Intertemporal preferences

- How should one compare intertemporal choice problems?

Examples

Problem 1: Should one take 1000€ today or 2000€ after three years?
Problem 2: Have a badly infected tooth operated today or wait a month with a risk of infecting the neighboring tooths as well?
Problem 3: Start preparing for an exam today or wait until you have too little time
Problem 4: Get 100€ today or a lottery of getting 50€ tomorrow or 150€ in a month

- Waiting means trade-off, but waiting is not always undesirable
- Problems where the DM chooses a timing of an incidence abound in economic context
- Why should we care?
  - Impatience
  - Uncertainty
  - Opportunity costs
  - Increased wealth
  - Modeling purposes

- How should we evaluate distinct time-outcome pairs (x, t)?
- Standard way: discounting - there is $\delta \in (0, 1)$ and a utility function u such that the current value of x at period t is

$$\delta^t u(x)$$

- Is there a behavioral justification?
- Preferences $\succsim$ on $X \times \mathbb{N}$ whose typical element (x, t) has the interpretation "outcome x is implemented in time t"
- What properties should one impose on $\succsim$?

- Axiomatic framework by Fishburn and Rubinstein (1982)
- Let the set of choices X be an interval in $\mathbb{R}$ whose elements admit the natural $\geq$ relation

FR1 (Weak order) $\succsim$ are transitive and complete

FR2 (Monotonicity) If $x > y$ then $(x, t) \succ (y, t)$, for all $x, y \in X$, for all $t \in \mathbb{N}$

  - (Un)desirability of an act does go away over time

FR3 (Time is valuable) If $x > 0$ and $t < s$ then $(x, t) \succ (x, s)$, for all $x \in X \backslash \{0\}$, for all $t, s \in \mathbb{N}$

- Having positive consumption $x > 0$ as soon as possible is desirable whereas with undesirable things $x < 0$ we want to procrastinate
- FR3 implies that the DM is impatient

FR4 (Continuity) $\{(y, s) : (y, s) \succsim (x, t)\}$ and $\{(y, s) : (x, t) \succsim (y, s)\}$ are closed (in the product topology)

- That is, for any sequences $\{(x_k, t)\}_k$ and $\{(y_k, s)\}_k$ such that $x_k \to x$ and $y_k \to y$, if $(x_k, t) \succsim (y_k, s)$ for all k, then $(x, t) \succsim (y, s)$

FR5 (Stationarity) $(x, t-1) \succsim (y, t)$ if and only if $(x, t) \succsim (y, t+1)$, for all $t \in \mathbb{N}$ and $x, y \in X$

- Stationarity requires that intertemporal tastes are not sensitive to the calender date - they only depend on the distance to the consumption as well as what is consumed

Theorem

$\succsim$ satisfies FR1-FR5 if and only if there is, for any $\delta \in (0,1)$, a Bernoulli utlity function u such that $(x, t) \succsim (y, s)$ if and only if $u(x)\delta^t \geq u(y)\delta^s$

- That is, the combinitaion of the utility function and the discount rate make the model falsifable

Proof.

Assume, without loss of generality, $X = [0, 1]$. We first claim that there is a unique function $f : X \to X$ such that $(f(x), 0) \sim (x, 1)$, for all $t \in \mathbb{N}$. FR3 implies $(x, 0) \succ (x, 1) \succ (0, 0)$. By FR2 and FR4, there is a unique $f(x)$ such that $(f(x), 0) \sim (x, 1)$. By FR2, f is strictly increasing.

Identify the sequence $\{x_k\}$ such that $f(x_k) = x_{k+1}$, for all $k = 0, 1, \ldots$ Then any x belongs to exactly one such maximal sequence. $\square$

(cont).

Define recursively $f^{k+1}(x) = f(f^k(x))$, for all x and for all $k \geq 1$, and $f^0(x) = x$. Take $\delta \in (0,1)$ and construct a function $u : X \to X$ as follows. Let $1 = u(1)$ and let u be a continuous and strictly increasing on $(f(1), 1]$ with $\lim_{x \downarrow f(1)} u(x) = \delta$. Let $\delta = u(f(1))$ and define u on $(f^2(1), f^1(1)]$ such that $u(f(x)) = \delta u(x)$ for all $x \in (f^2(1), f^1(1)]$. Continue recursively on all intervals $(f^{k+1}(1), f^k(1)]$ for all $k = 1, 2, \ldots$ to obtain values of u. Since

$$[0,1] = \cup_{k=0}^{\infty} \cup (f^{k+1}(1), f^k(1)]$$

it follows that u is well defined. Moreover, it satisfies

$$u(f(x)) = \delta u(x), \text{ for all } x \in [0,1]$$

$\square$

(cont.)

Noting that

$$u(f(f(x))) = \delta u(f(x)), \text{ for all } x \in [0,1]$$

it follows that

$$u(f^t(x)) = \delta^t u(x), \text{ for all } t \in \mathbb{N}, \text{ for all } x \in [0,1]$$

Since, by FR5,

$$(f^t(x), 0) \sim (x, t), \text{ for all } t \in \mathbb{N}, \text{ for all } x \in [0,1]$$

the result is implied, by FR1 and FR2. $\square$

- Note that $f(x) < x$ holds also for negative numbers x, i.e. procrastination with "bads" is completely rational
- The key benefit of the discounting model is that all the motives underlying an interetemporal choice can be comndenced into a single discounting parameter δ
- This makes the model particularly useful in applications:
  - Game theory
  - Corporate finance and asset pricing
  - Pensions
  - Public finance
- But: the model will not survive empirical testing

- The key problem: stationarity
- Stationarity guarantees that the nature of the per-period tradeoffs will not change as time evolves
- In particular, a tradeoff in the future looks today the same as it will look in the future
- Leads to dynamic consistency: every action that is optimal from a single period point of view will also be generally optimal
    - Separability of decision problems
    - Dynamic programming

Example

Which would you choose:

1. 100€ today or 110€ in one week
2. 100€ in ten weeks or 110€ in eleven weeks

- 100 today and 110€ in eleven weeks cannot be explained with a standard discounting model
- How to model the preference reversal?
- Discount factor time dependent?

- Hyperbolic discounting (the $\beta - \delta$ model): intertemporal payoff from outcome x at period t is defined by

$$\beta\delta^t u(x)$$

where $\beta, \delta < 1$
- Big cost $1 - \beta\delta$ of delaying consumption one day now but small cost $1 - \delta$ of delaying it after one year

Example

Savings: Let x_t be the consumption choice in period $t = 1, 2, 3$ with $x_1 + x_2 + x_3 \leq w$. Period 0 FOC x^0 satisfies

$$u'(x_1^0) = \delta u'(w - x_0^0 - x_1^0)$$

However, in period 1, FOC (x_1^1, x_2^1) implies

$$u'(x_1^1) = \beta\delta u'(w - x_0^0 - x_1^1).$$

Now $x_1^1 > x_1^0$ and $x_2^1 < x_2^0$ for strictly concave u. Thus the hyperbolic discounter wants to *reallocate* the savings at period 1 which is in conflict with the inital efficiency.

- Does discounting make sense?
  - Cognitive limitations
  - Unobserved factors
- Extensions
  - Habit formation
  - Utility from anticipation: choice dependent preferences
  - Multiple selves models