One-deviation principle and endogenous political choice[*]

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Abstract

We study social choice via an endogenous agenda setting process. At each stage, a status quo is implemented unless it is replaced by a majority (winning coalition) with a new status quo outcome. The process continues until the prevailing status quo is no longer replaced. We impose a one-time deviation restriction on the feasible policy processes. The key aspect of the solution is that it allows the process to depend on the history. A solution is shown to exists. Moreover, we show that the largest set of outcomes that can be implemented via a policy process that meets the on-deviation restriction coincides with the ultimate uncovered set. Finally, we show that our solution can be interpreted as a *stationary* Dynamic Condorcet Winner of Bernheim and Slavov (2009) in a model of repeated voting.

Keywords: voting, history dependence, one-deviation principle.

JEL: C71, C72.

1 Introduction

A recurring problem with political decision making is the lack of a Condorcet winner (e.g. Rubinstein 1979) - there is no status quo alternative that survives a majority contest against all other alternatives. For example, in the extensive literature on political institutions that focuses on the positional aspects of electoral campaigns, a Condorcet winner is guaranteed to exists only in the special one dimensional - single peaked case. In fact, the famous chaos theorems (McKelvey 1979, 1986; Bell, 1981; Schofield 1983) state that,

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with very relaxed conditions concerning how voters are distributed in the policy space, an agenda can be created where it is possible to start at any status quo alternative and, with a succession of majority comparisons, end at any other specified alternative in the policy space. Hence, without a presumed institutional structure there seems to be only little hope in reaching *any* predictions of the actual political choice. This is problematic since the outcome of a political process tends to be sensitive to the details of the structure. It also raises the natural question of where does the institutional structure come from.

Lack of farsightedness of agents is a well known and important limitation of with the chaos argument, however. With a forward moving agenda procedure studied by McKelvey and others, in which a status quo alternative is voted against a challenger and the winner becomes the new status quo alternative, farsighted voters should not vote for an alternative that triggers an undesirable path of status quos.[1] This questions whether all the dominance chains described by the chaos argument are really feasible. Indeed, as demonstrated by Banks (1985) and Shepsle and Weingast (1984), only a subset of outcomes turn out to be implementable under farsighted voting via a fixed, exogenous decision making procedure.

Modeling farsighted and *endogenous* political decision making has proved particularly challenging. The problem calls for modeling the dynamics of the agenda setting process, and there is no obvious way to do this (e.g. Banks and Duggan 2008; Dutta et al 2001a,b, 2002; Duggan 2006; Penn 2006a; Bernheim and Slavov 2009). The conceptual difficulty (see Ray 2007) stems from the open endedness of the problem. The profitability of a blocking of a status quo outcome can only be evaluated if one can predict the consequences of the blocking - the future blockings of the status quos. But since the later blockings should be evaluated according to the same criteria as the original one, there is no final stage from which to start the recursion. To obtain a well defined solution, and to guarantee its existence, the literature has often made demanding assumptions on the length of the agenda process, or on the underlying physical set up.

A natural way to solve the conceptual problem is to model policy making as an infinitely repeated policy process where the voters gain intertemporal payoff from the day-by-day decisions. This approach, recently adopted by Penn (2009), Roberts (2007), Konishi and Ray (2004), Duggan and Banks (2006), and, in particular, Bernheim and Slavov (2009), permits accounting for cyclic or randomized policy paths, which is often critical for the existence of the solution.

However, randomization and cycles are not an unproblematic way to solve the problem. First, computing randomized or cycling policy paths is difficult, and predictions based on them are less clear than desired. Second,

[1] The term "forward moving agenda" is due to Wilson (1986).

an ever changing policy requires much sophistication and coordination from the part of the voters. Finally, if blockings are interpreted as negotiation prior to a binding agreement, as is often the case in the one-shot voting context, it is not clear which outcome the voters could "agree upon" if there is no state in which the play stays permanently.[2]

Our aim is to characterize, and show the existence of, an endogenous and farsighted political decision making process that is not constrained by artificial bounds. In contrast to the above literature on infinite policy paths that allow infinite cycling or randomization, our focus is on policy processes that implement an outcome in finite time or, equivalently, converge to an *absorbing* state in finite time.

More concretely, we study the natural forward moving agenda of type McKelvey (1979, 1986), where, at each stage, a status quo may be replaced by a winning coalition (e.g. majority) with a new status quo outcome.[3] The process continues until the prevailing status quo is no longer challenged (and is implemented). Importantly, there are no bounds on how long the process may continue. Assuming only that the set of social alternatives is a compact metric space and that the social preferences are continuous, our set up encompasses, for instance, the case of finite set of social alternatives as well as the commonly studied spatial model where alternatives lie in a compact subset of finite dimensional Euclidean space.

As the solution concept we take the standard *one-deviation property,* equivalent to the solution used by Bernheim and Slavov (2009) in a framework where policy decisions are made repeatedly and future payoffs are discounted (they call a policy rule satisfying the property as a *Dynamic Condorcet Winner*). The solution demands that after each history of blockings of the status quos, the prescribed voting act is optimal for a winning coalition (e.g. a majority) in light of the continuation path that the action triggers. Thus the solution reflects farsightedness of the agents. The crucial feature of the process is that the voting act may *depend on the history.*

We study terminating policy programmes that implement an outcome in finite time after *any* history. By the chaos theorems, one is tempted to believe that termination and one-deviation property are not compatible properties of a policy programme. Our aim is to show that this conjecture is false.

We first give a characterization of terminating policy programmes that satisfy the one-deviation property, or equilibrium policy programmes for short. The characterization is directly in terms of the underlying dominance relation. That is, we identify a set of social alternatives that can be

[2] Assuming discounting, convexity of the payoff space can be used to guarantee the existence. Alternatively, one may focus on less stringent equilibrium notions (e.g. Herings et al 2004, Chwe 1994, or Greenberg 1990).

