Decision Theory

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FDPE

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This course

- Main material:
  - Lectures
  - Gilboa (2009), Kreps (1986)
  - Selected articles
- Excerises in both Fridays
- Time table
  - First week: Classical theory
  - Second week: Modern variations
- Requirements
  - Exam/problem sets
  - Term paper

Introduction - Methodology of decision theory

- The goal of decision theory is to understand human behavior, and to operationalize this understanding to the more general use
- Techniques formal but the emphasisis is in capturing empirical regularities

- Recent experimental and empirical evidence on human decision making, which often comes from psychology or neuroscience, has fostered much research in decision theory
- Many familiar behavioral patterns, e.g. temptation, time inconsistency or reference dependence, are seemingly in conflict with the standard decision theoretic framework
- This course surveys the central pieces of classical decision theory and some of the important recent developments
- The aim is to provide a cohesive and integrated view of the methods and arguments

- The dimensions of decision making that we are particularly interested in are its observable implications, choice under uncertainty, and intertemporal choice
- Our emphasis will be in the "rational" modeling of economic decision making which is a particularly useful approach from the perspective of applications
- We shall demonstrate that many of the seeming "biases" can be explained in this framework
- We will, however, also discuss the recent development in neuroeconomics and its potential contribution to the discipline of economics

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The fox was wandering in the forest and spotted a buch of grapes hangin in a high branch. The fox jumbed but failed to reach them. Giving up, the fox lifted its nose and said "they are probably sour anyway"

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- $=>$ desirability independent from feasibility

- Can choice be predicted?
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- Determinism vs. free will
- Free will may be illusion, but useful illusion
- Logical positivism: any used concept in theory should be related to observables

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- But then: theories are always wrong

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- But then: theories are always wrong
- Postmodern view of economics (science!): objective, accurate reality is not reachable and theories rethorical devices, stories
- Communication is the key

- To convey information, one should be clear what one means
- Behavioral assumptions behind DT:
- Revealed preference
    - Maximization
    - Context independence
- Decision matrix

Axiomatic method

- Why characterization?
- Since theories are not accurate, their representation matters
- Simplicity
- Testability
- "Scientific approach"
- Consistency and independency

- Normative approach
  - A mode of behavior that people would like to see themselves following, once exposed

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- Descriptive approach
  - Mode of behavior that we see people following
  - Tversky: "Given me an axiom and I'll design an experiment that refutes it"

From choices to preferences to utilities

- Set of alternatives X and a binary relation $R \subset X \times X$ written often xRy and having interpretation "x stands in relation to y" if xRy

Examples

$X = \{1, 2, 3\}$ and $R = \{(1, 2), (2, 3), (3, 1)\}$
$X =$Finnish citizens, $R = \{(x, y) : x$ is married to $y\}$
$X = \mathbb{R}$, $R = \geq$
$X = \{$commodity bundles$\}$, $R = \{(x, y) : x$ is at least as desirable as $y\}$

- Properties:

P1 (Complete) Either xRy or yRx for all $x, y \in X$

P2 (Transitive) If xRy and yRz, then xRz for all $x, y \in X$

P3 (Asymmetric) If xRy and yRx, then $x = y$ for all $x, y \in X$

P4 (Reflexive) xRx for all $x \in X$

P5 (Acyclic) If $x_0 R x_1 R \ldots R x_k$, then $x_0 \neq x_k$

- Binary relation $\succsim$ is a **preference relation** (weak order) if it is complete and transitive
- But: peanuts
- Completeness of $\succsim$ implies reflexivity
- The asymmetric part of $\succsim$, denoted by $\succ$, is called **strict** preference
- The strict preference relation $\succ$ is acyclic
- If $\succsim$ is complete and $\succ$ is acyclic, then $\succsim$ is a preference relation

- We say that a binary relation $\succsim$ on X is **represented** by a utility function $u : X \to \mathbb{R}$ if

$$u(x) \geq u(y) \quad \text{if and only if} \quad x \succsim y$$

Proposition Let X be a **finite** set. Binary relation $\succsim$ is representable by a utility function if and only if it is a preference relation (P1&P2).

Proposition Let X be a **countable** set. Binary relation $\succsim$ is representable by a utility function if and only if it is a preference relation (P1&P2).

Example (Lexigraphic preferences)

Let $X = [0,1]^2$ and define preferences $\succsim$ such that $(x_1, x_2) \succ (y_1, y_2)$ if $x_1 > y_1$ or $x_1 = y_1$ and $x_2 > y_2$. Then there is no utility function that represents preferences. For suppose that u is such function. Let $r(x_1)$ be a rational number such that $u(x_1, 1) > r(x_1) > u(x_1, 0)$ for all $x_1 \in [0,1]$. Since u represents $\succsim$, $r(x_1) > u(x_1, 0) > u(x_1', 1) > r(x_1)$ for all $x_1 > x_1'$. Thus r is an onto function from $[0,1]$ to a subset of rational numbers. But this is impossible since the cardinality of the set of rational numbers is countably infinite whereas and that of $[0,1]$ is continuum, i.e. uncountably infinite.

P6 (Separability) There exists a countable set $Z \subseteq X$ such that for all $x \succ y$ there is $z \in Z$ such that $x \succsim z \succsim y$

Theorem

Let X be a set. Binary relation $\succsim$ is representable by a utility function if and only if it is a separable (P6) preference relation (P1&P2).

Proof.

