x\}$, or adding an alternative x may make the DM better off in which case $A \cup \{x\} \succ A$, and x will be chosen from $A \cup \{x\}$
- The element $x \in X$ is an *opportunity* to $y \in X$, denoted $x \succ_O y$, if there exists $A \in 2^K \backslash \emptyset$ such that $y \in A$ and $A \cup \{x\} \succ A$.
- The element $x \in X$ is a *temptation* relative to $y \in X$, denoted $x \succ_T y$, if there exists $A \in 2^K$ such that $y \in A$ and $A \succ A \cup \{x\}$
- Opportunities and temptations could be thought as context dependent preferences

GP2 (acyclic opportunities and temptations) The binary relations $\succ_O$ and $\succ_T$ are acyclic

- Note that if GP2 holds, there are increasing functions v and w that represent $\succ_O$ and $\succ_T$, respectively, i.e. $w : X \to \mathbb{R}$, $v : X \to \mathbb{R}$ such that $x \succ_I y$ implies $w(x) > w(y)$ and $x \succ_O y$ implies $v(x) > v(y)$ (but not necessarily vice versa)
- We say that $U : 2^X \backslash \emptyset \to \mathbb{R}$ is a *temptation-self-control* utility if there exists (u, v, w) such that $u : w(X) \times v(X) \to \mathbb{R}$ where u is increasing in the first argument, decreasing in the second, and

$$U(A) = u(\max_{x \in A} w(x), \max_{y \in A} v(y))$$

- That is, temptation-self-control utilities reflect a tradeoff between opportunities and temptations when choosing the outcoem from which to choose in the later stage

Theorem

The preference relation $\succsim$ satisfies axiom GP1-GP2 if and only if it it can be represented by a temptation-self-control utility

- If there are no temptations, i.e. $\succ_T$ is empty, we may take v as a constant function and $U(A) \geq U(B)$ whever $B \subseteq A$
- Thus, without temptations one would maximize the flexibility to take advantage of opportunities

Proof.

We prove that the axioms imply the representation. Let v and w be as in the bullet point above.

Step 1: We show that if x maximizes w in A and y maximizes v in A, then $A \sim \{x\} \cup \{y\}$. If $\{x\} \cup \{y\} = A$, we are done. Otherwise, by our choice of w and v it follows that $z \not\succ_O x$ and $z \not\succ_T y$ for all $z \in A$. Hence, $\{x\} \cup \{y\} \sim \{x\} \cup \{y\} \cup \{z\}$. Continuing in this fashion we get $\{x\} \cup \{y\} \cup \{z\} \sim \{x\} \cup \{y\} \cup \{z, z'\}$, for $z' \in A$, etc.

Step 2: We show that $\max_A w \geq \max_B w$ and $\max_A v \leq \max_B v$ imply $A \succsim B$. □

Proof.

Let $x \in \arg\max_A w$ and $y \in \arg\max_B v$. Since $w(x) \geq w(y)$ it follows, by $y \not\succ_O x$, that $A \succsim A \cup \{y\}$. Since $v(x) \leq v(y)$ it follows, by $x \not\succ_T y$, that $B \cup \{x\} \succsim B$. By Step 1, we have $A \cup \{y\} \sim B \cup \{x\} \sim \{x\} \cup \{y\}$ and therefore

$$A \succsim A \cup \{y\} \sim B \cup \{x\} \succsim B.$$

Let $U : 2^X \backslash \emptyset \to \mathbb{R}$ be any function that represent $\succsim$. Define u such that

$$u(\max_A w, \max_A v) = U(A), \quad \text{for all } A$$

Then, by Step 2, u is increasing in the first argument and decreasing in the second. $\square$

- Finite set of alternatives: no technical axioms needed
- When lotteries are allowed, further assumptions are needed to pin down a characterization
- Let L denote the set of lotteries and A, B sets of lotteries in the set $2^L \setminus \emptyset$ of sets of lotteries
- Temptation is now described by a *set betweenness* -condition: If $X \succeq Y$, then $X \succeq X \cup Y \succeq Y$.
- Can be shown that GP2 implies set betweenness

Theorem

(Gul and Pesendorfer 2001): The binary relation $\succeq$ satisfies weak order, continuity, independence and set betweenness if and only if there are continuous linear functions U, u, v such that

$$U(A) := \max_{x \in A}(u(x) + v(x)) - \max_{y \in X} v(y), \text{ for all } X$$

and U represents $\succeq$

- A single decision maker
- Generalization: fully history dependent choices
- Question, what are the testable implications?
- Problematic welfare comparisons