Microeconomic Theory

Lecture 2

Consumer behavior under WA

- Use choice theory to derive positive implications for consumer theory.

- Derive the *Walrasian demand* from the observable primitives.

- The primitives: prices, income, and choices.

- How complete is the description of the consumer behavior that can be obtained by applying WA?

- Endogenous variables change in response to exogenous variables.

- Endogenous: Consumption choices.

- Exogenous: Prices and income.

- X the choice set is now interpret as the consumption set.

- We take $X = \mathbb{R}_{+}^{L}$, where $L \in \mathbb{N}$.

- $x = (x_1, ..., x_L)$, where each $x_l \in \mathbb{R}_{+}$ for each $l \in \{1, ..., L\}$.

- Goods are divisible, the choice set is convex.

- Conventions on matrices and operators: Let $x, y \in \mathbb{R}^L_+$ and $f : \mathbb{R}^L_+ \longrightarrow \mathbb{R}^L_+$. Then

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_L \end{bmatrix}, \; Df(x) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_L} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_L}{\partial x_1} & \cdots & \frac{\partial f_L}{\partial x_L} \end{bmatrix}.$$

Let $y \in \mathbb{R}^L_+$, and let X be an $L \times L$ matrix. Then

$$y^T x = y \cdot x = \sum_{l=1}^{L} y_l x_l.$$

- Nature of opportunity sets in general

- $B \in \mathcal{B}$ defines the *budget set* of a consumer.

- $\mathcal{B}$ gives the set of all possible budget situations.

- The feasible budget is defined by *prices* p and *disposable income* or wealth w.

- A budget feasible consumption is one that can be purchased with the disposable income.

- In classical consumer theory (and in this lecture), we assume that prices are *linear*:

  - The price of an additional unit of good l is independent of the amount of good l purchased.

  - The price of an additional unit of good k is independent of consumptions of goods $l \neq k$.

  - Rules out quantity discounts and offers like 'Buy CPU from us and get printer for 50% off'.

  - Rules out progressive taxes, exemptions etc.

  - $p \in \mathbb{R}^L_+, \; w \in \mathbb{R}_+$.

- Walrasian budget set:

$$\begin{aligned} B &= \{x \in X : p \cdot x \leq w\} \\ &= \left\{x \in X : \sum_{l=1}^{L} p_l x_l \leq w\right\} \end{aligned}$$

- Since B is determined by p and w, we write $B(p, w)$.

- Note: $B(p, w)$ rules out nonlinearities, indivisibilities, uncertainties, and interdependencies between individuals

- Walrasian demand correspondence $x(p, w)$: consumption choice given the budget set $B(p, w)$.

- Specifies the consumption of each commodity $l = 1, ..., L$.

- Is defined for all p and w. Hence

$$x : \mathbb{R}_+^L \times \mathbb{R}_+ \to \mathbb{R}_+^L.$$

- We *assume* that $x(p, w)$ is single valued function.

- Two additional assumptions on $x(p, w)$.

- Nothing's wasted:.

Assumption 1 The Walras' Law: $p \cdot x(p, w) = w,$ for all p, w.

- Sometimes called the adding-up restriction

- Only consumption matters.

- Since $B(p,w) = B(\lambda p, \lambda w)$:

Assumption 2 Homogeneity: $x(\lambda p, \lambda w) = x(p,w)$, for all $\lambda > 0$ and all p, w.

- That is, $x(p,w)$ is homegenous of degree 0 in (p,w).

- No money illusion.

- The effect of units on the consumer's perception of opportunities.

- Notice that $p \cdot x\,(p,w)$ is a function of $L+1$ variables, i.e. $p \cdot x\,(p,w) : \mathbb{R}_+^{L+1} \rightarrow \mathbb{R}_+$.

- Denote by $D_p x\,(p,w)$ the derivative of $x\,(p,w)$ with respect to p and by $D_w x\,(p,w)$ the derivative w.r.t. w. Then

$$\begin{aligned} D_p x(p,w) &: \mathbb{R}_+^{L+1} \rightarrow \mathbb{R}_+^{L \times L}, \\ D_w x(p,w) &: \mathbb{R}_+^{L+1} \rightarrow \mathbb{R}_+^{L}. \end{aligned}$$

- Implications of Walras' law (which is an identity):
  - Engel aggregation: increased wealth is consumed
$$p \cdot D_w x(p,w) = 1. \tag{1}$$
  - Cournot aggregation: total expedinture independent of prices
$$p \cdot D_p x(p,w) + x(p,w) = 0. \tag{2}$$

