Advanced Microeconomic Theory
Fall 2014
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Problem Set 3

1. Argue that, in the Second Wefare Theorem, if the consumers' preferences are *strictly* convex, the Pareto-efficient x is the *unique* Walrasian allocation that results from the initial endowment x.

2. Consider n agent exchange economy with n consumers with identical and strictly concave utility functions. Let there be some initial endowment vector ω. Show that equal division is a Pareto-efficient allocation.

3. Consider one consumer, one firm economy (Robinson Crusoe) where the consumer's utility function is of the Cobb-Douglas form $u(x, \ell) = x\ell$, where x is is the consumption good and ℓ is the amount of leisure. The firm's production function is linear $x = y^{\beta}$, where y is the labor inputand $\beta \in (0, 1)$. The resource constraint on the amount of leisure and labor is $\ell + y = 1$. The consumer owns the firm.

   (a) Write down the consumer's and the firms optimization problems, given the prices p of the output and w of the labor input.
   (b) Find the demand function of the consumer and the production function of the firm (as a function of p and w)
   (c) Solve the Walrasian equilibrium prices p^* and w^*
   (d) Compute the equilibrium utility of the consumer. What happens to the utility when β changes?

4. Suppose that in the previous exercise, the consumer is an entrepreneur who participates a large market, and adjusts her production and consumption to the exogenously given prices p' and w'. Show that he utility does not decrease relative to the Robinson Crusoe case.

5. Let $X = \{x_1, x_2, x_3\}$ be the set of pure outcomes where vNM preferences $\succsim$ satisfy $x_1 \succ x_2 \succ x_3$. A lottery 1_k offers x_k with certainty. Show that there is $\alpha \in [0, 1]$ such that $1_2 \sim \alpha \cdot 1_1 + (1 - \alpha) \cdot 1_3$.

6. Let a risk averse agent with Bernoulli utility function $u(\cdot)$ decide his insurance coverage $x > 0$. The probability of an accident is $\beta > 0$ and the corresponding montery loss is L. Coverage x reflects the compensation from the insuance company in the case of an accident. Let the constant marginal cost of insurance be exactly β. What coverage should the agent choose?

7. By using the formal definition of risk-aversion, show that an agent is risk-averse has a strictly concave Bernoulli utility function.

8. Argue that if a risk averse decision maker rejects a fixed favorable bet, i.e. one whose expected value is larger than the cost of participating the bet, at all levels of initial wealth, then the Bernoulli utility of the decision maker is bounded from above.