[3] This problem has been recently analysed by Roberts (2007) and Penn (2009). In their models, however, the challenging policy is exogenously determined.

implemented with an equilibrium policy programme, and show that for any such set of alternatives one can construct an equilibrium policy programme that is consistent with implementing outcomes in this set. Importantly, we show that an equilibrium policy programme always exists. This is due to the freedom that comes with the programme being conditioned on the history. The existence proof is by showing that an equilibrium programme can be built on a version of the *ultimate uncovered set*, resulting from infinitely iterating (our version of) the uncovered set.[4] We show that this set is the largest set of outcomes that can be implemented with *any* equilibrium programme.

As the ultimate uncovered set is a subset of the uncovered set, our results refine the conventional wisdom that, under variety of institutional settings, it is the uncovered set that describes the outcomes that can be implemented (Miller 1980; Shepsle and Weingast 1984; Banks 1985). Indeed, we show it is precisely the ultimate uncovered set that can be implemented in the natural non-cooperative equilibrium via the procedure that has been routinely analyzed in the literature. A second contribution of the paper is to extend the existing results on the uncovered set and its derivatives in general domains, an issue which has received some attention recently (see Penn 2006a; Banks et al 2006; Dutta et al 2005).[5] The crucial observation is that the uncovered set - given our version of the covering relation that is implied by the underlying one-deviation restriction - is compact. Without compactness, the general existence result cannot be extended to the iterations of the uncovered set. We prove that (our version of) the uncovered set and its iterations are closed and hence also nonempty. Moreover, we extend the result of Dutta (1988) by tying the ultimate uncovered set to the covering set in a general domain.

Finally, we show that a terminating policy programme satisfying the one-deviation restriction are equivalent, in real terms, to some *stationary* Dynamic Condorcet Winner of Bernheim and Slavov (2009) when there is no discounting and the associated majority relation is continuous (also the converse holds). Thus our existence result also proves the existence of a stationary Dynamic Condorcet Winner. This complements the existence result of Bernheim and Slavov (2009) which requires randomization.[6] Our characterization also connects stationary Dynamic Condorcet Winners tightly to the other solutions in the voting literature, in particular to the uncovered set and its iterations as well as to the covering set of Dutta (1988).

The current paper is related to the literature on endogenous agenda

[4]The uncovered set is due to Fishburn (1977), and Miller (1980). The ultimate uncovered set is studied by Dutta (1988). Coughlan and LeBreton (1999) study how to implement (in) this set.

[5]See also Bordes et al. (1992).

[6]They show the existence of a Dynamic Condorcet Winner in a model without randomization but cannot guarantee its stationarity.

formation. Duggan (2006) provides a general existence result for a game of endogenous agenda formation in which the agenda is formed by an ex ante known finite sequence of proposers. The constructed agenda is then voted upon. This generalizes the result in Banks and Gasmi (1987) in which three players take turns proposing a single alternative each. Dutta et al. (2002, see also 2001a,b) consider endogenous agendas in a less structured setting, imposing only consistency conditions on the outcomes of the process. Importantly, also they assume a bounded maximum length of the resulting agenda which again permits iterating the solution backwards.[7] To our knowledge, Penn (2008) is the only paper that allows unbounded proposal process. Players stop amending the agenda only when the constructed agenda is stable against changes, given the forthcoming voting under the agenda. Penn (2008) shows that, in the divide-the-dollar set up, the set of feasible outcomes is a subset of the vNM stable set associated to the problem.

The solution concept of this paper is related to equilibrium process of coalition formation by Konishi and Ray (2003) and Vartiainen (2010) who study coalition formation in the general framework of Chwe (1994). The existence results and characterizations in these papers do not, however, extend to the current set up for two reasons. First, Konishi and Ray (2003) assume history independent processes but allow randomization which transforms the existence question to the one of fixed point in a convex, compact set. Here, however, the rule is deterministic and it is history dependence that creates the necessary freedom to obtain the fixed point. No convexity assumptions are made. Vartiainen (2010) assumes finite outcome space which rules out, e.g. the spatial model. Second, and more importantly, the in the current model the solution is not only required to be robust against one-time deviation by the currently active coalition but against *all* decisive coalitions, , e.g. all majority coalitions. This makes the current solution much more demanding, and also means that the results in Konishi and Ray (2003) and Vartiainen (2010) do not apply.

The paper is organized as follows. Section 2 introduces the model and defines the solution concept. In Section 3, the solutions is characterized. Section 4 derives the existence result and, in Section 5, the connection to the model of Bernheim and Slavov (2009) is demonstrated. Section 6 concludes with discussion.

2 The Set Up

Let there be a set of *social alternatives* X. Preferences of individuals of the society are summarized by a *social preference ordering* $R \subseteq X \times X$, with the asymmetric part P. For instance, R could be the majority relation

[7] See also Penn (2006b).

(see below). We typically write xRy when $(x, y) \in R$. Denote the *lower contour set* and the *strict lower contour set* of R at x, respectively, by $L(x) = \{y \in X : xRy\}$ and $SL(x) = \{y \in X : xPy\}$. Furthermore, let $L^{-1}(x) = \{y \in X : yRx\}$. The *indifference set* of x is then defined by $L(x) \cap L^{-1}(x)$.

We make the following assumptions concerning the underlying physical structure:

A0 X is a compact metric space.

Relation R is *complete* if either xRy and/or yRx, for all $x, y \in X$.

A1 R is complete.

By A1, mapping L is nonempty valued. Also, by A1, $SL(x) \cup L^{-1}(x) = X$.

A correspondence is *continuous* if it is both upper and lower hemicontinuous.

A2 L and L^{-1} are continuous.[8]

We abstract from the details of how the preferences are aggregated but, by A1, a natural interpretation of R is the majority relation: xRy if at least one half of the voters prefer x over y. A0 permits all finite scenarios but also the case of multidimensional spatial preferences studied e.g. by McKelvey (1979). A2 is a technical assumption, guaranteeing that R is closed and that a small shift in an outcome does not increase dramatically the set of outcomes that are preferred.