- Note that, by homogeineity, for all $l = 1, ..., L$,
$$\begin{aligned} \left.\frac{d}{d\lambda} x_l(\lambda p, \lambda w)\right|_{\lambda=1} &= \\ D_p x_l(p,w) \cdot p + D_w x_l(p,w)\, w &= 0. \end{aligned} \tag{3}$$

- Denote the budget share of l by

$$b_l(p,w) = \frac{p_l x_l(p,w)}{w}, \text{ for all } l = 1, ..., L.$$

Denote the price and income elasticities by

$$\begin{aligned} \varepsilon_{lk}(p,w) &= \frac{\partial x_l(p,w)}{\partial p_k} \frac{p_k}{x_l(p,w)}, \text{ for all } l = 1, ..., L, \\ \varepsilon_{lw}(p,w) &= \frac{\partial x_l(p,w)}{\partial w} \frac{w}{x_l(p,w)}. \end{aligned}$$

Then, by the Cournot and Engel aggregation rules,

$$\begin{aligned} \sum_{l=1}^{L} b_l(p,w)\,\varepsilon_{lk}(p,w) + b_k(p,w) &= 0, \\ \sum_{l=1}^{L} b_l(p,w)\,\varepsilon_{lw}(p,w) &= 1. \end{aligned}$$

By homogeneity, for all $l = 1, ..., L$,

$$\sum_{k=1}^{L} \varepsilon_{lk}(p,w) + \varepsilon_{lw}(p,w) = 0.$$

- Recall: WA if $x, y \in B$ and $x \in c(x)$, then $x, y \in B'$ and $y \in c(B')$ implies $x \in c(B')$.

- In the context of Walrasian budget sets, this has the form:

Axiom 1 $x(p, w)$ *satisfies WA if*

$$p \cdot x(p', w') \leq w \text{ and } x(p, w) \neq x(p', w'),$$

implies

$$p' \cdot x(p, w) > w',$$

for any two budget situations (p, w) *and* (p', w').

- Do you see the relation to WA? (recall that x is single valued)

- Recall the *Law of Demand*: $x(p,w)$ and p move to in opposite directions.

- Intuition: The more something costs, the less one can afford it.

- However, not obvious: An increase in p_l changes relative prices (slope of the budget line) and effective wealth (i.e. is not feasible with new prices).

- Wealth effect not necessarily positive which implies overall ambiguity.

- To isolate the substitution effect we consider *compensated* price changes.

- Idea: Look at the effects of relative price changes by offsetting the associated wealth change, i.e. forcing the original consumption point to lie on the new budget line.

- Formally, (p', w') is a compensated price change from (p, w) if

$$p' \cdot x(p, w) = w'.$$

Proposition 2 *Suppose $x(p, w)$ satisfies Assumptions 1-2. Then $x(p, w)$ satisfies WA if and only if, for any compensated price change (p', w'),*

$$(p' - p) \cdot [x(p', w') - x(p, w)] \leq 0 \qquad (4)$$

where the inequality is strict whenever $x(p', w') \neq x(p, w)$.

- This might be called as the *compensated law of demand:* compensated demand and price move to opposite directions.

- Define the *substitution* or the *Slutsky matrix* $S(p, w)$ of $x(p, w)$:

$$S(p, w) = [D_p x(p, w) + D_w x(p, w) x(p, w)^T],$$

whose lk-component

$$\frac{\partial x_l(p,w)}{\partial p_k} + \frac{\partial x_l(p,w)}{\partial w} x_k(p,w)$$

decomposes the effect of p_k's price change to l's demand into a *substitution effect* and an *income effect*.

- The compensated law of demand, has implications for $S(p,w)$: the differential analog $dp \cdot dx \leq 0$ of (4) implies

$$dp \cdot S(p,w) dp \leq 0.$$

- This says that the $L \times L$ matrix $S(p,w)$ is *negative semidefinite*.

Proposition 3 *Suppose $x(p,w)$ satisfies Assumptions 1-2 and the WA. Then, at any (p,w) the Slutsky matrix $S(p,w)$ is negative semidefinite.*

- Local properties.

- Giffen goods.

- Does negative semidefinite $S(p, w)$ generated by $x\,(p, w)$ satisfying the Walras Law and Homogeneity restiction imply that WA is also satisfied?

- $S(p, w)$ is not symmetric in general.