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Von Neumann–Morgenstern stable sets, discounting, and Nash bargaining

Klaus Kultti^a, Hannu Vartiainen^{b,*}

^aDepartment of Economics, University of Helsinki, Finland ^bYrjö Jahnsson Foundation, Ludviginkatu 3-5, 00130 Helsinki, Finland

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Abstract

We establish a link between von Neumann–Morgenstern stable set and the Nash solution in a general n-player utility set. The stable set-solution is defined with respect to a dominance relation: payoff vector u dominates v if one player prefers u even with one period delay. We show that a stable set exists and, if the utility set has a *smooth* surface, any stable set converges to the Nash bargaining solution when the length of the period goes to zero.

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1. Introduction

Von Neumann and Morgenstern [11] propose internal and external stability conditions that can be imposed on an acceptable solution to any collective choice problem. The stability conditions are defined with respect to a dominance relation, and a solution characterized by the conditions, the *stable set*, is dependent on the choice of this relation. In this note we show that in the general *n*-player bargaining problem, defined as an *n*-dimensional convex, compact and comprehensive utility set, there is a natural dominance relation that gives an interpretation to the Nash [6] bargaining solution in terms of stable sets.

Our dominance relation is motivated as follows. Any player may impose an objection to a division of utilities by demanding a new division. It takes one period before any such demand

* Corresponding author. Fax: +3589605002.

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E-mail address: hannu.vartiainen@yjs.fi (H. Vartiainen).

may materialize. A division u of utilities dominates division v if and only if the discounted value of u exceeds the current value of v for *some* player.

With a short enough period there are no undominated divisions. We focus on a subset of all divisions, the stable set, defined by the following properties: no element in the stable set is dominated by an element in the set, and any element outside the stable set is dominated by some element in the set. We characterize the stable set, establish its existence, and show that all stable sets converge to the Nash bargaining solution as the time interval tends to zero. The latter result bears similarity to Binmore et al. [1], who demonstrate the convergence in the two-player bargaining game á la Rubinstein [7].

2. The set up

There is a set $N = \{1, ..., n\}$ of players with generic elements *i* and *j*, and a compact, convex, and comprehensive utility possibility set $U \subset \mathbb{R}^n_+$.^{1,2} The vector of utilities is denoted by $u = (u_1, ..., u_n)$, or $u = (u_i, u_{-i})$. For any $v \in U$, let D(v) be the points that Pareto dominate $v \in U$:

$$D(v) := \{ u \in U : u \ge v \}.$$

$$\tag{1}$$

For any $v \in U$, D(v) is a compact and v-comprehensive set. Pareto-optimal outcomes P are then defined by $P := \{u \in U : D(u) = \{u\}\}$.

Bargaining takes place through objections against a potential division of utilities. An objection is a specification for a new division. However, there is a one-period delay before the objection may become effective. Delay is costly: the present value of player *i*'s next period utility u_i is $u_i \delta_i^{\Delta}$, where $0 < \delta_i < 1$ is the discount factor and $\Delta > 0$ is the length of the period.

A stable set is defined with respect to a domain of alternatives and a dominance relation on this set. We let the domain be U. Dominance relation > is defined as follows: u > v if $u_i \delta_i^A > v_i$, for some $i \in N$, for $u, v \in U$. A set $G \subset U$ is *stable* if:

• (External stability) $u \notin G$ implies there is $v \in G$ s.t. $v \succ u$,

• (Internal stability) $u \in G$ implies there is $no v \in G$ s.t. $v \succ u$.

3. Characterization and existence

Without loss of generality, we let $\Delta = 1$ in this section. Take $u = (u_1, \ldots, u_n)$, and call $(\delta_i^{-1}u_i, u_{-i})$ the δ_i -extension of $u \in U$.³ Denote by \underline{u} a typical point whose all δ_i -extensions lie on the Pareto-frontier, i.e.

$$(\delta_i^{-1}\underline{u}_i, \underline{u}_{-i}) \in P \quad \text{for all } i \in N.$$
⁽²⁾

Occasionally, <u>u</u> is called a "minimal point".