A0-A2 are needed when we establish the existence of the solution. In the remainder of this paper, they are assumed without further notice.

Policy Programme A *path* is a *finite* sequence $\bar{x} = (x_0, ..., x_K)$ of outcomes. Denote the final element of a path $(x_0, ..., x_K)$ by

$$\mu[(x_0, ..., x_K)] = x_K.$$

Denote the set of paths, i.e. *histories,* by $H = \cup_{k=0}^{\infty} X^k$. Following Bernheim and Slavov (2009), *policy programme* σ specifies a social action given the history of past actions $\sigma : H \to X \cup \{\text{STOP}\}$. The interpretation of a policy programme is that if $\sigma(h, x) = y \in X$, then after a history h of status quos, the current status quo x is successfully challenged by a winning coalition (e.g. majority) with outcome y which the becomes the new status quo, and if $\sigma(h, x) = \text{STOP}$, then all the winning coalitions agree on implementing

[8]See Banks and Duggand (2000) and Banks et al (2006) on domains in which the condition holds.

x and this action is put in force. Thus, a policy programme specifies how the sequence of status quos evolves and which outcome - if any - eventually becomes implemented.

Denote, in the usual way, by $\sigma^t(h)$ the t^{th} iteration of σ starting from h, *i.e.*, $\sigma^0(h) = \sigma(h)$ and $\sigma^t(h) = \sigma(h, \sigma^0(h), ..., \sigma^{t-1}(h))$, for all $t = 1, ...$. A policy programme σ is *terminating* if, for any $h \in H$ there is $T < \infty$ such that $\sigma^{T+1}(h) = \text{STOP}$ (T may depend on h). That is, after history h, the policy programme will eventually implement the outcome $\sigma^T(h)$.

Our focus will be on terminating policy programmes. That is, we preclude at the outset complex dynamics such as infinite cycling. Terminating programmes are easy to interpret if the political process concerns a one-shot policy decision. With a terminating programme, political actions could reflect negotiation prior to a binding one-shot agreement. Terminating programmes are also easier to describe and compute.

One should note that the requirement that an agreement has to achieved in finite time reduces the flexibility of the political process. This makes it in general harder - not easier - to find a solution that meets the desired stability properties.

Let $\bar{\sigma}(h)$ denote the sequence of status quos in X that is induced by the programme σ from the history h onwards

$$\bar{\sigma}(h) = (\sigma^0(h), \sigma^1(h), ...).$$

If σ is terminating, then $\bar{\sigma}(h)$ is finitely long and $\mu[\bar{\sigma}(h)]$ is well defined, for all h. Specifically, for a terminating policy programme σ, if a policy action $a \in X \cup \{\text{STOP}\}$ is chosen at history $(h, x) \in H$, then

$$\mu[\bar{\sigma}(h, x, a)] = \begin{cases} \mu[\bar{\sigma}(h, x, y)], & \text{if } a = y \in X, \\ x, & \text{if } a = \text{STOP}. \end{cases} \tag{1}$$

In particular, $\mu[\bar{\sigma}(h, \sigma(h))] = \mu[\bar{\sigma}(h)]$.

The Solution Our equilibrium condition, which is just a version of the standard one-deviation principle, is defined next.

Definition 1 (One-Deviation Property) *A history dependent terminating policy programme σ satisfies the* one-deviation property *if*

$$\mu[\bar{\sigma}(h)] R \mu[\bar{\sigma}(h, a)], \quad \textit{for all } a \in X \cup \{\text{STOP}\}, \textit{ for all } h \in H.$$

That is, after each history, a winning coalition will not want to change the prescribed action given the consequences of the action and its counterfactual. Since the programme is terminating, the consequences are always well defined. It is important to note that the one-deviation restriction is imposed on all histories - that is, also on off-equilibrium histories. This means that at the final stage of any finitely long deviation sequence the final deviation violates the one-deviation property. Hence the property implies robustness against *finite* deviations.

In a framework where policy decisions are made repeatedly and future payoffs are discounted, Bernheim and Slavov (2009) introduce the concept of *Dynamic Condorcet Winner* (DCW) which is equivalent to the one-deviation property, adjusted to their framework. Since cycling or more complex dynamics is difficult to interpret in the standard one-shot social choice framework, a particular focus of Bernheim and Slavov (2009) is on *stationary* DCWs in which a policy converges immediately to an absorbing state after any history. However, Bernheim and Slavov (2009) find stationary a very demanding property. Existence of a stationary DCW is established under rather heavy domain conditions.

While stationarity is more stringent requirement than being terminating, we argue in Section 4 that together with the one-deviation property they are essentially equivalent.[9] This implies that all our results are transferable to the framework of Bernheim and Slavov (2009). In particular, we our results will establish the existence of a stationary DCW under rather weak conditions. We also characterize the feasible policy programmes.

Implementable outcomes and Condorcet consistency Note that an active winning coalition can always guarantee the status quo x by choosing "STOP". Therefore, the one-deviation property implies that

$$\mu[\bar{\sigma}(h,x)]Rx, \qquad \text{for all } (h,x) \in H. \tag{2}$$

That is, the outcome that becomes implemented if the equilibrium path is followed must not be majority dominated by any element along the path.

We say that the set Y of alternatives is *implementable* via a dynamic policy programme σ if

$$Y = \{x \in X : \sigma(h,x) = \text{STOP}, \text{ for some } h \in H\}.$$

That is, for each element x of Y there is a history (h,x) such that x is implemented. What the initial status quo is may affect the alternative that will be implemented in Y but not the set Y itself. The sets of implementable outcomes are the main object of our study.

Before going to the main results of the paper, we observe that our solution passes the test of being Condorcet consistent. An outcome x is a *Condorcet winner* if xRy, for all outcomes y. It is a *strong Condorcet winner* if xPy, for all outcomes $y \neq x$.

Proposition 2 *(a) Let z be a Condorcet winner. Then there is a terminating policy programme σ meeting the one-deviation property such that z is implementable via σ.*

[9] This is particularly true in the case of no discounting.

