Advanced Microeconomic Theory
Fall 2014
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**Problem Set 2**

1. Define the *elasticity of a factor* $k$ at $x$ by

$$\varepsilon_k(x) = \frac{\partial f(x)}{\partial x_k} \frac{x_k}{f(x)}$$

   (a) Interprete elasiticity.
   (b) Let the production function be of Cobb-Douglas form $f(x_1, x_2) = \beta x_1^{\alpha} x_2^{1-\alpha}$, $0 < \beta$, $0 < \alpha < 1$. Compute the elasticities of each factor.
   (c) Let the production function be of CES form $f(x_1, x_2) = (x_1^{\rho} + x_2^{\rho})^{1/\rho}$, $\rho > 0$. Compute the elasticities of each factor.

2. A production function $f$ is homothetic if $f(\cdot) = g(h(\cdot))$ where is $h : \mathbb{R}_+^K \to \mathbb{R}_+$ has the property that $h(tx) = th(x)$ for all $x$, and $g : \mathbb{R}_+ \to \mathbb{R}_+$ is a monotonic function ($g(y) \geq g(y')$ if $y \geq y$). Show that the marginal rate of technological subsitution of a homothetic $f$ at $x$ is the same as at $tx$, for any $t > 0$. Show that the Cobb-Douglas and CES production functions (previous problem) are homothetic.

3. Prove Hotelling's Lemma (see slides 3).

4. Let the production function be of Cobb-Douglas form $f(x_1, x_2) = \beta x_1^{\alpha} x_2^{1-\alpha}$. Let prices of the ouput be $p$ and the inputs $w_1, w_2$.

   (a) Identify the Lagrangean associated with the cost minimizing problem.
   (b) Compute the cost function.
   (c) Identify the optimal production.

5. Consider a partial equilibrium in a merket with identical consumers and identical firms (for simplicity, let the quantity of both be 1). Let the price in the market be $p$. Assume that utility function of the consumer is $\ln q - pq$ and the profit function of the firm is $pq - q^2$. Compute the associated indirect utility function of the consumer and the associated

profit function of the firm. Show that their sum is a *decreasing* function of $p$, reaching its minum at the (partial) equilibrium $p^*$. Interprete the observation. Compute $p^*$.

6. Consider a value added tax in the previous problem: share $p(1-t)$, where $t \in (0,1)$, of the price $p$ remains as the revenue of the firm. Compute the equiliberium price and qantity sold. What happens to them as $t \to 1$? Sketch the magnitude of the welfare loss associated to the tax.

7. (Edgeworth Boxes) Consider a two person, two good exchange economy. The initial endowments are $\omega^1 = (1,0)$ and $\omega^2 = (0,1)$. The utility functions $u^1$ and $u^2$ are depicted below. For each of the following two cases, find the set of Pareto optimal allocations and the Walrasian equlibria and illustrate them in an Edgeworth box.

   (a) $u^1(x_1^1, x_2^1) = x_1^1 + x_2^1,\ u^2(x_1^2, x_2^2) = (x_1^2)^{\alpha}(x_2^2)^{1-\alpha}$.

   (b) $u^h(x_1^1, x_2^1) = \min\{x_1^1, x_2^1\},\ u^2(x_1^2, x_2^2) = (x_1^2)^{\alpha}(x_2^2)^{1-\alpha}$.