Advanced Microeconomic Theory
Fall 2014

Problem Set 1

1. Show that if preferences $\succsim$ on $\mathbb{R}^L_+$ satisfy continuity, and $x \succsim z \succsim y$, then there is an m in the line segment connecting x and y such that $m \sim z$

2. Show that if preferences $\succsim$ on $\mathbb{R}^L_+$ satisfy continuity and monotnicity, then the function t such that $x \sim (t(x), ..., t(x))$ is continuous.

3. Let preferences $\succsim$ on $\mathbb{R}^2_+$ be of Leontief form: $(x_1, x_2) \succsim (y_1, y_2)$ if $\min\{x_1, x_2\} \geq \min\{y_1, y_2\}$. Find a utility representation for these preferences. Identify the Marshallian demand function $x(p, w)$ and the indirect utility function.

4. Let concumer's preferences be represented by a utility function u reflecting constant elasticity of subsitution (CES)

$$u(x_1, x_2) = (x_1^\rho + x_2^\rho)^{1/\rho}, \text{ where } \rho \in (0, 1)$$

   Let product prices be p_1 and p_2 and income w.

   (a) Construct a Lgrangean that reflects the consumer's optimization problem.
   (b) Find the first order conditions for optimality.
   (c) Identify the Marshallian demand function.
   (d) Construct the indirect utility function.
   (e) Identify the Hicksian demand function.
   (f) Construct the expedinture function

5. Preferences $\succsim$ are *homothetic* if $x \succsim y$ implies $tx \succsim ty$ for all consumption bundles x, y. Let $x(\cdot, \cdot)$ be the Marshallian demand function. Show that:

   (a) $x(p, tw) = tx(p, w)$, for all $t > 0$ (homogenous of degree 1 in w)
   (b) $x(tp, w) = t^{-1}x(p, w)$, for all $t > 0$ (homogenous of degree -1 in p)
   (c) $x(tp, tw) = x(p, w)$, for all $t > 0$ (homogenous of degree 0 in (p, w))

6. The government finances public expenditure of magnitude g by collecting taxes. In this question, you are invited to think about the optimal ways of collecting taxes. Suppose that there are goods x and y. The government can finance g by choosing either a tax on income t_w or by taxing consumption of good x by rate t_x. The government budget constraint for the two cases reads: $t_w w = g$ and $t_x x\left(p_x, p_y, t_x\right) = g$. Show that the consumer prefers an income tax in this case.

7. Let preferences $\succsim$ be represented by utility fucntion u. Show $\succsim$ are convex if and only if the utility fucntion u representing $\succsim$ is *quasiconcave*:

$$u(tx + (1-t)y) \geq \min[u(x), u(y)]$$

   for every $x, y \in X$,and $0 \leq t \leq 1$.

8. Preferences are said to be additively separable if they can be represented by a utility function of the form;

$$u\left(x\right) = \Sigma_{i=1}^{L} u_i\left(x_i\right).$$

   (a) If $u_i''\left(x_i < 0\right)$ for all i and for all $x_i \geq 0$, show that all goods are normal.

   (b) Show also that

$$\frac{\partial x_i\left(p,w\right)/\partial p_k}{\partial x_j\left(p,w\right)/\partial p_k} = \frac{\partial x_i\left(p,w\right)/\partial w}{\partial x_j\left(p,w\right)/\partial w}.$$

   (c) Suppose that $u_1(x_1) = x_1$ and $p_1 = 1$ (i.e. the consumer can *save* her wealth by consuming x_1). What happens to consumption of goods when w becomes large ($\geq \max\{p_2, ..., p_L\}$)?