(b) Let z be a strong Condorcet winner. Then z is the only outcome that is implementable via any terminating policy programme σ meeting the one-deviation property.

Proof. (a): Construct a policy programme σ such that $\sigma(h,x) = z$ if $x \neq z$, and $\sigma(h,x) = \text{STOP}$ if $x = z$. We show that σ meets the one-deviation property. Since $\mu[\sigma(h,z,x)] = z$, a one-time deviation at (h,z) is not profitable, for any $h \in H$. Since $\mu[\sigma(h,z,\text{STOP})] = z$, since σ is terminating, and since z is a Condorcet winner, there is no profitable one-time deviation.

(b): Let σ be a policy programme that satisfies the one-deviation property. Suppose, on the contrary of the proposition, that $\sigma(h,x) = \text{STOP}$ for some $x \neq y$. Since $\sigma(h,x) = z$ is not a profitable one-time deviation, it must be that $\sigma(h,x,z) \neq \text{STOP}$. But then, since σ is terminating and since z is a strong Condorcet winner, $\sigma(h,x,z) = \text{STOP}$ is a profitable one-time deviation at history (h,x,z). ■

2.1 Characterization

In this section, we characterize terminating policy programmes meeting the one-deviation property. The characterization is given directly in terms of outcomes that are implementable via them. For this purpose, we define the following solution concept for social choice problems.

Definition 3 (Consistent Choice Set) *A nonempty set $C \subseteq X$ is a* consistent choice set *if, for any $x \in C$ and for any $y \in X \backslash \{x\}$, there is $z \in C$ such that $z \in L(x) \setminus SL(y)$.*

That is, if x is in C, then for any outcome y there is another outcome z - possibly x itself - *in* C such that xRz and zRy.[10] Hence, any element x in the choice set is reachable from any other element y with at most two dominance steps such that *also* the intermediate step, z, is in the set. While neither implies the other, there is a relationship between the notion of consistent choice set and that of the uncovered set (Fishburn 1977; Miller 1980), as will be seen in the next section However, in a current set up a consistent choice set is a *consistent set* of Chwe (1994), but not vice versa.[11]

A priori, the existence of a set C is not clear. This will be proven in the next section.

A consistent choice set contains only the set of Condorcet winners whenever this set is nonempty. Whenever a Condorcet winner does *not* exist, a

[10] Recall that R is complete.

[11] To see that a consistent choice set C is a consistent set, let $x \in C$ and $y \notin L(x)$. Then there is (x,y,z) that directly dominates (x,y) but does not indirectly dominate (x) such that $z \in C$. Hence $z \in L(x) \setminus SL(y)$.

consistent choice set contains at least three elements (apply the definition to any pair y, z in C), and is a strongly connected component of X.[12]

Now we characterize terminating policy programmes meeting the one-deviation property through the concept of consistent choice set.

Lemma 4 *Let terminating policy programme σ satisfy the one-deviation property. Then the set Y of outcomes that are implementable via σ is a consistent choice set.*

Proof. We show that Y satisfies Definition 3. Take any $(h, x) \in H$ such that $\sigma(h, x) = \text{STOP}$. Then $\mu[\bar{\sigma}(h, x)] = x \in Y$. Take any $y \in X$, and let $z = \mu[\bar{\sigma}(h, x, y)] \in Y$. By (2), zRy, or $z \notin SL(y)$. By Definition 1, xRz, or $z \in L(x)$, as desired. ■

We now show that the converse of this result holds too by constructing a terminating policy programme that meets the one-deviation property, and implements outcomes that form a consistent choice set. Fix a consistent choice set C and an alternative $\alpha \in C$. Let us describe a policy programme as a deterministic Markov chain $(\sigma^C : \tau^C, Q^C)$, where Q^C is a set of states, indexed by the elements of C such that

$$Q^C = \{q_x : x \in C\}. \tag{3}$$

Function $\tau^C : Q \times X \to Q$ is a transition function between states, function $\sigma^C : Q^C \times X \to X$ is the strategy that is conditional only on the outcome of the table and the current state. Note that Q^C partitions H based on the transition function τ.

Construct a function $z : X \times X \to X$ such that for any $x \in C$ and for any $y \notin C$,

$$z(x, y) \in C \cap L(x) \setminus SL(y). \tag{4}$$

By Definition 3, $C \cap L(x) \setminus SL(y)$ is non-empty and $z(x, y)$ is well defined.[13] Let the transition rule τ satisfy

$$\tau^C(q_x, y) = \begin{cases} q_y, & \text{if } y \in L(x) \cap C, \\ q_{z(x,y)}, & \text{if } y \notin L(x) \cap C. \end{cases} \tag{5}$$

Finally, given the function z, let the agenda setting strategy σ satisfy

$$\sigma^C(q_x, y) = \begin{cases} \text{STOP}, & \text{if } y \in L(x) \cap C, \\ z(x, y), & \text{if } y \notin L(x) \cap C. \end{cases} \tag{6}$$

We now give a verbal interpretation of the constructed policy programme. For the sake of the argument, think R as the majority relation. The policy

[12] There is a directed path from any element in X to any other element in X.

[13] Appealing to the Axiom of Choice.

programme is constructed so that any deviating majority coalition will become punished. The punishment is achieved by implementing an outcome that the deviating coalition does not prefer relative to the outcome that was originally to become implemented. The role of a state in the construction is to store in memory which majority is to be punished. The z-function specifies the majority whose job it is to implement the punishment (by stopping the programme). The transition function τ determines when and how the majority that is to be punished should be changed. The circularity in punishments eventually makes the programme robust against profitable majority deviations in all states, *i.e.*, after all histories.

Of course, the construction is feasible only due to the assumed characteristics of the consistent choice set C. The existence of a set with such characteristics is a separate issue, and established in the next section.

We will now prove formally that $(\sigma^C : \tau^C, Q^C)$ satisfies the one-deviation property. To this end, we state an intermediate result.