For any nonempty set $X \subset U$, define the supremum of *i*'s feasible payoffs in X by

$$m_i(X) = \sup\{u_i : u \in X\}.$$

¹ Vector notation: $x \ge y$ if $x_i \ge y_i$ for all $i, x \ge y$ iff $x \ge y$ and not $x_i = y_i$ for all i, and x > y iff $x_i > y_i$ for all i.

² $X \subset \mathbb{R}^{k}_{+}$ is comprehensive if $x \in X$ and $x \ge y \ge 0$ imply $y \in X$.

³ The concepts are taken from Thomson and Lensberg [10, Chapter 8].

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Theorem 1. A set $G \subset U$ is stable if and only if G = D(u).

Proof. "If": Assume that $G = D(\underline{u})$. By construction, $u_i \ge \underline{u}_i = m_i(G)\delta_i \ge v_i\delta_i$, for all *i*, for all $u, v \in D(u)$. Thus, internal stability is met. Take $u \notin D(u)$. Then there is a player i such that $\underline{u}_i > u_i$. This implies that also $m_i(G)\delta_i > u_i$. Since $m_i(G) \in \{u_i : u \in D(\underline{u})\}$, external stability is met.

"Only if": Suppose G is a stable set. By external stability, if $v_i < m_i(G)\delta_i$ for some i, then $v \notin G$. By internal stability, if $v_i \ge m_i(G)\delta_i$ for all *i*, then $v \in G$. Thus

$$\bigcap_{i\in\mathbb{N}} \{u\in U: m_i(G)\delta_i\leqslant u_i\}=G.$$

Since U is a comprehensive set, there is $\underline{u} = (\underline{u}_1, \dots, \underline{u}_n) \in U$ such that $m_i(G)\delta_i = \underline{u}_i$ for all *i*. By construction, $G = \{u : u \ge u\}$. Then G = D(u) for D meeting (1), and u meeting (2), as required. \Box

A stable set is characterized by a minimal point $\underline{u} = (\underline{u}_1, \dots, \underline{u}_n)$: points in U above \underline{u} constitute a stable set. Moreover, a stable set is convex, and contains n "maximal points" u^1, \ldots, u^n that induce the highest possible payoff in the stable set for each *i*. Given the minimal point \underline{u} , player *i*'s maximal point satisfies $u^i = (\delta_i^{-1} \underline{u}_i, \underline{u}_{-i})$. Also, if u^i is an *i*-maximal point of a stable set G, then $m_i(G) = u_i^i$.

We prove that in our domain the existence of a stable set is guaranteed. Recall that for any $u = (u_i, u_{-i}),$

$$m_i(D(u_i, u_{-i})) = \max\{u'_i : (u'_i, u_{-i}) \in U\}$$

If $u \in P$, then $m_i(D(u)) = u_i$.

Theorem 2. A stable set exists.

Proof. Define function $g_i : U \to \mathbb{R}_+$

$$g_i(u) := \delta_i m_i(D(u)) \quad \text{for all } (u_i, u_{-i}) \in U, \text{ for all } i \in N.$$
(3)

By convexity of U, function g_i is continuous. Let $g(\cdot) := (g_1(\cdot), \ldots, g_n(\cdot))$, and define function $\bar{x}: U \to \mathbb{R}_+$ such that

 $\bar{x}(u) := \max\{x \in \mathbb{R} : xg(u) \in U\}$ for all $u \in U$.

By compactness of U, \bar{x} is well defined. Construct function $\hat{g}_i : U \to \mathbb{R}_+$

 $\hat{g}_i(u) := g_i(u) \min\{\bar{x}(u), 1\}$ for all $u \in U$.