Define, for any x, $Z^*(x) = \{z \in Z : z \succ x\}$ and $Z_*(x) = \{z \in Z : x \succ z\}$. Order the elements of Z by $z_0, z_1, \ldots$. Define $r(z_k) = 2^{-k}$ for all $k = 0, 1, \ldots$, and let

$$u(x) = \sum_{z_k \in Z_*(x)} r(z_k) - \sum_{z_k \in Z^*(x)} r(z_k).$$

Since $Z^*(x)$ and $Z_*(x)$ are enumerable, both sums are well defined. Since $x \succsim y$ implies $Z^*(x) \subseteq Z^*(y)$ and $Z_*(y) \subseteq Z_*(x)$, and, by P6, $Z^*(x) = Z^*(y)$ and $Z_*(y) = Z_*(x)$ only if $x = y$, u represents $\succsim$. $\square$

- Correspondence F from an Euclidean space to another is continuous if it is both upper hemi continuous and lower hemi continuous

P7 (Continuity) Preference relation $\succsim$ is continuous on a metric space X if $\{y \in X : y \succsim x\}$ and $\{y \in X : x \succsim y\}$ are continuous correspondences of x

Theorem

Let X be a compact subset of an Euclidean space. Binary relation $\succsim$ is representable by a utility function if and only if it is a continuous (P7) preference relation (P1&P2).

Proof.

Denote by $\|y - x\|$ the Euclidean distance between x and y, and let

$$u(x) = \int_{\{y:x \succsim y\}} dy - \int_{\{y:y \succsim x\}} dy.$$

By transitivity and completeness of $\succeq$, $\{y : y \succsim x\} \subseteq \{y : y \succsim x'\}$ and $\{y : y \succsim x\} \subseteq \{y : y \succsim x'\}$ if $x' \succsim x$. Thus, since $\{y : y \succsim x\}$ and $\{y : x \succsim y\}$ are closed, $u(x) > u(x')$ whenever $x > x'$. Thus u represents $\succsim$. $\square$

(Proof cont.)

Finally, since $\{y : y \succsim x\}$ and $\{y : x \succsim y\}$ are continuous correspondences, for any $\{x_k\}$ such that $x_k \to_k x$,

$$
\begin{aligned}
\int_{\{y : x_k \succsim y\}} dy &\to_k \int_{\{y : x \succsim y\}} dy \\
\int_{\{y : y \succsim x_k\}} dy &\to_k \int_{\{y : y \succsim x\}} dy
\end{aligned}
$$

Thus u is continuous. $\square$

Utility maximization – WARP and all that

- The data concerning the behavior of the decision maker DM is captured by a choice function
- Prreferences, and hence utilities, can only be observed via choices
- A choice function reflects what the DM would choose in each context $Y \subseteq X$ - it is a mapping $c : 2^X \backslash \emptyset \rightarrow 2^X$ such that $c(Y) \subseteq Y$ for all $Y \subseteq X$
- That is, preferences are independent of the context, i.e. desirability independent of feasibility

- Define the optimal choices for the binary relation R (with asymmetric part P) by

$$c_R(Y) = \{x \in Y : \text{not } yPx \text{ for all } y \in Y\}$$

- c_R is a choice function if it is always empty

Proposition Let X be a finite set. Then c_R is a choice function if and only if P is acyclic.

- Thus observations as such do not imply completeness nor transitivity of R

- What extra properties c_R need to satisfy for R to qualify as a preference relation?

  Sen's α If $x \in c(Y)$ and $x \in Z \subseteq Y$ then $x \in c(Z)$

- This is equivalent to Nash's (1950) Independence of Irrelevant Alternatives and, when applied to the consumption set up

  Sen's β If $x, y \in c(Y)$ and $Y \subseteq Z$ and $y \in c(Z)$, then $x \in c(Z)$

- The combination of Sen's α and β is called the Weak Axiom of Revealed Preference (WARP), after Samuelson (1938)
- Conditions are, in principle, testable

Theorem

Let X be a finite set. Choice function c satisfies Sen's α and β if and only if $c = c_{\succsim}$ for some preference relation $\succsim$. Moreover, this preference relation is unique.

Proof.

[Proof sketch] Define $\succsim$ such that $x \succsim y$ iff $x \in c(\{x, y\})$. If $x \in c(Y)$, then $x \in c_{\succsim}(Y)$ by α. If $x \notin c(Z)$, then $x \notin c_{\succsim}(Z)$ by β. $\square$

- Caveat: context independence the crucial meta-assumption

Example (Reference dependence)

Let preferences depend on the anticipatored choice x such that, when x is chosen preferences are $\succsim_x \subset X \times X$. Optimal choice need not exist.

- Dynamic considerations
- Behavioral economics: relax context dependence
- Multiple motivations $=>$ social choice

Example (Multi-attribute decisions)

Let DMs preferences concerning a care depend on the price, reliability, and coolness. Car x is preferred to y if x is better in terms of two of the criteria. Let preferences be

Rank	Price	Reliability	Coolness
1.	x	y	z
2.	y	z	x
3.	z	x	y

No maximal choice exists.

- May's theorem: with two alternatives, the majority rule is the only anonymous with respect ti criteria, neutral with respect to the names of the alternatives, and monotone choice function
- But: Arrow!

Utility and happiness

- Recently it has become fashionable to evaluate human well being through reflect happiness measures
- Could utility functions be replaced with "happiness functions"?
- Problematic questionners
  - The order of questions
  - Correlation with weather but not when the weather is pointed out
  - Meaning of life not evaluated

- National well being is often measured through GDP or equivalent
- Can happiness be measured by wealth?
  - Easterlin paradox
  - Stimulus effect
  - Keeping up with the Joneses
- Neuroimaging