Lemma 5 *Let policy programme $(\sigma^C : \tau^C, Q^C)$ be constructed as in (3) - (6). Then $\mu[\bar{\sigma}^C(q_x, y)] \in L(x) \cap C$, for all $x, y \in X$.*

Proof. Starting from any $(q_x, y) \in Q^C$, it takes at most two periods to implement an outcome. Applying (5) and (6),

$$\mu[\bar{\sigma}^C(q_x, y)] = \begin{cases} y, & \text{if } y \in L(x) \cap C, \\ \mu[(q_{z(x,y)}, z(x,y))], & \text{if } y \notin L(x) \cap C. \end{cases}$$

Since (4) and (6) imply $\sigma^C(q_{z(x,y)}, z(x,y)) = \text{STOP}$, it follows that

$$\mu[\sigma^C(q_x, y)] = \begin{cases} y, & \text{if } y \in L(x) \cap C, \\ z(x,y), & \text{if } y \notin L(x) \cap C. \end{cases}$$

Thus, by (4), the result follows. ■

Lemma 6 *Policy programme $(\sigma^C : \tau^C, Q^C)$ satisfies the one-deviation property.*

Proof. Take any $(q_x, y) \in Q^C \times X$. It suffices to show that a one-time deviation from $\sigma^C(q_x, y)$ is not profitable. There are two cases.

1. Let $y \in L(x) \cap C$. Then $\sigma^C(q_x, y) = \text{STOP}$ and hence $\mu[\bar{\sigma}^C(q_x, y)] = y$. A deviation to $w \in X$ changes the state to $\tau^C(q_x, y) = q_y$. Then

$$\mu[\bar{\sigma}^C(\tau^C(q_x, y), w)] = \mu[\bar{\sigma}^C(q_y, w)]. \tag{7}$$

Applying Lemma 5 to $\mu[\bar{\sigma}^C(q_y, w)]$,

$$\mu[\sigma^C(q_y, w)] \in L(y) \cap C.$$

Thus, by (7), $\mu[\bar{\sigma}^C(\tau^C(q_x, y), w)] \in L(y)$, implying that the deviation is not profitable.

2. Let $y \notin L(x) \cap C$. Then $\sigma^C(q_x, y) = z(x, y)$ and $\tau^C(q_x, y) = q_{z(x,y)}$. Thus

$$\mu[\bar{\sigma}^C(q_x, y)] = z(x, y).$$

There are two kinds of deviations. (i) A deviation to "STOP" implements y. By (4), $z(x, y) \notin SL(y)$, thus the deviation is not profitable. (ii) A deviation $w \in X \setminus \{z(x, y)\}$ changes the state to $\tau^C(q_x, y) = q_{z(x,y)}$. Hence

$$\mu[\bar{\sigma}^C(\tau^C(q_x, y), w)] = \mu[\bar{\sigma}^C(q_{z(x,y)}, w)]. \tag{8}$$

Applying Lemma 5 to $\mu[\bar{\sigma}^C(q_{z(x,y)}, w)]$,

$$\mu[\bar{\sigma}^C(q_{z(x,y)}, w)] \in L(z(x, y)) \cap C. \tag{9}$$

Thus, by (8), $\mu[\bar{\sigma}^C(\tau^C(q_x, y), w)] \in L(z(x, y))$, implying that the deviation is not profitable. ■

By Lemma 4, a set Y of alternatives is implementable via a terminating policy programme meeting the one-deviation property only if Y is a consistent choice set. Conversely, by Lemma 6, outcomes of any consistent choice can be implemented via a terminating policy programme meeting the one-deviation property. We compound these observations into the following characterization.

Theorem 7 *Set Y of alternatives is implementable via a terminating policy programme that satisfies the one-deviation property if and only if Y is a consistent choice set.*

This result does not, however, tell anything about the existence of a consistent choice set nor how it can be identified. The existence of a consistent choice set is proven and an algorithm for identifying the maximal consistent choice set is provided in the next section.

3 Existence

We shall use the following version of the well known relation. Given $B \subseteq X$, we say that y *covers* x *in* B if $x, y \in B$, yPx, and xRz implies yPz, for all $z \in B$.[14] Since, by A1, xRx, we can state this more succinctly: y covers x in B if

$$L(x) \cap B \subseteq SL(y) \cap B \quad \text{and} \quad x, y \in B.$$

This relation was introduced by Duggan (2006) and Duggan and Jackson (2005) who call it *deep covering*.

[14] There are many versions of the covering operation in the literature. See below.

The covering relation in B is transitive. Denote the maximal elements of the covering relation in B by $UC(B)$, the *uncovered* set of B (cf. Fishburn, 1977; Miller, 1980). That is, $UC(B)$ comprises alternatives that are not covered in B by any element in B. The following important result is discussed by Duggan and Jackson (2005).

Lemma 8 *Let B be a closed subset of X. Then $UC(B)$ is nonempty and closed.*

Proof. First we show that $UC(B)$ is nonempty. Since, by A0, X is compact, B is compact. Let, by the Hausdorff Maximal Principle, $M \subseteq B$ be a maximal subset of B that is totally ordered by the covering relation. Since B is compact, A2 implies that there is $z \in B$ such that $L(z) \cap B = \cap_{x \in M} L(x) \cap B$. By the construction of z, either $M = \{z\}$ or z covers any element in M. In either case, z is not covered by any element in M. Since M is a maximal totally ordered subset of B, z is uncovered in B.