If min{ $\bar{x}(u), 1$ } = 1, then $\hat{g}(u) = g(u) \in U$, and if min{ $\bar{x}(u), 1$ } = $\bar{x}(u)$, then $\hat{g}(u) = \bar{x}(u)g(u) \in U$ U. Thus,

$$\hat{g}(u) = (\hat{g}_1(u), \dots, \hat{g}_n(u)) : U \to U.$$

By convexity of U, function \bar{x} is continuous. Thus, $\hat{g}: U \to \mathbb{R}^n_+$ is a continuous function. By the Brouwer's Theorem, there is a $\underline{u} \in U$ such that

$$\hat{g}\left(\underline{u}\right) = \underline{u}.$$
 (4)

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If also

$$g\left(\underline{u}\right) \in U,$$
(5)

then, $g(\underline{u}) = \underline{u}$. This implies that \underline{u} satisfies condition (2), and that $D(\underline{u})$ is a stable set. Thus, condition (5) needs to be checked.

Suppose (5) does not hold. Then

$$\bar{x}(\underline{u}) < 1. \tag{6}$$

By (4) and (6),

$$\underline{u} = g(\underline{u})\bar{x}(\underline{u}) \in P. \tag{7}$$

This implies that $m_i(D(\underline{u})) = \underline{u}_i$, for all $i \in N$. By (3) and comprehensiveness of U we have

$$g(\underline{u}) = (\delta_1 m_1(D(\underline{u})), \dots, \delta_n m_n(D(\underline{u})))$$

= $(\delta_1 \underline{u}_1, \dots, \delta_n \underline{u}_n)$
 $\in U,$

a contradiction. Thus, $g(\underline{u}) \in U$, as required. \Box

4. Relationship with the Nash solution

We now demonstrate the relation between the stable set-solution and the Nash bargaining solution. Let $\{G_{\Delta}\}$ be a collection of stable sets, one for each Δ .⁴ We study the limit behavior of G_{Δ} , when Δ becomes small.

First, introduce a vector of weights $\alpha = (\alpha_1, \ldots, \alpha_n)$ where

$$\alpha_i = \frac{-1}{\ln \delta_i}$$
 for all $i \in N$.

Denote the α -weighted Nash solution by

$$u^{\alpha} := \arg \max_{u \in U} \prod_{i \in N} u_i^{\alpha_i}.$$
(8)

Also, denote the α -weighted hyperbola that contains u by

$$H(u) := \left\{ (v_1, \ldots, v_n) \in \mathbb{R}^n : \prod_{i \in \mathcal{N}} v_i^{\alpha_i} = \prod_{i \in \mathcal{N}} u_i^{\alpha_i} \right\}.$$

For any $\Delta > 0$, take a stable set G_{Δ} and identify its minimal point $\underline{u}(\Delta)$ and maximal points $u^{1}(\Delta), \ldots, u^{n}(\Delta)$. Then, for all i,

$$u^{i}(\varDelta) = (\delta_{i}^{-\varDelta}\underline{u}_{i}(\varDelta), \underline{u}_{-i}(\varDelta)).$$

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⁴ Uniqueness needs not hold under Δ . See Kultti and Vartiainen [3], or Thomson and Lensberg [10, pp. 121–124], for an example.



Fig. 1.

It is now easy to see that all the maximal points lie on the same hyperbola: for any j,

$$\prod_{i\in N} u_i^j(\varDelta)^{\alpha_i} = \delta_j^{-\varDelta\alpha_j} \prod_{i\in N} \underline{u}_i(\varDelta)^{\alpha_i} = e^{\varDelta} \prod_{i\in N} \underline{u}_i(\varDelta)^{\alpha_i}.$$
(9)

The last expression is independent of j.

