# Advanced Microeconomic Theory: Decision Theory and Markets

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- What is economic theory?

1. Set of theories that can (or should) be tested
2. Bag of tools to be used by economic agents
3. A framework through which professional and academic economists view the world
4. Arena for the investigation of concepts we use in thinking about economics in real life

- **Methodological individualism**: a principle according to which social phenomena can only be understood by examining how they result from actions of individual agents
- Microeconomics: models in which the primitives are details about the behavior of units called **economic agents**
- Microeconomic models investigate assumptions about economic agents' activities and about interactions between these agents
- Models in microeconomic theory are, as in any honest scientific enterprise, formal
  - Permits clear insight
  - Makes models comparable and integrable
  - Rules out faulty logic
  - Comparative static exercises
  - Facilitates testing the model

## Lecture notes 1: Choice theory

- The only thing that is even in principle observable from the agent is his behavior
- What does observed (economic) behavior tell us about the decision maker? => **Her preferences**
- Obs.: "utility" cannot be observed!
- Observations without a model meaningless - finding the right model crucial
- In economics, the model is that of a **rational agent** (what does rationality mean?)

- Rationality precludes biases, delusions, and inconsistencies

**Example (Aesop's fox)**

The fox was wandering in the forest and spotted a bunch of grapes hanging in a high branch. The fox jumped but failed to reach them. Giving up, the fox lifted its nose and said "they are probably sour anyway"

**Example (Groucho Marx)**

I never care to join a club that accepts people like me as its members

### Example (Money pump)

The agent is willing to pay 1€ to replace an apple to banana, 1€ to replace an banana to orange, and 1€ to replace an orange to apple. Whenever, she has $x$ at her hand, she is thus willing to pay 50c to replace it to something else. Soon, she is in financial troubles.

- Precluding inconsistencies of this sort, i.e. violations of transitivity, can perhaps be justified on evolutionary grounds

# From choice to preferences

Four elements:

1. The known choice set $X$
2. Observed feasible set $A \subseteq X$
3. Choice rule
4. Behavioral assumption

## Set of possible outcomes $X$

- $X$ is the universe of alternative choices
- Examples:
  1. Lunch from a menu
  2. Consumption over time
  3. Speeding or not speeding a car
  4. Occupational choice
  5. $\mathbb{R}_+^n$

## Feasible Set $A$

- Achievable choices, a subset of $X$
- Given by external conditions
- Examples:
  - Budget set $B(p, m) = \left\{ x \in \mathbb{R}_+^L : \sum_{l=1}^{L} p_l \cdot x_l \leq m \right\}$ with $L$ commodities, prices $p_0, ..., p_L$ and budget $m$
  - In a normal form game, $X = X_1 \times \cdots \times X_N$ each player $i$ chooses independently from his strategy set in $X_i$, i.e. $B_i(x_{-i}) = \{(x_i, x_{-i}) : x_i \in X_i\}$
- Why separate $A$ and $X$?

## Choice function

- How is choice made when $A$ is given?
- Let $\mathcal{A}$ denote the collection of all possible feasible sets in $X$, call $\mathcal{A}$ a **context**
- A **choice function** $c$ assigns to each set $A$ in the context $\mathcal{A}$ a unique element $c(A) \in A$ with the interpretation that $c(A)$ is chosen if $A$ happens to be the choice problem at hand
- $c$ is the information that we get of the agent in the context $\mathcal{A}$

## Behavioral assumption

- In economics, decisions in $c$ are made through "rational deliberation"
- What would rationality imply for $c(A)$?

**Axiom (Independence of irrelevant alternatives, IIA)**

*If* $B \subseteq A$ *and* $c(A) \in B$, *then* $c(A) = c(B)$

- Removing nonchosen outcomes will not affect the choice
- A version of what is called the *Weak Axiom of Revealed Preferences*
- Our aim is to show that if the agent chooses according to IIA, then he behaves **as if** he has rational preferences that he maximizes (and conversely)

# What are preferences?

- Preferences reflect the summary of all judgements of the agent, how he compares distinct alternatives against one another
- Independent of the context, i.e. desirability does not depend on feasibility
- **Preference relation** $\succsim$ is a binary relation, a subset of $X \times X$, but written for convenience $x \succsim y$ when $(x, y) \in \succsim$
- Other binary relations derived from $\succsim$:
  - Indifference part: $x \sim y$ if $x \succsim y$ and $y \succsim x$
  - Strict part: $x \succ y$ if $x \succsim y$ and not $y \succsim x$

# Rational preferences

**Axiom (Completeness)**

*For all* $x, y \in X$ *either* $x \succsim y$ *or* $y \succsim x$

**Axiom (Transitivity)**

*For all* $x, y, z \in X$, *if* $x \succsim y$ *and* $y \succsim z$, *then* $x \succsim z$

- Complete and transitive preferences are called **rational**
- Below we simplify exposition by also ruling out indifferences

**Axiom (Strictness)**

*For all* $x, y \in X$, *if* $x \succsim y$ *and* $y \succsim x$, *then* $x = y$

- Rationality thus means nothing but that the agent can **order** the alternatives

- Independence of the frame is crucial

**Example (Reference dependence)**

Let preferences depend on the anticipated choice $x$ such that, when $x$ is chosen preferences are $\succsim_x \subset X \times X$. Optimal anticipated choice need not exist.