We now show that $UC(B)$ is closed. Suppose that $UC(B)$ is not closed. Then there is a converging sequence $\{x_k\} \subseteq UC(B)$ and $x \notin UC(B)$ such that $x_k \to x$. Since x is covered in B, there is $y \in B$ such that $L(x) \cap B \subset SL(y) \cap B$. Equivalently, $L(x) \cap L^{-1}(y) \cap B = \emptyset$. Since $x_k \in UC(B)$ for all k, also $L(x_k) \cap B \not\subset SL(y) \cap B$ for all k. That is, there is z_k such that $z_k \in L(x_k) \cap L^{-1}(y) \cap B$, for all k. Find a converging subsequence $\{z_{k(j)}\}_j$ and $z \in B$ such that $z_{k(j)} \to_j z$. Then also $x_{k(j)} \to_j x$. By A2, $z \in L(x) \cap L^{-1}(y) \cap B$. But then y does not cover x in B, a contradiction. ■

The compactness of $UC(B)$ owes to the asymmetry in the relations that define the covering relation, i.e. that the covering element's lower contour should be contained in the *strict* lower contour set of the element that is covered. To the author's knowledge, there are no corresponding results - and probably cannot be - in the standard case when covering is defined with respect to the "Miller relation" $L(x) \subseteq L(y)$ (e.g. Miller 1980; Banks 1985, and Dutta et al. 2004) or with respect to the "Gillies relation" $SL(x) \subseteq SL(y)$ (e.g. Shepsle and Weingast 1984). For a more comprehensive discussion and analysis of these versions of the uncovered set, see Bordes (1992), Penn (2006a), or Duggan et al. (2006).

Compactness of the uncovered set is, however, instrumental in one being able to iterate the concept. Our aim is to show that through iteration of the uncovered set -operator one eventually reaches a fixed point where the set coincides with the uncovered set derived from it. We show that this limit set also satisfies the properties of a consistent choice set.

The iterated version of the uncovered set, the *ultimate uncovered set,* is defined recursively as follows. Set $UC^0 = X$, and let $UC^{k+1} = UC(UC^k)$, for all $k = 0, \ldots$. By Lemma 8, since a closed subset of a compact metric space is itself a compact metric space, UC^{k+1} is closed and nonempty for all $k = 0, \ldots$. The ultimate uncovered set UUC is then obtained in the limit

$$UUC := UC^{\infty}.$$

The ultimate uncovered set UUC is nonempty and closed since it is an intersection of nested closed and nonempty sets.[15]

Lemma 9 *The ultimate uncovered set UUC is nonempty and closed.*

By construction, no element in UUC is covered in UUC. The next result extends the result of Dutta (1988) into general compact domains: the set UUC is a *covering set* in the sense that any element z outside UUC is covered in $UUC \cup \{z\}$, and that $UUC = UC(UUC)$.

Lemma 10 *Let $y \in X \diagdown UUC$. Then there is $z \in UUC$ such that $L(y) \cap UUC \subset SL(z) \cap UUC$.*

Proof. Choose $y = z_0$ and, for all $j = 0, \ldots$, find k_j such that z_{j+1} covers z_j in UC^{k_j} and $z_j \in UC^{k_j} \setminus UC^{k_j+1}$. Since the covering relation is transitive, such element exists by Lemma 8.

Since $L(z_0) \cap UC^{k_0} \subseteq SL(z_1) \cap UC^{k_0}$, and since $UC^{k_1} \subseteq UC^{k_0}$, it follows that $L(z_0) \cap UC^{k_1} \subseteq SL(z_1) \cap UC^{k_1}$. As the same relation holds for z_1 and z_2, we have, by chaining the relations, $L(z_0) \cap UC^{k_2} \subseteq SL(z_2) \cap UC^{k_2}$. By induction on $0, \ldots, j$, it follows that

$$L(z_0) \cap UC^{k_j} \subseteq SL(z_j) \cap UC^{k_j}. \tag{10}$$

Since X is compact there is z such that for a subsequence $\{z_k\}$ of $\{z_j\}$ we have $z_k \to z$. Since $z_k \in UC^k$ for all k, and $\cap_{k=0}^{\infty} UC^k = UUC$ is closed, it follows that $z \in UUC$. By (10), $L(z_0) \cap UUC \subseteq SL(z_k) \cap UUC$, for all k. Suppose that there is $w \in UUC \cap L(z_0) \setminus SL(z)$. By A2, there is an open neighborhood $E \subseteq X \times X$ of (z, w) such that $E \cap R = \emptyset$. Since $z_k \to z$, there must be k such that $(z_k, w) \in E$. But then $w \in UUC \cap L(z_0) \setminus SL(z_k)$, a contradiction. ■

That the ultimate uncovered set is a covering set means that if one moves away to y from an element x in the ultimate uncovered set, then it takes at most one (weak) dominance step from y to some z to return to the ultimate uncovered set. However, this does not yet mean that the arrival outcome z in the ultimate uncovered set is (weakly) dominated by the outcome x from the departure originally took place. And this is the property that is needed for the ultimate uncovered set to be also a consistent choice set. The next theorem, which is the main result of the paper, shows that the ultimate uncovered indeed has the desired property.

[15] The ultimate uncovered set is analysed in the finite case e.g. by Miller (1980), Dutta (1988) and Laslier (1998). The infinite case has not, to the best of our knowledge, been analysed before.

Theorem 11 *The ultimate uncovered set UUC is a consistent choice set.*

Proof. Take $x \in UUC$ and let $y \in X$. We find an element z in UUC such that $z \in L(x) \setminus SL(y)$. If $y \in UUC \cap L(x)$, then $y = z$ qualifies as such element. Thus let $y \notin UUC \cap L(x)$.

By Lemma 10, there is $z \in UUC$ such that $L(y) \cap UUC \subseteq SL(z) \cap UUC$. Since $z \notin L(y)$, we are done if $z \in L(x)$. Suppose, on the contrary, that $z \notin L(x)$. Since $x, z \in UUC$, and $UC^\infty = UUC$, it follows that $L(x) \cap UUC \nsubseteq SL(z) \cap UUC$. Thus there is $w \in UUC$ such that $w \in L(x) \setminus SL(z)$. Since $L(y) \cap UUC \subseteq SL(z) \cap UUC$, and $w \in UUC \setminus SL(z)$, we have that $w \notin L(y)$. Thus $w \in L(x) \setminus SL(y)$, as desired. ■

In fact, it is easy to see that *any* covering set is also a consistent choice set. However, the converse is not true. Moreover, as opposed to the case of covering sets, a minimal consistent choice set (in the sense of set inclusion) may not be unique (see Vartiainen 2006).

