Microeconomic Theory

Lecture notes 8

Production Economies

- Index consumers by $h \in \{1, ..., H\}$.
- Commodities are indexed as before by $k \in \{1, ..., K\}$.
- Utility functions: $u^h : \mathbb{R}^K_+ \to \mathbb{R}$.
- Initial endowments $\omega^h \in \mathbb{R}^K_+$.
- Firms are indexed by $j \in \{1, ..., J\}$.

- Technologies given by production sets $Y^j \subset \mathbb{R}^K$.

- Consumer h owns share θ^{hj} of firm j.

- An allocation is $(x, y) = \left(x^1, ..., x^H, y^1, ..., y^J\right)$ such that $x^h \in \mathbb{R}_+^K$ for all h and $y^j \in Y^j$ for all j.

- An allocation is feasible if

$$\sum_h x^h = \sum_h \omega^h + \sum_h y^h.$$

- An allocation (x, y) is Pareto-efficient if there is no other feasible allocation (x', y') such that

$$u^h(x^h) \geq u^h(x'^h) \text{ for all } h \text{ and } u^h(x^{h\prime}) > u^h(x'^{h'}) \text{ for some } h'.$$

Definition 1 *A competitive equilibrium in a production economy consists of a price vector* $p^* \in \mathbb{R}_+^K$ *and an allocation* (x^*, y^*) *such that:*

1. y^{*j} solves

$$\max_{y \in Y^j} p \cdot y \text{ for all } j.$$

2. x^{*h} solves

$$\begin{aligned} &\max_x u(x) \\ \text{s.t. } p \cdot x \ &\leq \ p \cdot \omega^h + \sum_j \theta^{hj} \left(p \cdot y^j \right). \end{aligned}$$

3.

$$\sum_h x^{*h} = \sum_h \omega^h + \sum_j y^{*j}.$$

- Existence and welfare properties can be established along similar lines to the discussion in exchange economies.

Extending the model:

- A sequence of dates, e.g. macro and dynamic finance models.

- A number of contingencies important in applied work.

- Externalities and public goods.

Problems arise with:

- Missing markets.

- Incomplete information.

- More realistic firm behavior (e.g. voting by shareholders).

General Equilibrium Theory - Special cases

Exchange Economies

- $\left(u^1, \omega^1, ..., u^H, \omega^H\right)$.

- Solve for the Walrasian demand $x^h\left(p, \omega^h\right)$ for each h.

- Find $p \in \mathbb{R}_+^K$ such that $\sum_h x^h\left(p, \omega^h\right) = \sum_h \omega^h$.

Example, Cobb-Douglas economy with two goods.

$$u^h\left(x^h, y^h\right) = \alpha^h \ln x^h + \left(1 - \alpha^h\right) \ln y^h.$$

Prices: $p = (p_x, p_y)$.

Individual demands:

$$x^h\left(p, \omega^h\right) = \frac{\alpha^h\left(p_x \omega_x^h + p_y \omega_y^h\right)}{p_x}.$$

Market clearing for x :

$$\sum_{\iota=1}^{H} \frac{\alpha^h\left(p_x \omega_x^h + p_y \omega_y^h\right)}{p_x} = \sum_{\iota=1}^{H} \omega_x^h.$$

Rearranging this yields:

$$\sum_{\iota=1}^{H} \left(1 - \alpha^h\right) p_x \omega_x^h = \sum_{\iota=1}^{H} \alpha^h p_y \omega_y^h.$$

Thus we have

$$\frac{p_x}{p_y} = \frac{\Sigma_h \alpha^h \omega_y^h}{\Sigma_h \left(1 - \alpha^h\right) \omega_x^h}.$$

Since market clears for good x and the budget constraints hold as equalities, market also cleares for y at these prices. (You can verify this by direct calculation).

The equilibrium allocation is found by substituting the equilibrium relative prices into the demand functions:

$$x^h = \alpha^h \omega_x^h + \frac{\alpha^h \omega_y^h}{\Sigma_h \alpha^h \omega_y^h} \sum_h \left(1 - \alpha^h\right) \omega_x^h.$$

Similarly,

$$y^h = \left(1 - \alpha^h\right)\omega_y^h + \frac{\left(1 - \alpha^h\right)\omega_x^h \sum_h \alpha^h \omega_y^h}{\sum_h \left(1 - \alpha^h\right)\omega_x^h}.$$

It is easy to calculate the effects of e.g. increasing the aggregate endowment of one good on the prices and equilibrium consumption and to do other comparative statics exercise based on this simple model.

Exchange Economy under Uncertainty

H consumers.

S states of the world, $s \in \{1, ..., S\}$.

A single physical good, x.

Consumption of x in state s by consumer h, x_s^h.

Endowment of consumer h in state s is ω_s^h.

von Neumann-Morgenstern utility functions of the consumers:

$$U^h\left(x_1^h, ..., x_S^h\right) = U^h\left(x^h\right) = \sum_{s=1}^{S} q(s) u^h\left(x_s^h\right),$$

where $u^h : \mathbb{R}_+ \longrightarrow \mathbb{R}$ is a strictly increasing, strictly concave and differentiable function.

Note that $q(s)$ should be interpreted as the commonly held probability of state s being realized.

Thus we normalize (without loss of generality) $\sum_s q(s) = 1$.

Assume that there is no aggregate risk, i.e.

$$\sum_{h=1}^{H} \omega_s^h = \overline{\omega} \text{ for all } s.$$

The relevant consumption set for the consumers in this example is $\mathbb{R}_+^S$.