- In particular, economic agents *do not regret*

## Example (Multi-attribute decisions)

Let agent's preferences concerning a cars $x$, $y$, and $z$ depend on the price, reliability, and coolness. A car is preferred to anther if it is better in terms of two of the criteria. Let criteria based ranking be

| Rank | Price | Reliability | Coolness |
|---|---|---|---|
| 1. | $x$ | $y$ | $z$ |
| 2. | $y$ | $z$ | $x$ |
| 3. | $z$ | $x$ | $y$ |

By majority relation $x \succ y$, $y \succ z$, $z \succ x$. Hence no maximal choice exists.

- Given the observed choice function $c(\cdot)$, we can define the **revealed preference relation** $\succsim^*$:

$$x \succsim^* y \text{ if } x, y \in A \text{ and } x = c(A), \text{ for some } A \in \mathcal{A}$$

- "$x \succsim^* y$" means "$x$ is at least as good as $y$" or "$y$ is not preferred to $x$"

**Proposition**

*Let context $\mathcal{A}$ include all subsets of $X$ containing two or three elements. If $c(\cdot)$ satisfies IIA on $\mathcal{A}$, then the induced revealed preference relation $\succsim^*$ is rational and strict*

- That is, $\succsim^*$ **rationalizes** $c$ if $c$ meets IIA

- Why is the restriction on the sets in the previous proposition important?
- Example 1: $X = \{x, y, z\}$, $\mathcal{A} = \{\{x, y\}, \{y, x\}, \{x, z\}\}$
- Example2: As Ex. 1 but add $X$ to $\mathcal{A}$

- To obtain the other direction, assume that $\succsim$ is a strict preference relation: either $x \succ y$ or $y \succ x$ for all $x \neq y$
- Since strict rational preferences $\succsim$ put alternatives into a linear order, each subset $A$ of $X$ contains a unique $\succsim$ $-$maximal element denoted by $c^*(A, \succsim)$

**Proposition**

*If $\succsim$ is a strict rational preference relation, then the choice function $c^*(\cdot, \succsim)$ induced by $\succsim$ satisfies IIA*

**Interpretation**:

- If the sample of observations is sufficiently rich ($\mathcal{A}$ includes all subsets of $X$ with two or three elements), rationality (strict, complete, and transitive preferences) is **equivalent to** Independence of Irrelevant Alternatives
- Taking rational preferences as the starting point means that the analysis is based on (potentially) **observable** characterisitics of the decision maker (assuming IIA)
- Conversely, rejecting rationality would imply rejection on IIA – plausible?
- In principle testable hypothesis

# Alternative approaches to decision making

- Psychological elements such as feelings, emotions, anxiety, excitement do **not** affect the rational choice theory as such: there is no reason why the preference relation $\succsim$ could not summarize the effect of these as well
- Psychological effects may have an impact if they affect the decision making procedures of the agent: **how** she deliberates and chooses
- Resulting models, which emphasize the frictions implied by the procedure, reflect **bounded rationality**

## Example (Satisficing)

(Herbert Simon): the agent arranges the alternatives in $A$ into an ordering, and starts checking the value of the candidates in this order. The first alternative whose value exceeds a threshold value is chosen.

- The the ordering in the list is the same across $B$s, the observed choice function $c^*$ meets IIA, and is made **as if** there is a rational preference ordering that is maximized
- The the ordering in the list varies between $B$s, the observed choice function $c^*$ does **not** meet IIA, and cannot imitated by a rational choice model

- Satisficing one of the very few models of decision making that meet the IIA
- However, super sensitive to the underlying assumptions (how to choose listing order), and hence more complicated and arbitrary than rationality

## Examples (Framing)

(Kahnemann and Tversky): An outbreak of a disease will cause 600 deaths. One of two emergency programs may be executed:

1. 400 people will die
2. with prob. $1/3$, no-one dies and with prob. $2/3$, all die

Another way to describe the decision problem:

1'. 200 people will be saved

2'. with prob. $1/3$, all will be saved and with prob. $2/3$, no-one will be saved

Experminetal subjects typically choose 2 and 1'

# Utility representation for rational preferences

- Real utility or happiness, if it exists, is not used in nor required by economics models
- However, we often work with a **utility functions** for convenience: it can be easily manipulated, and it nicely summarizes the information contained in preferences
- Then utility function **represents** preferences
- Is it OK to let a real-valued function to represent potentially complicated preferences over the choice set?
- What are we exactly assuming when taking this approach?
- Our objective: reveal the relationship between the axioms and the utility function

