Microeconomic Theory

FDPE 2007

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Lecture 1: Choice Theory

- What does observed (economic) behavior tell us about decision maker?

- Observations without data organizing assumptions meaningless. What to assume?

- What is rationality? What does that imply on choice behavior?

From choice to preferences

Four elements:

1. The known choice set X.

2. Observed feasible set $B \subset X$.

3. Choice Rule.

4. Behavioral assumption.

Set of possible outcomes X

- X is the universe of alternative choices

- Examples:

  1. Admissions to a Ph.D. programme in economics.
  2. Consumption over time.
  3. Speeding or not speeding.
  4. Occupational choice.
  5. $\mathbb{R}^n_+$

Feasible Set B

- Achievable choices
- May be dependent on external conditions.
- Examples:
  - Budget set $B(p, m) = \left\{ x \in \mathbb{R}_+^L : \sum_{l=1}^{L} p_l \cdot x_l \leq m \right\}$,
  - In a normal form game, $X = X_1 \times \cdots \times X_N$ each player i chooses independently from his strategy set in X_i. Then $B_i(x_{-i}) = \{(x_i, x_{-i}) : x_i \in X_i\}$.
- Why separate B and X?

Choice Rule

- How is choice made when B is given?

- Let $\mathcal{B}$ denote the collection of all possible feasible sets.

- $c(B)$ is the observed choice correspondence such that $c(B) \subset B$ for all $B \in \mathcal{B}$.

- $(\mathcal{B}, c(\cdot))$ is called a choice structure.

Behavioral assumption

- Nonemptyness: $c(B) \neq \emptyset$.

- What does the observed choice $c(B)$ tell us?

Axiom 1 *(Weak axiom of revealed preference, WA): If $x, y \in B$ and $x \in c(B)$, then $x, y \in B'$ and $y \in c(B')$ imply $x \in c(B')$.*

- Reflects "stationarity" or "context indepence" of choices.

- Implies the Independence of Irrelevant Alternatives: if $x \in c(B) \cap B'$ such that $B' \subset B$, then $x \in c(B')$.

- For a given $(\mathcal{B}, c(\cdot))$, we can define the *revealed preference relation* $\succeq^*$ by $x \succeq^* y$ if and only if $x, y \in B$ and $x \in c(B)$, for some $B \in \mathcal{B}$.

- "$x \succeq^* y$" means "x is at least as good as y" or "y is not preferred to x".

From prefences to choice

Four elements:

1. The choice set X.

2. Feasible set $B \subset X$.

3. Preference relation $\succeq$ on X

4. Behavioral assumption $c(B, \succeq)$.

What is a preference relation?

- Preference relation $\succeq$ is a binary relation, a subset of $X \times X$, but written for convenience $x \succeq y$ if and only if $(x, y) \in \succeq$.

- A strict relation $\succ$ denotes the asymmetric part of $\succeq$. That is, $x \succ y$ if $x \succeq y$ but not $y \succeq x$.

- Interpretation

Rational choice

Axiom 2 *(Completeness): For all $x, y \in X$ either $x \succeq y$ or $y \succeq x$.*

Axiom 3 *(Transitivity): For all $x, y, z \in X$, $x \succeq y$ and $y \succeq z$ imply that $x \succeq z$.*

Other binary relations derived from $\succeq$:

- Indifference: Write $x \sim y$ if $x \succeq y$ and $y \succeq x$.

- Strict preference: Write $x \succ y$ if not $x \succeq y$ and not $y \succeq x$.

Other properties can be derived for rational preferences:

- $\succeq$ is reflexive: For all $x \in X$, $x \succeq x$.

- $\succ$ is asymmetric: For all $x, y \in X$, $x \succ y$ implies $not\ y \succ x$.

- $\succ$ is negatively transitive: For all $x, y, z \in X$, $x \succ z$ implies $y \succ z$ or $x \succ y$.

Note

- Problem 1: Framing

- Problem 2: Judgements that are hard

- Problem 3: Aggregation

Behavioral assumption

- The choice is induced from preferences according to the following:

$$c^*(B, \succeq) = \{x \in B : x \succeq y, \text{ for all } y \in B\}.$$

- This defines the decision-maker's most preferred alternatives in B.