In the two-player case, the fact that all maximal points lie on the same hyperbola implies that $u^{\alpha} \in G_{\Delta}$, for all $\Delta > 0$. The distance between the maximal points, $[\sum_{i=1,2} (\delta_i^{-\Delta} - 1)\underline{u}_i(\Delta)]^{1/2}$, converges to zero as Δ tends to zero which implies that the stable set shrinks to a one point set in the limit. ⁵ Since $u^{\alpha} \in \bigcap_{\Delta > 0} G_{\Delta}$ it follows that the stable set actually shrinks to the Nash solution. With more than two players this need not hold as a stable set may not contain the Nash solution. However, a weaker convergence result holds, as we next demonstrate.

We say that a sequence $\{G_{\Delta}\}$ of stable sets converges to $\{u\}$ in the Hausdorff metric as Δ tends to zero if for any open ball with radius *r* around $u \in U$, denoted by $B^{r}(u)$, there is $\Delta_{r} > 0$ such that $G_{\Delta} \subset B^{r}(u)$, for all $\Delta \in (0, \Delta_{r})$.

Theorem 3. Let P be smooth. Then any sequence of stable sets converges to $\{u^{\alpha}\}$ as Δ tends to 0.

To see the intuition for the proof consider the case with three players (see Fig. 1). Think of the surface P of U as a chart of one-dimensional curves, each reflecting an intersection of P and a hyperbola. As Δ becomes small, a (sub)sequence of stable sets shrinks to a point u^* on P. If u^* is distinct from u^{α} , then, since P is smooth, the envisioned chart over P is locally homeomorphic to an open disk that is permeated by a collection of line segments, each corresponding to a hyperbola. Any neighborhood of u^* also contains the maximal points of the stable set for small enough Δ . Under any Δ , the maximal points lie on the same hyperbola, and they span a two-dimensional simplex T. Thus, it follows that T becomes embedded into a line segment as Δ tends 0, which leads to a contradiction.

Convergence need not hold if P is not smooth (for an example, see [3, 10, Chapter 8.2]).

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⁵ It can be shown that the stable set is unique in the two-player case.

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Fig. 2.

Proof. Let $\{G_{\Delta}\}_{\Delta>0}$ be a collection of stable sets. For any Δ , let $\underline{u}(\Delta)$ be the minimal point of G_{Δ} , and $u^{1}(\Delta), \ldots, u^{n}(\Delta) \in P$ the corresponding maximal points. By (9), there is a unique (n-1)-dimensional hyperplane $L(\Delta)$ that contains $u^{1}(\Delta), \ldots, u^{n}(\Delta)$. Since $(\delta_{i}^{-\Delta} - 1)\underline{u}_{i}(\Delta)$ tends to zero as Δ becomes small, there is a subsequence $\{\Delta\}$ converging to zero such that, for some $u^{*} \in P$, and some hyperplane L^{*} , ⁶

$$u^{i}(\Delta) \to u^{*}$$
 for all i , (10)
 $L(\Delta) \to L^{*}$. (11)

Denote the hyperplane that supports H(u) at $u \in U$ by $L^{H}(u)$. Since P is smooth, (10) implies that

$$L^{H}(u^{i}(\Delta)) \to L^{*}$$
 for all *i*. (12)

Denote by V(u) the (n - 2)-dimensional hyperplane that supports H(u) at $u \in L(\Delta)$ in the subspace $L(\Delta)$. Then

$$V(u^{i}(\Delta)) = L^{H}(u^{i}(\Delta)) \cap L(\Delta)$$
 for all *i*.

Suppose, to the contrary of the theorem, that $u^* \neq u^{\alpha}$. Since $V(u^i(\Delta))$ and $V(u^j(\Delta))$ support the same hyperbola in the same subspace and, by (11) and (12), they approach the same limit, we have

$$\frac{\min_{v}\left\{\left\|u^{j}(\varDelta) - v\right\| : v \in V(u^{i}(\varDelta))\right\}}{\left\|u^{j}(\varDelta) - u^{i}(\varDelta)\right\|} \to 0 \quad \text{for all } j \neq i,$$

$$(13)$$

as depicted in Fig. 2.