- We say that a utility function $u : X \to \mathbb{R}$ represents rational preferences $\succsim$ if it holds that

$$u(x) \geq u(y) \text{ if and only if } x \succsim y$$

- No additional interpretation associated to $u$, in particular, $u$ does not reflect the level of satisfaction nor "happiness"

**Proposition**

*If there exists a utility function representing $\succsim$, then $\succsim$ is rational*

- Note: If $u$ represents $\succsim$, then so does $f \circ u$ for **any** increasing $f : \mathbb{R} \to \mathbb{R}$
- $=>$ Utilities here do not have any interpretation as the level of satisfaction or "happiness"

- When the underlying environment is countable, one can always **construct** a utility function step-by-step, starting from a specific outcome and adding or substracting utility when moving upwards or downwards in preferences

**Proposition**

*If the choice set $X$ is countable and $\succsim$ is rational, then $\succsim$ has a utility representation.*

- One can imagine noncountable situations where utility representation does exist: e.g. consumtion of a single desirable good
- Are there situations where a utility representation does not exist?

## Example

Let preferences.on $X = [0,1] \times [0,1]$ be **Lexicographic** such that

$$(x_1, x_2) \succsim (y_1, y_2)$$
$$\text{if and only if}$$
$$x_1 \geq y_1 \text{ or } [x_1 = y_1 \text{ and } x_2 \geq y_2].$$

Assuming a representation $u$ for these preferences leads to impossibility:
Suppose $u$ represents preferences. Then $u(a,1) > u(a,0) > u(b,0)$, for any $a, b \in [0,1]$ such that $a > b$. For any $a$, choose a rational number $f(a)$ such that $u(a,1) > f(a) > u(a,0)$. Then $f$ is a strictly monotonic function from $[0,1]$ to the set of rational numbers, i.e. there is a 1-1 mapping from a continuum to a subset of rational numbers, a contradiction.

- Implication: further restrictions on the preference relation are needed
- Let $X = \mathbb{R}^L_+$, e.g. the set of commodity bundles.
- Define the **upper contour set** (or simply upper set) at $x$ by

$$\succsim(x) = \{y \in X : y \succsim x\}$$

- Similarly, the **lower contour set** (or simply lower set) at $x$ is given by

$$\precsim(x) = \{y \in X : x \succsim y\}$$

- and the **indifference set** at $x$ is denoted by

$$I(x) = \{y \in X : x \succsim y \text{ and } y \succsim x\}$$

- The set $Y \subseteq X$ is **closed** if for all sequences $\{y_n\}$ such that $y_n \rightarrow y$ and $y_n \in Y$, we have $y \in Y$
- If $\precsim(x)$ and $\succsim(x)$ are closed, so is their intersection $I(x)$

- Note that a path from $y \in \succsim (x)$ to $z \in \precsim (x)$ passes through a point of indifference

**Axiom (Continuity)**

*Preferences $\succsim$ are* **continuous** *if, for all $x \in X$, the sets $\succsim (x)$ and $\precsim (x)$ are closed*

- If the agent strictly prefers $x$ to $y$, and preferences are continuous, then a small perturbation of $x$ (or $y$) does not affect the ranking
- The next result states that, in a consumer choice context, rational preferences have a utility function characterization under very general conditions

## Theorem

*(Debreu) Let $X = \mathbb{R}_+^L$. If $\succsim$ is rational and continuous, then there exists a continuous utility function $u(\cdot)$ that represents $\succsim$.*

## Proof.

[Sketch] Let $Y$ be a dense subset of $X$ (such exists). Let $v$ be the utility function on $Y$ (such exists by the previous proposition). Choose $u(x) = \sup\{y \in Y : x \succ y\}$, for all $x \in X$. We claim that $u(x) \geq u(y)$ if $x \succsim y$ for all $x, y \in X$. Since $X$ is dense in $Y$, by continuity of preferences, $u(x) = u(y)$ if $x \sim y$. Let $x \succ y$. Then there are $z_1, z_2 \in Y$ such that $x \succ z_1 \succ z_2 \succ y$ (see Rubinstein p.19). By construction $u(x) \geq v(z_1) > v(z_2) \geq u(y)$. □

- Does not require assumptions regarding tastes (convexity, monotonicity)

# Utility and happiness

- Recently it has become fashionable to evaluate human well being through reflect happiness measures
- Could utility functions be replaced with "happiness functions"?
- Problematic questioners
  - The order of questions
  - Correlation with weather but not when the weather is pointed out
  - Meaning of life not evaluated

- National well being is often measured through GDP or equivalent
- Can happiness be measured by wealth?
  - Easterlin paradox
  - Stimulus effect
  - Keeping up with the Joneses
- Neuroimaging