But to the other direction we can say more. The next result shows that UUC is the maximal consistent choice set in the sense of set inclusion.

Theorem 12 *The ultimate uncovered set UUC is the maximal consistent choice set.*

Proof. Let C be a consistent choice set. We show that $C \subseteq UUC$. By the definition of a consistent choice set, $C \cap L(x) \setminus SL(y)$ is nonempty, for all $x \in C$ and for all $y \in X$. Thus, for any $B \subseteq X$ such that $C \subseteq B$,

$$L(x) \cap B \nsubseteq SL(y) \cap B, \quad \text{for all } x \in C, \text{ for all } y \in B. \tag{11}$$

Choosing $B = X = UC^0$ in (11), it follows by the definition of covering that $C \subseteq UC(UC^0) = UC^1$. By induction, $C \subseteq UC(UC^k) = UC^{k+1}$, for all $k = 0, 1, \ldots$. Thus $C \subseteq UC^\infty = UUC$. ■

Finally, we are able to tie the existence results concerning consistent choice sets to the existence issue of policy programmes that satisfy the one-deviation property. By Theorems 7 and 12, we have shown that a terminating policy programme that has the one-deviation property does exist, and that the set outcomes that are implementable via any such programme is contained in the ultimate uncovered set.

Corollary 13 *There is a terminating policy programme meeting the one-deviation property that implements outcomes in the ultimate uncovered set. Moreover, the ultimate uncovered set is the maximal set of outcomes that can be implemented via any terminating policy programme meeting the one-deviation property.*

Thus it is without loss of generality to focus on the ultimate uncovered set UUC if one is interested in the welfare consequences of a dynamic political decision making.

4 Relationship to Bernheim and Slavov (2009)

In this section we interpret our results in the framework of Bernheim and Slavov (2009). The clearest connection can be made when X is a finite set.

The model is captured by $\{1,...,n\}$ individuals. The set X is now interpreted as social *states* which may change in dates $t = 0, 1, ...$. The per-period utility functions of the players are written as $u_i : X \to \mathbb{R}$ for all $i = 1, ..., n$, which induces a per-period utility possibility set $U = \{u(x) \in \mathbb{R}^n : x \in X\}$. Given that X is a finite set, the set U also contains finitely many elements. This implies that the convexity conditions of Bernheim and Slavov (2009), which are required for their existence result, need not be met.

To complete the analysis relating our solution concept to that of Bernheim and Slavov (2009), we focus is on the generic finite case where indifferences are ruled out. We assume that n is *odd* and that preferences over per period payoffs are strict: $u_i(x) \geq u_i(y)$ and $x \neq y$ implies $u_i(x) > u_i(y)$, for all i and for all x, y.

Under these assumptions, X satisfies condition A0 and the majority relation $M \subset X \times X$ such that

$$xMy \quad \text{if and only if} \quad \#\{i : u_i(x) \geq u_i(y)\} \geq \frac{n}{2},$$

satisfies conditions A1 and A2. Moreover, M is asymmetric. This guarantees that M is robust against small changes in the agents' payoffs.[16]

Policy making is now an ongoing process where the individuals gain benefits from the policy choices in each period $t = 0, 1, ...$. Letting H denote the set of all possible finite paths of social alternatives - the set of histories - a dynamic policy programme is now a function $p : H \to X$, capturing the transitions from one history to another. Let H be the set of all histories of states $(x_0, ..., x_t)$ such that $x_0 = x^\alpha$. These transitions will be induced by winning majority coalitions who stand to benefit from them. Let $p^0(h) = p(h)$ and $p^t(h) = p(h, p^0(h), ..., p^{t-1}(h))$, for all $t = 1, ...$.

Let the intertemporal payoffs be evaluated by discounted sum of per period payoffs

$$v_i(p(h)) = \sum_{t=0}^{\infty} u_i(p^t(h))\delta^t.$$

A policy programme p is a *Dynamic Condorcet Winner* (DCW) of Bernheim and Slavov (2009) if

$$\#\{i : v_i(p(h)) \geq v_i(p(h, y))\} \geq \frac{n}{2}, \text{ for all } h \in H, \text{ for all } y \in X.$$

That is, if a majority of agents prefers action $p(h)$ over any action y at any history h, given the continuation path the action triggers.

[16] xMy and yMx imply $x = y$.

To highlight the relationship of our solution to that of Bernheim and Slavov (2009), let us focus on policy programmes that are *stationary* in the sense that

$$p(h) = p(h, p(h)), \quad \text{for all } h \in H. \tag{12}$$

That is, after all histories, the policy path converges immediately to an *absorbing* state in which it stays permanently. Stationary rules are simple and intuitive as they do now exhibit complex dynamics or cycles.

As Bernheim and Slavov (2009) discuss, stationarity is a desirable property of a choice rule but also quite demanding. Their existence result concerning stationary DCWs require convexity assumptions, e.g. randomization over X. Our aim is to show that randomization is not needed.

For any stationary policy programme p, the limit of the intertemporal payoff of i as δ tends to unity is well defined, and satisfies $\lim_{\delta \to 1} v_i(p(h)) = u_i(p(h))$, for all h. Given that n is odd and the per period preferences are linear, there is $\bar{\delta} \in (0,1)$ such that a stationary policy programme p is a DCW for all $\delta \geq \bar{\delta}$ if and only if

$$p(h) M p(h, y), \text{ for all } h \in H, \text{ for all } y \in X. \tag{13}$$

To prove that for each stationary DCW p there is an equivalent terminating policy programme σ meeting the one-deviation property (defined with respect to M), construct σ from p by letting, for all $h \in H$,

$$\sigma(h,x) = \begin{cases} \text{STOP} & \text{if} \quad p(h,x) = x, \\ p(h,x) & \text{if} \quad p(h,x) \neq x. \end{cases}$$

Since p is a DCW, σ satisfies the one-deviation property, by (13).