For any $c \in \mathbb{R}^n_+$, denote by A_c the linear transformation matrix

 $A_{c} = \begin{bmatrix} c_{1} & 0 & \cdots & 0 \\ 0 & c_{2} & \vdots \\ \vdots & \ddots & 0 \\ 0 & \cdots & 0 & c_{n} \end{bmatrix}.$

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⁶ Since, for any Δ , there are $p \in [0, 1]^n$ and $u^i(\Delta) \in U$ such that $L(\Delta) = u^i(\Delta) + \{v \in \mathbb{R}^n : v \cdot p = 0\}$, the set of parameters defining $\{L(\Delta)\}_{A>0}$ is bounded.

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Abusing the notation, $A_c x = (c_1 x_1, \dots, c_n x_n)$ and $A_c X = \{A_c x' : x' \in X\}$, for any $x \in X \subset \mathbb{R}^n_+$. Given Δ , choose $c(\Delta) = (c_1(\Delta), \dots, c_n(\Delta))$ such that

$$c_i(\Delta) = (\delta_i^{-\Delta} - 1)\underline{u}_i(\Delta) \quad \text{for all } i \in N.$$
(14)

Denote the (n-1) -dimensional standard simplex by $T = \{x \in \mathbb{R}^n_+ : \sum x_i = 1\}$. Now ⁷

$$\operatorname{co}\{u^{1}(\varDelta),\ldots,u^{n}(\varDelta)\} = \underline{u}(\varDelta) + A_{c(\varDelta)}T \quad \text{for all } \varDelta.$$
(15)

Fix some player *i*. Define

$$j(\varDelta) = \arg \max_{j} \left[\min_{v} \left\{ \left\| u^{j}(\varDelta) - v \right\| : v \in V(u^{i}(\varDelta)) \right\} \right].$$

Take any $\varepsilon > 0$. By (13), there is \varDelta_{ε} such that, for all $\varDelta \in (0, \varDelta_{\varepsilon})$,

$$\frac{\min_{v}\left\{\left\|u^{j(\varDelta)}(\varDelta) - v\right\| : v \in V(u^{i}(\varDelta))\right\}}{\left\|u^{j(\varDelta)}(\varDelta) - u^{i}(\varDelta)\right\|} < \varepsilon.$$

$$(16)$$

By (15) and (16), for all $\Delta \in (0, \Delta_{\varepsilon})$,

$$\underline{u}(\varDelta) + A_{c(\varDelta)}T \subset \left\{ u : \|u - v\| < \varepsilon \left\| u^{j(\varDelta)}(\varDelta) - u^{i}(\varDelta) \right\|, \text{ for } v \in V(u^{i}(\varDelta)) \right\}.$$
(17)

Since, by (14),

$$\left\|A_{c(\varDelta)}^{-1}u^{j(\varDelta)}(\varDelta) - A_{c(\varDelta)}^{-1}u^{i}(\varDelta)\right\| = \sqrt{2},$$

condition (17) reduces to

$$T \subset \left\{ u : \|u - v\| < \varepsilon \sqrt{2}, \text{ for } v \in V(A_{c(\varDelta)}^{-1}[u^i(\varDelta) - \underline{u}(\varDelta)]) \right\} \text{ for all } \varDelta \in (0, \varDelta_{\varepsilon}).$$

I.e., *T* is contained by the $\varepsilon \sqrt{2}$ -neighborhood of the hyperplane $V(A_{c(\Delta)}^{-1}[u^i(\Delta) - \underline{u}(\Delta)])$, defined with respect to the utility possibility set $A_{c(\Delta)}^{-1}[U - \underline{u}(\Delta)]$. But since $\varepsilon > 0$ is arbitrarily small, this means that the (n-1)-dimensional simplex *T* is contained by an (n-2)-dimensional hyperplane, a contradiction. \Box

5. Discussion

The current paper is closely related to Thomson [9], Lensberg [4], and Thomson and Lensberg [10, Chapters 7 and 8], where the notion of "Nash-like" solution is developed. It can be shown that the minimal point of the stable set is a particularly parametrized Nash-like solution.

