Microeconomic Theory: Basic Game Theory
FPPE Spring 2007
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**Problem Set 1**

1. (Guess the average). Consider the $n-$ player game where all players announce simultaneously a number in the set $\{1, ..., K\}$ and a price of \$1 is split equally among all the players having the guess closest to $\frac{2}{3}$ of the average of the announced numbers. Show that this game has a unique outcome that survives the iterated deletion of weakly dominates strategies.

2. (by Rubinstein) Harper (1982) experminented ducks at Cambridge bond. Two expermineter positioned themselves in two banks at the bond. They were throwing balls of bread onto the ground at fixed rate. Experiminter 1 threw ball every 10 seconds and experminer 2 every 5 seconds. Soon the 12 ducks distributed between the experminers in the ratio 8:4.

   Desribe the situation as a game and analyse its Nash equilibria. Can you give an explanation for the result?

3. (Traveler's dilemma) Two identical businessmen are returning home from a trip. Their luggages are lost. The airline knows that each lugagge is worth of at least 180. Both travelers make a claim (in -terms) on their luggage, an amount of at least 180. The airline believes that the right claim is the lower one, and hence pays this to both travelers. However, a penalty of 5 is transferred from the traveler whose claim is higher to the traveler with lower claim. In the case that both travelers claim the same amount, they both receive that amount and no transfer is made.

(a) Model the situation as a game.

(b) Identify the pure strategy Nash equilibria.

(c) Construct a mixed strategy Nash equilibrium in which both player impose claim 180, $180 + 1$, $180 + 2$, ..., with positive probability.

(d) What happens if the airline can pay at most 1 000 000 000 000 for one piece of luggage?

4. (Location game).There are two firms, 1 and 2, located in the ends of a main street of a town (1 to the left end 2 to the right). The length of the street is 1. Production is costless: demand $y$ and price $p$ generate the firm profit $py$. Citizens in the town (mass 1) are distributed uniformly on the main street. Each consumer has unit demand for a good. Consumption value of either firm's good is equally (very) high. However, walking is painful: distance $t$ to the left costs $w_1 t$ to the consumer and $t$ to the right costs $w_2 t$. Consumer's total cost is the sum of the price he pays and his walking cost. Firms compete by setting prices $p_1$ and $p_2$.

   (a) Identify the demand firm $i$ faces under prices $p_1$ and $p_2$. (Hint: Given $p_1$ and $p_2$, when should buyer $x \in [0, 1]$ buy from firm $i$?)

   (b) Identify firm $i$'s reaction function, and solve the Nash equilibrium

   (c) Compute firms' equilibrium profits. Discuss how the profits are affected by changes in $w_1$ and $w_2$.

5. (Auctions with complete information) An indivisible object is for sale. There are two buyers 1 and 2 with valuations $v_1 > v_2 > 0$ for the object. Buyers submit bids $b_i$ simultaneously. The winner is always decided as follows. Buyer $i$ wins the object if $b_i \geq b_j$ for all $j \neq i$ (break the tie in favor of, say, 1). The following rules are used for deciding the amount to be paid.

(a) (Regular first price auction) The winner pays the highest bid.

(b) (Regular second price auction) The winner pays the second highest bid.

(c) (All-pay first price auction) Both players pay their bids.

(d) (All-pay second price auction) Both players pay the second highest bid

Conmstruct at least one Nash equilibrium for each game. Which of the games have efficient (buyer 1 always gets the object) Nash equilibria? Which of the games have inefficient Nash equilibria? Which games you have seen being played?

6. Let there be three players with some preferences over set $A$ of alternatives. Construct a normal form game that supports all Pareto-optimal outcomes, and only them, in Nash equilibrium.