For the other direction, let C be a consistent choice set and construct a policy programme $(p^C : \tau^C, Q^C)$ such that

$$\tau^C(q_x, y) \quad = \quad \begin{cases} q_y, & \text{if } y \in L(x) \cap C, \\ q_{z(x,y)}, & \text{if } y \notin L(x) \cap C, \end{cases}$$

and

$$p^C(q_x, y) \quad = \quad \begin{cases} y, & \text{if } y \in L(x) \cap C, \\ z(x,y), & \text{if } y \notin L(x) \cap C. \end{cases}$$

The only difference of this programme to the one defined in (3) - (6) concerns the choice $p^C(q_x, y)$ when $y \in L(x) \cap C$. Since

$$p^C(\tau^C(q_x, y), y) = p^C(q_y, y) = y$$

and

$$\tau^C(q_y, y) = q_y,$$

it follows that the policy programme $(p^C : \tau^C, Q^C)$ is stationary: starting from any configuration, the programme starts repeating the status quo at most after one period lag. By Lemma 6, it is clear that the programme also meets (13).

By Theorem 7, we may compound the above observations in a proposition.

Proposition 14 *Let X be finite, n odd, and the agents' preferences over X strict. Then the set B of states is a consistent choice set with respect to the majority relation M if and only if there is $\delta_B \in (0,1)$ such that B is the set of absorbing states of a stationary DCW for all $\delta \geq \delta_B$.*

By the previous proposition, and by Theorems 11 and 12, we can conclude that a stationary DCW always exists, and that the ultimate uncovered set completely characterizes the stationary states that can be supported by a DCW.

Corollary 15 *Let X be finite, n odd, and players' preferences over X strict. There is $\bar{\delta} \in (0,1)$ such that a stationary DCW exists for all $\delta \geq \bar{\delta}$. Moreover, there is $\delta_{UUC} \in (0,1)$ such that UUC is the set of possible absorbing states of any stationary DCW, for all $\delta \geq \delta_{UUC}$.*

That is, UUC serves as a reliable prediction of a political process when the parties seek to converge to a stable state that can be supported in the long run. This result complements the existence result in Bernheim and Slavov (2009) which requires convexity assumptions. The leading motivation for the convexity assumption is randomization which may be difficult to motivate in the context of a political process where the parties cannot commit to the status quo outcome. First, it is not clear in what sense could a majority coalition choose a randomized action. Second, since the lack of commitment is a primitive of the model, i.e. that the parties cannot commit not to change status quo, it is natural to think that they need not commit to a randomized devices either. That is, after uncertainty related to the randomized device has resolved, they have an option to rethink their choice. Third, stationarity as a concept loses some of its appeal if the per period state is randomly determined.

Note on the no-discounting case The above results are stated under discounting, to relate our model to that of Bernheim and Slavov (2009). However, they extend without complications to the limiting case, where the payoff streams are evaluated by the time-average criterion:

$$v_i(p(h)) = \frac{1}{T}\sum_{t=0}^{T} u_i(p^t(h)).$$

Under this payoff specification, the results of this section can be stated without the restriction that M is asymmetric. Also finiteness of X can be relaxed but M then has to be assumed continuous.

Furthermore, under this intertemporal payoff specification the results can also be extended by stating them in terms of *absorbing* policy programmes. A programme p is absorbing if, after all h there it t_h such that $p^t(h) = p^{t+1}(h)$, for all $t > t_h$. Under the time-average payoff criterion, absorbing policy programmes can be interpreted as terminating ones, where an outcome or state is implemented when the policy process absorbs to it. Thus, by Section 3, the set B of states is a consistent choice set with respect to the majority relation M if and only if B is the set of absorbing states of a stationary DCW. Note that stationary programmes are absorbing but not vice versa.

5 Conclusion

In this paper, we study farsighted political decision making when the voting acts may be conditioned on the history. We abstract from the details of the voting procedure and assume that individual preferences are aggregated by a continuous social preference (e.g. majority) ordering. Choices are made on the basis of binary comparisons - the current status quo may be challenged with another outcome and the status quo is implemented if it is not defeated by any challenger. The key aspect of the model is farsightedness: the agents foresee the consequences of the blocking behavior. The solution we apply is the standard one-deviation principle.

Our results contribute to the voting literature in three dimensions. First, we show that the one-deviation property, which is a synonym for sequential rationality in many non-cooperative set ups, is a natural way to model collective decision making in the canonical social choice scenario: a dynamic policy programme meeting the one deviation property always exists and has an interpretation in terms of well known solution concepts in the social choice literature. In particular, our model bridges the one-deviation property to the concept of ultimate uncovered set, the infinitely iterated version of a version of the uncovered set. The uncovered set has been one of the central solution concepts in the literature of agenda formation and voting (e.g. Miller 1980; Shepsle and Weingast 1984; Banks 1985).

Second, as our domain restrictions are rather weak - we assume that the set of social alternatives is a compact metric space - our results extend the literature on the uncovered set and its derivatives. In particular, we introduce a new version of the covering operation that has an interpretation in terms of the one-deviation property. Importantly, we show that the uncovered set that is derived with respect to this covering relation is compact.[17]

[17]E.g. Bordes et al. (1992) provide conditions under which the Miller's and Gillies'

As a consequence, we can show that the iterations of the uncovered set and, in particular, the ultimate uncovered set does exist. We also show that, with this definition of covering, the ultimate uncovered set is a covering set of Dutta (1988).

Third, we demonstrate that terminating policy programmes meeting the one-deviation property are, in real terms, equivalent to *stationary* Dynamic Condorcet Winners (DCWs) of Bernheim and Slavov (2009). Thus our results, in particular on the existence, are directly transferable to their framework. The existence of a stationary DCW is not clear *a priori* since stationarity is a demanding condition. Moreover, our analysis allows one to interpret the important solution of Bernheim and Slavov (2009) in a general class of political domains.

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