Thomson and Lensberg also discuss the convergence properties of the Nash-like solution. They point out that in a smooth problem the Nash-like solution converges to the Nash solution. Their analysis is driven by the assumption that the solution remains unchanged in all two-player utility projections.

Rubinsten et al. [8] characterize the Nash solution in a model whose motivation bears similarity to our framework. They use a system of objections and counterobjections to define the Nash solution. There are important differences to our approach, though. First, the Rubinstein et al.

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⁷ co *X* is a convex hull of $X \subset \mathbb{R}^n$.

model relies on an asymmetry between objections and counterobjections. Hence their solution does not conceptually relate to the stable set (except in terms of convergence). Second, Rubinstein et al. analyze the two-player case, and it is not clear how to extend it to a multi-player scenario. A straightforward extension would allow only bilateral objections/counterobjections.⁸ Along the argument made in this paper, that would lead to the Nash solution in the class of smooth problems.

The stable set solution is also related to equilibria of the n-player unanimity bargaining game (cf. [2]) where, at any stage, the offer of a proposer becomes implemented if all other players accept the offer (in sequence).⁹ The responder who rejects the offer first becomes the proposer in the next round. Let 1 be the first proposer. Then an outcome is implemented in a *stationary* equilibrium if and only if it is the 1-maximal point of a stable set.¹⁰ Since any stable set converges to the Nash bargaining solution u^{α} as the time interval becomes small, also *all* stationary equilibria converge to u^{α} .

The intuition is that in any stationary equilibrium players are indifferent between rejecting the equilibrium offer of the other players, given that in the next round they get their equilibrium offers. Thus, the equilibrium outcome must satisfy the same properties as a maximal point. Since, indeed, in a maximal point of a stable set players are indifferent between receiving their minimal point payoff today and waiting for the next round to get their maximal point payoff, a maximal point can be supported as an equilibrium.

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References

- K. Binmore, A. Rubinstein, A. Wolinsky, The Nash bargaining solution in economic modelling, Rand J. Econ. 17 (1986) 176–188.
- [2] M. Herrero, A strategic theory of market institutions, Ph.D. Dissertation, London School of Economics, 1985.
- [3] K. Kultti, H. Vartiainen, Von Neumann–Morgenenstern stable sets and the noncoopertive solution to the bargaining problem, HECER Discussion Paper No. 114, 2006.
- [4] T. Lensberg, Stability and the Nash solution, J. Econ. Theory 54 (1988) 101-116.
- [6] J. Nash, The bargaining problem, Econometrica 18 (1950) 155–162.
- [7] A. Rubinstein, Perfect equilibrium in a bargaining model, Econometrica 50 (1982) 97-109.
- [8] A. Rubinstein, Z. Safra, W. Thomson, On the interpretation of the Nash bargaining solution and its extension to non-expected utility preferences, Econometrica 60 (1992) 1171–1186.
- [9] W. Thomson, A study of choice correspondences in economies with a variable number of agents, J. Econ. Theory 46 (1988) 237–254.
- [10] W. Thomson, T. Lensberg, Axiomatic Theory of Bargaining with a Variable Number of Agents, Cambridge University Press, Cambridge, 1989.
- [11] J. von Neumann, O. Morgenstern, Theory of Games and Economic Behavior, John Wiley and Sons, New York, 1944.

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⁸ Keeping the other players' payoffs fixed.

⁹ Assume here that U is strictly convex.

¹⁰ Where each player makes the same proposal whenever he serves as the proposer, and his acceptance decision depends only on the current offer.