A competitive equilibrium for this economy is (p, x), where $p \in \mathbb{R}_+^S$ and $x = \left(x^1, ..., x^H\right) \in \mathbb{R}_+^{NS}$ such that

h) For all h, x^h maximizes $U\left(x^h\right)$ s..t $p \cdot x^h \leq p \cdot \omega^h$.

$ii)$ For all s, $\sum_{h=1}^{H} x_s^h = \sum_{h=1}^{H} \omega_s^h = \overline{\omega}$.

To characterize the competitive equilibria of the model, use the welfare theorems.

First characterize the Pareto optimal allocations.

Consider an arbitrary feasible allocation x.

Suppose that $x_s^h \neq x_{s'}^h$ for some h and for some s, s'.

Compare this allocation to x^*, where $\left(x_s^h\right)^* = \sum_s q(s) x_s$.

In this allocation, each consumer consumes her average consumption in all states.

Then

$$
\begin{aligned}
U^h\left(\left(x^h\right)^*\right) &= \sum_s q(s) u^h\left(\sum_s q(s) x_s^h\right) \\
&= u^h\left(\sum_s q(s) x_s^h\right) \\
&\geq \sum_s q(s) u^h\left(x_s^h\right) \\
&= U\left(x^h\right)
\end{aligned}
$$

The inequality follows from the concavity of u^h and it is strict if $x_s^h \neq x_{s'}^h$ for some s, s'.

Thus x^* Pareto dominates x if we can show that x^* is also feasible.

To see this, note that

$$\begin{aligned}\sum_{h=1}^{H}\left(x_s^h\right)^* &= \sum_h \sum_s q(s) x_s^h \\ &= \sum_s q(s) \sum_h x_s^h \\ &= \sum_h x_s^h \\ &= \overline{\omega}.\end{aligned}$$

Hence the only Pareto optimal allocations have all individuals perfectly insured.

By first welfare theorem, we know that all competitive allocations must then have $x_s^h = x_{s'}^h$ for all h, s, s'.

The first order conditions for optimal consumer demand imply:

$$\frac{q(s)u'\left(x_s^h\right)}{q(s')u'\left(x_{s'}^h\right)} = \frac{p_s}{p_{s'}}.$$

But then, we must have

$$\frac{p_s}{p_{s'}} = \frac{q(s)}{q(s')}.$$

2×2 **Production Economy**

Two outputs: y_1, y_2.

Two factors: z_1, z_2.

Production technologies:

$$y_h = f^h\left(z_{i1}, z_{i2}\right) \text{ for } h = 1, 2,$$

where z_{ij} denotes the amount of factor j used in the production of output h.

Factor endowments: $\overline{z}_j$ for $j = 1, 2$.

Definition 2 *A vector* $y = (y_1, y_2)$ *is* output efficient *if there is no other vector* y' *such that* $y' > y$ *and such that*

$$y'_h = f^h\left(z'_{i1}, z'_{i2}\right) \text{ for } h = 1, 2 \text{ and } z_{1j} + z_{2j} \leq \overline{z}_j \text{ for } j = 1, 2.$$

We can draw a production Edgeworth Box to demonstrate the set of production efficient output vectors.

Equilibrium with production:

Assume a small open economy: Output prices are fixed in the world market: p_1, p_2 and factors do not move across borders. However, factor prices, $w_1$and w_2 are determined endogenously.

Calculate factor demands:

$$z_{ij}\left(p_1, p_{2,} w_1, w_2\right) \text{ for } h = 1, 2 \text{ and } j = 1, 2.$$

Market clearing: Find w_1, w_2 such that

$$\sum_h z_{ij}\left(p_1, p_{2,} w_1, w_2\right) = \overline{z}_j \text{ for } j = 1, 2.$$

Example: CRS Cobb-Douglas technology:

$$y_h = z_{i1}^{\alpha^h} z_{i2}^{1-\alpha^h} \text{ for } h = 1, 2.$$

Only the ratio of factor demands for each output is determined for each firm. (Why?).

From FOC's, we get

$$r_h = \frac{z_{i1}}{z_{i2}} = \frac{\alpha^h w_2}{\left(1 - \alpha^h\right) w_1}.$$

For interior solutions to the firms' problems, we must have a zero profit condition:

$$p_h y_h - w_1 z_{i1} - w_2 z_{i2} = 0 \text{ for } h = 1, 2.$$

But dividing both sides by z_{i2}, we get:

$$p_h r_h^{\alpha^h} - w_1 r_h - w_2 = 0 \text{ for } h = 1, 2.$$

Substituting gives:

$$p_h \left(\frac{\alpha^h w_2}{\left(1 - \alpha^h\right) w_1} \right)^{\alpha^h} = \left(\frac{1}{1 - \alpha^h} \right) w_2 \text{ for } h = 1, 2.$$

Solving from this we get the equilibrium ratio of factor prices as:

$$\frac{w_2}{w_1} = \left[\left(\frac{1 - \alpha^1}{1 - \alpha^2} \right) \left(\frac{p_1}{p_2} \right) \left(\frac{\alpha^1}{1 - \alpha^1} \right)^{\alpha^1} \left(\frac{\alpha^2}{1 - \alpha^2} \right)^{-\alpha^2} \right]^{\frac{1}{\alpha^2 - \alpha^1}}.$$

From this expression, we see that the factor endowments have no effect on the relative factor prices.

The comparative statics of the model with respect to output price increases (Stolper-Samuelson Theorem) and factor endowments (Rybczynski Theorem) are easily analyzed graphically.