Connections between the choice- and preference-based approaches

- From preferences to WA:

Proposition 4 *If* $\succeq$ *is a rational preference relation, then the strucure* $(\mathcal{B}, c^*(\cdot, \succeq))$ *induced by* $\succeq$ *satisfies WA.*

- From WA to preferences:

Proposition 5 *Let* $\mathcal{B}$ *include all subsets of* X *with two or three elements. If* $(\mathcal{B}, c(\cdot))$ *satisfies WA, then the induced revealed preference relation* $\succeq^*$ *is rational. Moreover,* $\succeq^*$ *is the unique preference relation that induces* $c(\cdot)$, *i.e.* $c(B) = c^*(B, \succeq^*)$ *for all* $B \in \mathcal{B}$.

- Why is the restriction on the sets in Proposition 2 important?

- Example 1: $X = \{x, y, z\}$, $\mathcal{B} = \{\{x, y\}, \{y, x\}, \{x, z\}\}$

- Example2: As Ex. 1 but add X to $\mathcal{B}$.

Conclusion:

- If the sample of observations is sufficiently rich ($\mathcal{B}$ includes all subsets of X with two or three elements), then the Weak Axiom of Revealed Preference is *equivalent* to rationality, i.e. completeness and transivity of preferences.

- Taking rational preferences as the starting point means that the analysis is based on (potentially) *observable* chabracterisitics of the decision maker (assuming WA).

- Empirically testable?

Utility representation

- In most models we work with a utility function for convenience: it can be easily manipulated, and it nicely summarizes the information contained in preferences.

- Then utility function *represents* preferences.

- Is it OK to let a real-valued function to represent potentially complicated preferences over the choice set?

- What are we exactly assuming when taking this approach?

- Our objective: reveal the relationship between the axioms and the utility function

Representation for $\succeq$

- We are looking for numerical representation of rational $\succeq$, which is a function $u : X \to \mathbb{R}$ such that

$$u(x) \geq u(y) \text{ if and only if } x \succeq y. \tag{1}$$

Proposition 6 *If there exists a utility function representing $\succeq$, then $\succeq$ is rational.*

Proposition 7 *If the choice set X is finite and $\succeq$ is rational, then $\succeq$ has a representation.*

Notes

- If u represents $\succeq$, then so does $f \circ u$ for any increasing $f : \mathbb{R} \to \mathbb{R}$

- Note that u maps into real line on which the complete and transtive binary relation "$\geq$".

- The restriction on X in Proposition 4 is sometimes too demanding

- Example: Lexicographic preferences. $X = [0,1] \times [0,1]$. Let

$$(x_1, x_2) \succeq (y_1, y_2)$$
$$\text{if and only if}$$
$$x_1 \geq y_1 \text{ or } [x_1 = y_1 \text{ and } x_2 \geq y_2].$$

Assuming a representation u for these preferences leads to a contraction. Implication: we need further restrictions on the preference relation.

- To ask whether a utility function exists equivalent to asking whether the $\succeq$ -ordered set X is similar to a (sub)set of reals.

- A subset $S \subset X$ is *order dense* in X if, for any $x, y \in X$, $x \succ y$ implies that there is $z \in S$ such that $x \succeq z$ and $z \succeq y$.

- Note that the set of rationals is order dense in the set of reals.

Proposition 8 *Let $\succeq$ be a rational preference relation on X. Then $\succeq$ has a utility representation u if X has a* countable *order dense subset.*

- Let $X = \mathbb{R}_+^L$, e.g. the set of commodity bundles.

- Define the *upper contour set* (or simply upper set) at x as follows

$$U(x, \succeq) = \{y \in X : y \succeq x\}.$$

- Similarly, the *lower contour set* (or simply lower set) at x is given by:

$$L(x, \succeq) = \{y \in X : x \succeq y\}.$$

- The set $U(x, \succeq)$ is closed if and only if for all sequences $\{y_n\}$ such that $y_n \to y$ and $y_n \in U(x, \succeq)$, we have $y \in U(x, \succeq)$. Such a preference relation is continuous, that is, it is preserved under limits.

- Note that a path from $y \in U(x, \succeq)$ to $z \in L(x, \succeq)$ passes through a point of indifference.

Axiom 9 *Preferences* $\succsim$ *are* continuous *if, for all* $x \in X$*, the sets* $U(x, \succeq)$ *and* $L(x, \succeq)$ *are closed.*

Proposition 10 *If* $\succeq$ *is rational and continuous, then there exists a continuous* $u(x)$ *that represents* $\succeq$*.*

- Does not require assumptions regarding tastes (convexity, monotonicity)